Disk Dynamics & Spiral Structure

Observational Background:

Spiral types abound:

- Grand design spirals - 2 very symmetric, long spiral arms
- Flocculent spirals - pieces of spiral arms, not continuous
- Multiple arms
- Barred spirals - spiral arms come off central bar
Properties of spiral arms:

- Very prominent in blue wavelengths, in Hα line emission, in radio continuum emission

- Often have strong dust lanes

- Velocity perturbations ~ 20-30 km/s along spiral arms (from HI data)

- In red light, spiral arms are smoother, broader & lower in amplitude than in blue light

What do we learn from this?
Angles

Logarithmic spirals, \( \phi \propto u \), \( u \equiv \ln(R) \). This observation indicates that the brightness \( I \) of a disk at a pitch angle \( \psi \) of a spiral of pitch angle \( \psi \) is:

\[
m(p)e^{i(pu+m\phi)}
\]

\( n(p) \) peaks strongly at \( m \), but the galaxy has pitch angle \( \psi \) estimated for many galaxies (e.g. 93) for details. They increase \( 8^\circ \) at Sc.

Figure 4.56 shows that the \( (B-V) \) more than \( \pm0.2 \) mag across a.

Schweizer (1976) found that stars contribute about 17% of the brightness at 15 kpc, 15% and 50% stronger 1. The underlying disks are.

For the disks in his sample, in many old galactic clusters, the central parts of the arms are bluer than the disk with increasing distance to shock-compresion of interstellar gas, blaze briefly to become slightly bluer at all.

Interstellar media of galaxies described above are fairly possible to determine within the galaxy (e.g. 4.4.1), very blue veiling. That is, stars and leave them on their course.

1d of the spectrum is sensitive and the nearer side of a galaxy. The gas 4622 has both leading and trailing.

0.56 Spiral structure in M81. \( U, B, \) and \( O \) azimuthal profiles at \( r = 475 \) arcsec. The angle \( \phi \) is measured from the apparent major axis in the direction of rotation. The profile has been continued periodically beyond \( \phi = 360^\circ \). Notice how the \( U \) profile consists of narrow peaks superposed on a broader wave pattern which resembles the \( B \) and \( O \) profiles. Much of the small fluctuation in the \( (B + O) \) color can be accounted for by an instrumental malfunction. [After Schweizer (1976)]

The presence of short-lived stars. These form on the inner edges of spiral arms as a response to shock-compression of interstellar gas, blaze briefly to very blue objects as they traverse the peak of the arm, and are then veiled. Surface brightness in the red and infrared is determined by a sublince of stars, including objects that do not evolve significantly in the line it takes a star to drift across a spiral arm. Hence \( O \) and \( I \) surface brightness can give truer impressions of the enhancement in the mass density and numerical magnitude of the color of the broad components in Figure 4.56 is of some importance since it very strongly suggests that the disk stars (of relatively late spectral types) contribute about 40% of the increased brightness found near the spiral arms. If we accept this conclusion, it follows that spiral structure is not an entirely gas-dynamic
The Winding Problem

paint a radial stripe on a rotating galaxy

the equation of that stripe will be

$$\phi(R,t) = \phi_0 + \Omega(R) t$$

Angular rotation frequency

$$\Omega = \frac{V(R)}{R}$$

→ what is the condition for the stripe to stay straight?

→ do galaxies obey this condition?
Define pitch angle to be angle between the tangent point of an arm and a circle.

\[
\cot i = R \left| \frac{d\phi}{dR} \right| = R \Omega \left| \frac{d\Omega}{dR} \right|
\]

\[i \sim 2^\circ \times \left( \frac{1 \text{ Gyr}}{t} \right)\]

From Kennicutt 1981

Figure 6-8. How a material arm winds up in a differentially rotating disk. The rotation law is \(\Omega(R) \propto R^{-1}\).
Epicycles ... a small review

Describes motion of disk stars to a good approximation

\[ \text{Derivation of epicyclic motion assumes that orbital angular momentum } L_z = R \cdot v_\phi \text{ is conserved.} \]
Will this be strictly true in a real galaxy?

Cylindrical coords \( R, \phi, z \).

