How far will the dark matter (and associated baryons) collapse?

As things fall into the growing gravitational potential wells, they gain kinetic energy. Eventually they settle into 'virial equilibrium,' and obey the virial theorem:

\[ 2K + U = 0 \]

\[ \uparrow \]

\[ \uparrow \text{potential energy} \]

\[ \text{kinetic energy} \]

Let our idealized sphere collapse to half the size at turnaround (when it stops expanding and begins to contract) and conserve energy (dark matter doesn't radiate & cool)

\[ \text{Half radius at turnaround} = r_{\text{max}} \]

Uniform sphere has potential energy

\[ U = -\frac{3GM^2}{5r} \]

At half maximum size, with energy conserved, gain in k.e. = loss in p.e. = \( \frac{3GM^2}{5(r_{\text{max}}/2)} \)

and \( 2K + U = 0 \Rightarrow \text{virialized.} \)
Galaxies are considerably smaller than half the radius of their associated halos at turnaround.

How does the collapse go further?
Non-linear collapse of density fluctuations

As the density continues to increase, linear approximation fails. "non-linear regime"

It does not collapse entirely (into a black hole) because it will gain kinetic energy as it loses potential energy.

dark matter $\rightarrow$ violent relaxation, reaches equilibrium ("virialized")

gas $\rightarrow$ potential energy turns to thermal energy, gas heats ($\sim 10^6 \, K$ in Milky Way-type hole)

.... needs to cool before it can form stars

At high temperatures, gas cools via free-free interactions

- bremsstrahlung radiation
- (ion - electron encounters, $\sim$ not bound)
- (how we detect hot gas in clusters)

At lower temperatures, gas cools by bound-free or bound-bound transitions

$\rightarrow$ recombination or collisional excitation

give off emission lines in optical

Depends on metallicity of gas
<table>
<thead>
<tr>
<th>Temperature</th>
<th>Cooling process</th>
<th>Spectral region</th>
</tr>
</thead>
<tbody>
<tr>
<td>$&gt;10^7$ K</td>
<td>Free-free</td>
<td>X-ray</td>
</tr>
<tr>
<td>$10^7$ K $&lt; T &lt;$ $10^8$ K</td>
<td>Iron resonance lines</td>
<td>X-ray</td>
</tr>
<tr>
<td>$10^5$ K $&lt; T &lt;$ $10^7$ K</td>
<td>Metal resonance lines</td>
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<tr>
<td>$8000$ K $&lt; T &lt;$ $10^5$ K</td>
<td>C, N, O, Ne forbidden lines</td>
<td>IR, optical</td>
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<tr>
<td>Warm neutral gas: $\sim8000$ K</td>
<td>Lyman-$\alpha$, [O]i</td>
<td>1216 Å, 6300 Å</td>
</tr>
<tr>
<td>$100$ K $&lt; T &lt;$ $1000$ K</td>
<td>[O]i, [C]ii, H$_2$-$\rightarrow$</td>
<td>Far IR: 63 μm, 158 μm</td>
</tr>
<tr>
<td>$T \sim 10-50$ K</td>
<td>CO rotational transitions</td>
<td>Millimeter-wave</td>
</tr>
</tbody>
</table>

Cooling rate depends on density of gas, and on metallicity.

Why? Which cools faster, zero metal gas or metal-rich gas?

For rapid collapse (to make stars before we all lose interest)

\[
\text{cooling time} < \frac{1}{\text{free fall time}}
\]

\[
\propto \frac{1}{\sqrt{gp}}
\]

\[
\propto \frac{1}{\sqrt{n}} \quad n \text{ is number density of particles}
\]

So, high density regions cool & collapse fast.

Very low density regions will neither cool nor collapse.
Cooling rate for an astrophysical plasma of unit density, for various abundances that range from solar to depleted cases. In the depleted cases, ratios of heavy element abundances are used that correspond to the ratios of stars. New cooling cross-sections have been incorporated by Sutherland [294] to give plotted cooling rates.

(a) Plot of gas number density versus electron temperature. (c) Plot of gas number density versus dynamical time-scale, cooling time-scale = free-fall time (solid lines) and cooling time-scale = Hubble time (dashed lines). (a) Locii of constant virial mass (in $M_\odot$) and the condition that dynamical time equals a Hubble time.
The above plot (from Silk & Wyse 1993) shows gas number density vs. temperature, and also shows the loci where the cooling time = grav collapse time and where both equal the Hubble time. Lines have constant mass (how object would collapse & stay virialized)

(i) What regions of density & temperature effectively never cool? Never collapse gravitationally?

(ii) Which regions will collapse rapidly?

(iii) Will it be easier for gas in massive dark halos ($10^{14} M_\odot$, galaxy cluster size) to cool & collapse, or gas in small halos? What does this tell us about galaxy cluster formation?
Early (pre-dark-matter) ideas on galaxy formation

Eggen, Lynden-Bell & Sandage (1962) was one of the first papers on this.

What they got wrong: they claimed that the collapse which formed the Galaxy was very rapid, a few $\times 10^8$ years.

What they got right: they pointed out that old stars near the Sun have quite radial orbits, and that this would happen quite naturally in a free-fall collapse.

They also discussed the way a dissipative collapse (with energy lost by radiation) which conserved angular momentum could produce our Galaxy's disk, with stars on near-circular orbits.
EVIDENCE FROM THE MOTIONS OF OLD STARS THAT THE GALAXY COLLAPSED

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Carnegie Institution of Washington, California Institute of Technology
Received May 17, 1962

ABSTRACT

The \((U, V, W)\)-velocity vectors for 221 well-observed dwarf stars have been used to compute the eccentricities and angular momenta of the galactic orbits in a model galaxy. It is shown that the eccentricity and the observed ultraviolet excess are strongly correlated. The stars with the largest excess (i.e., lowest metal abundance) are invariably moving in highly elliptical orbits, whereas stars with little or no excess move in nearly circular orbits. Correlations also exist between the ultraviolet excess and the \(W\)-velocity. Finally, the excess and the angular momentum are correlated; stars with large ultraviolet excesses have small angular momenta.

These correlations are discussed in terms of the dynamics of a collapsing galaxy. The data require that the oldest stars were formed out of gas falling toward the galactic center in the radial direction and collapsing from the halo onto the plane. The collapse was very rapid and only a few times \(10^8\) years were required for the gas to attain circular orbits in equilibrium (i.e., gravitational attraction balanced by centrifugal acceleration). The scale of the collapse is tentatively estimated to be at least 10 in the radial direction and 25 in the \(Z\)-direction. The initial contraction must have begun near the time of formation of the first stars, some \(10^9\) years ago.

I. INTRODUCTION

Beginning with Strömgren's early investigations of stellar motions, it has become increasingly clear that the galaxy contains many types of objects with a large range in kinematic properties. The most elementary classification by kinematic properties is that introduced by Oort (1926), who divided the stars into "low"- and "high"-velocity objects. The boundary value between the two groups was taken as the velocity where a marked asymmetry appears in stellar motions—63 km/sec. But the data accumulated in subsequent years show that the kinematic situation is more complex and much richer in detail than this simple division indicates. Various kinematic and spatial subsystems—such as young main-sequence stars in the galactic disk, old globular clusters, RR Lyrae variables, and extreme subdwarfs—form a near continuum from the very lowest to the very highest space velocities. Historically, the idea of a continuum in the velocity distribution was developed from the work of Strömgren and Oort by Vyssotsky and Williams (1948) through their extended investigations of proper motions, and by Schwarzschild (1952) who examined the possible origins of the high velocity stars.

It is now recognized that a study of these subsystems allows us partially to reconstruct the galactic past because the time required for stars in the galactic system to exchange their energies and momenta is very long compared with the age of the galaxy. Hence knowledge of the present energy and momenta of individual objects tells us something of the initial dynamic conditions under which they were formed. Furthermore, results from the theory of stellar evolution allow us to estimate the age of the stars in the various subgroups. This provides a method of investigating changes in the dynamic structure of the galaxy with time, as indicated by differences in the kinematic behavior of stars ordered in a time sequence. This paper attempts to show how far one can go in reconstructing the history of the galactic system by using observational data now available.

The results of Section IV show that remarkable correlations exist between the chemical composition of the individual stars, the eccentricity of their galactic orbits, their

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We now see from equation (26) that the actual motion in $R$ is as though the potential in the galactic plane was not $\psi(R)$ but $\psi(R)$. If $\rho(R, 0)$ decreases with $R$, this $E_0$-dependent addition to the potential increases both apogalactic and perigalactic distance. The effect is small for $E_0$ small compared with $E_0$ and has been neglected in the computations in Section IV. Even when the neglect is not valid, the planar "eccentricity" as defined by equations (14), (15), and (16) still classifies orbits into "near circular" and "near rectilinear" groups.

### III. DYNAMICS IN A CONTRACTING GALAXY

Later we will have to consider times before the potential field of the galaxy was independent of time. It is important to know what properties of a stellar orbit are preserved throughout its history and how other properties are changed by changes in the galactic field. Since at earlier times much of the galaxy was gas, it is equally important to discuss some of the properties of the gas dynamics. We shall make two assumptions: (1) the gravitational potential of the galaxy is axially symmetrical at all times, and (2) masses whose angular momenta (about the galactic axis) are significantly different from one another do not exchange angular momenta through pressure gradients or magnetism. Assumption 1 is really the assumption that our Galaxy was never a barred spiral. Assumption 2 is probably correct if the scale of any turbulence was always much smaller than the size of the galaxy. We assume the truth of both assumptions for all times since the first stars in our Galaxy were formed.

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*Fig. 3. Same as Fig. 2. The more circular orbit is for HD 29587 with $\delta = -0^\circ.13$. The more elliptical orbit is for Ross 106 with $\delta = +0^\circ.26$. The orbit for Ross 106 is retrograde.*

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In practice, the distribution of angular momentum with mass, as...