Two harmonic oscilations, independent

(i) \[ \ddot{z} = \frac{\dot{z}^2}{z} = -\frac{2}{\dot{z}}(R_g)z \] \quad \text{vertical motion}

\[ z = Z \cos \left( \frac{2}{\dot{z}} t + \theta \right) \]
\[ \text{const} \quad \text{const} \]

(ii) \[ R = \frac{R_g}{\dot{z}} + x \] \quad \text{radial motion}

\[ \text{guiding center radius} \]
\[ \ddot{x} = -\frac{\dot{x}^2}{x} \] \quad \text{so } x = X \cos \left( \frac{\dot{x}^2}{x} t + \varphi \right) \]
\[ \text{const} \]
Then azimuthal coordinate $\phi$ given by

$$\phi(t) = \phi_0 + \frac{2\pi R_g}{c} t - \frac{1}{2} \frac{2\pi}{R_g} \frac{1}{K} \sin(Kt + \psi)$$

Near the Sun, $K \approx 1.4 \, \text{yr}$

So its orbit around the galactic center is not closed.

**Special cases**

Point mass $K = \frac{2}{c^2}$, closed orbits,

ellipses with point mass
at one focus

(Platyne only considered circular
specycles, not elliptical ones)
Spiral Density Waves

$\rightarrow$ remember orbital frequency $\Omega$

$\Rightarrow$ epicyclic frequency $\kappa$

in a frame rotating at $\Omega$, we see the epicycle $\phi$

in a frame rotating at $\Omega - \kappa$, we see an offset oval $\circ$

in a frame rotating at $\Omega - \frac{\kappa}{2}$, we see a on-center oval $\circ$
we can set up orbits nested in radius that form a long-lived pattern if it rotates at the right frequency

Figure 5.28 Left, oval orbits nested to form a two-armed spiral; the equation of the pattern is $R = R_g \{1 + 0.075 \cos[2(5 - 5R_g + \phi)]\}^{-1}$, and $0.3 < R_g < 1$. Right, a one-armed spiral, with $R = R_g \{1 + 0.15 \cos[(5 - 5R_g + \phi)]\}^{-1}$.

Pattern has $\Omega_p$ different from orbits of individual stars

- but we need this pattern speed to be constant $\omega R$. is it?
Qualitatively:

If in an epicyclic timescale, a star "skates" across one perturbation to another, the perturbation is stable.

Or: if $\sigma_r \Rightarrow \text{large} \Rightarrow \text{stability}$

If $\Sigma \Rightarrow \text{large} \Rightarrow \text{instability}$

$Q \gg 1 \Rightarrow \text{hard to make perturbations grow "hot disk" (no spiral)}$

$Q \ll 1 \Rightarrow \text{very unstable towards growth of "cold disk" structure}$

What happens to a disk $w/ Q \ll 1$?
Where do we run into problems? 
⇒ the Lindblad resonances

- a star "sees" the pattern with a frequency \( z \left| \Omega_p - \Omega \right| \)

- it only responds strongly (i.e., reinforces the pattern) if the forcing frequency is slower than the epicyclic frequency:
  \[
  z \left| \Omega_p - \Omega \right| < K
  \]
  or

\[
\Omega_p = -\Omega - \frac{K}{2} \quad \rightarrow \quad \Omega + \frac{K}{2}
\]

\( \uparrow \)

inner Lindblad resonance

\( \uparrow \)

outer Lindblad resonance
important points

- stars move independently of the pattern

- winding problem not solved completely since $\Omega - \frac{K}{r} \neq \text{constant}$ but winding is much slower

- gravity can amplify/stabilize the pattern

- for self gravity to work, random motions can't be too large

$$Q = \frac{K_0}{3.36 \sigma} < 1$$

Toomre (1964)
So the epicycle is an oscillation around circular motion. Notes:

- The epicycle is **retrograde**

- The frequency of this oscillation is known as the epicyclic frequency $K$

- The epicyclic frequency & the angular speed of the guiding center are both governed by the potential & are relatable:

  $$K^2 = \left[ R \frac{d\Omega^2}{dR} + 4 \Omega^2 \right]$$

- Epicycle has axis ratios

  $$\frac{x}{r} = \frac{K}{2\Omega}$$
What about the Milky Way?

\[
\begin{align*}
\sigma_R &\sim 30 \text{ km/s} \\
\Sigma &\sim 50 \text{ Mo/pc}^2 \\
H &\sim 35 \text{ km/s/kpc}
\end{align*}
\]

\(Q \approx 2 \leftarrow \text{stable}\)

\(\rightarrow \sigma, \Sigma, H \) all vary w/ radius so \(Q\) can vary w/ radius

\(\rightarrow\) is \(Q\) "observable"?