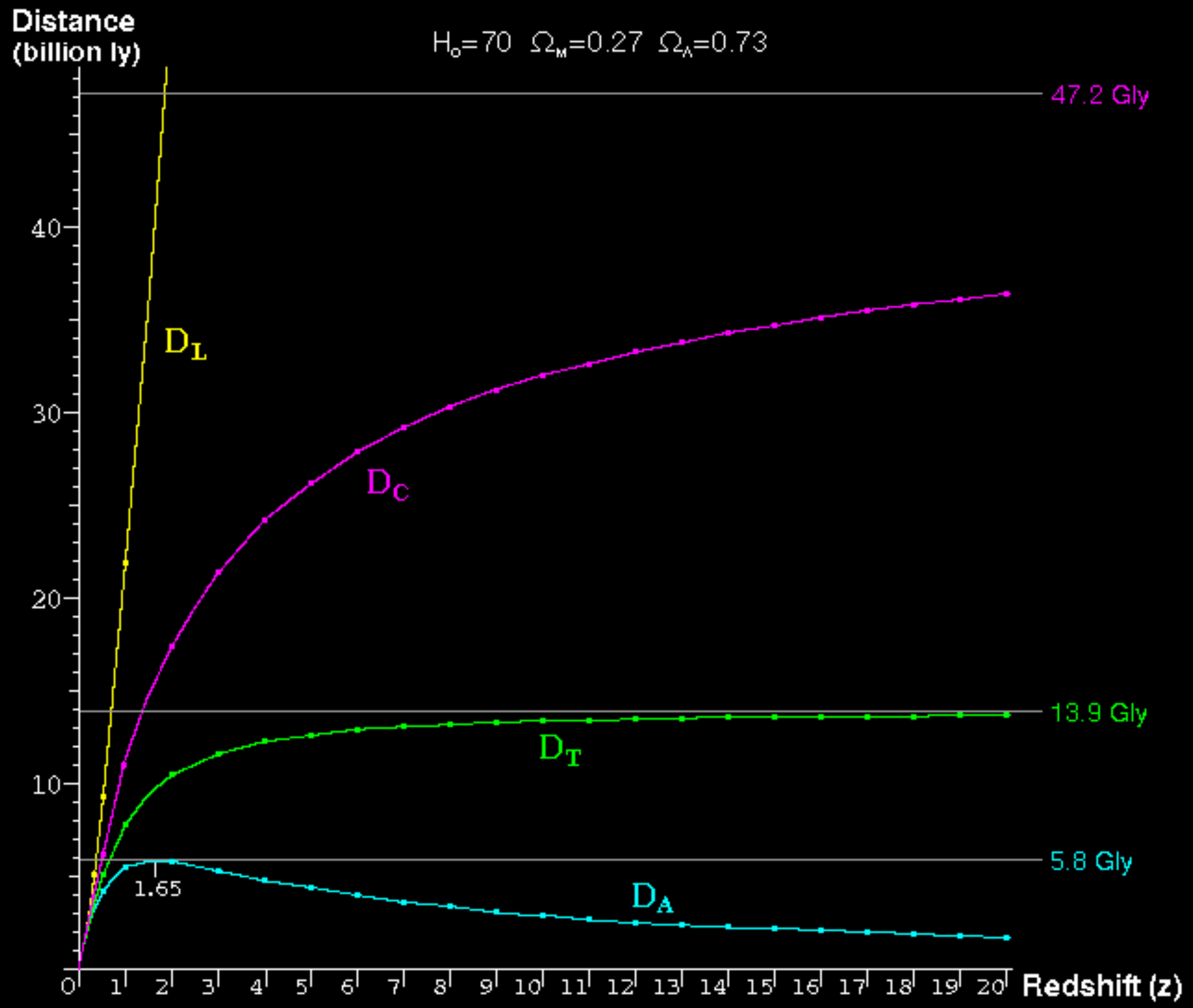
http://www.atlasoftheuniverse.com/redshift.html

This is the problem of defining a distance in an expanding universe: Two galaxies are near to each other when the universe is only 1 billion years old. The first galaxy emits a pulse of light. The second galaxy does not receive the pulse until the universe is 14 billion years old. By this time, the galaxies are separated by about 26 billion light years; the pulse of light has been travelling for 13 billion years; and the view the people receive in the second galaxy is an image of the first galaxy when it was only 1 billion years old and when it was only about 2 billion light years away.

http://www.atlasoftheuniverse.com/redshift.html



$$1 + z = \sqrt{\frac{1 + v/c}{1 - v/c}}$$

D_C = **"comoving distance"**

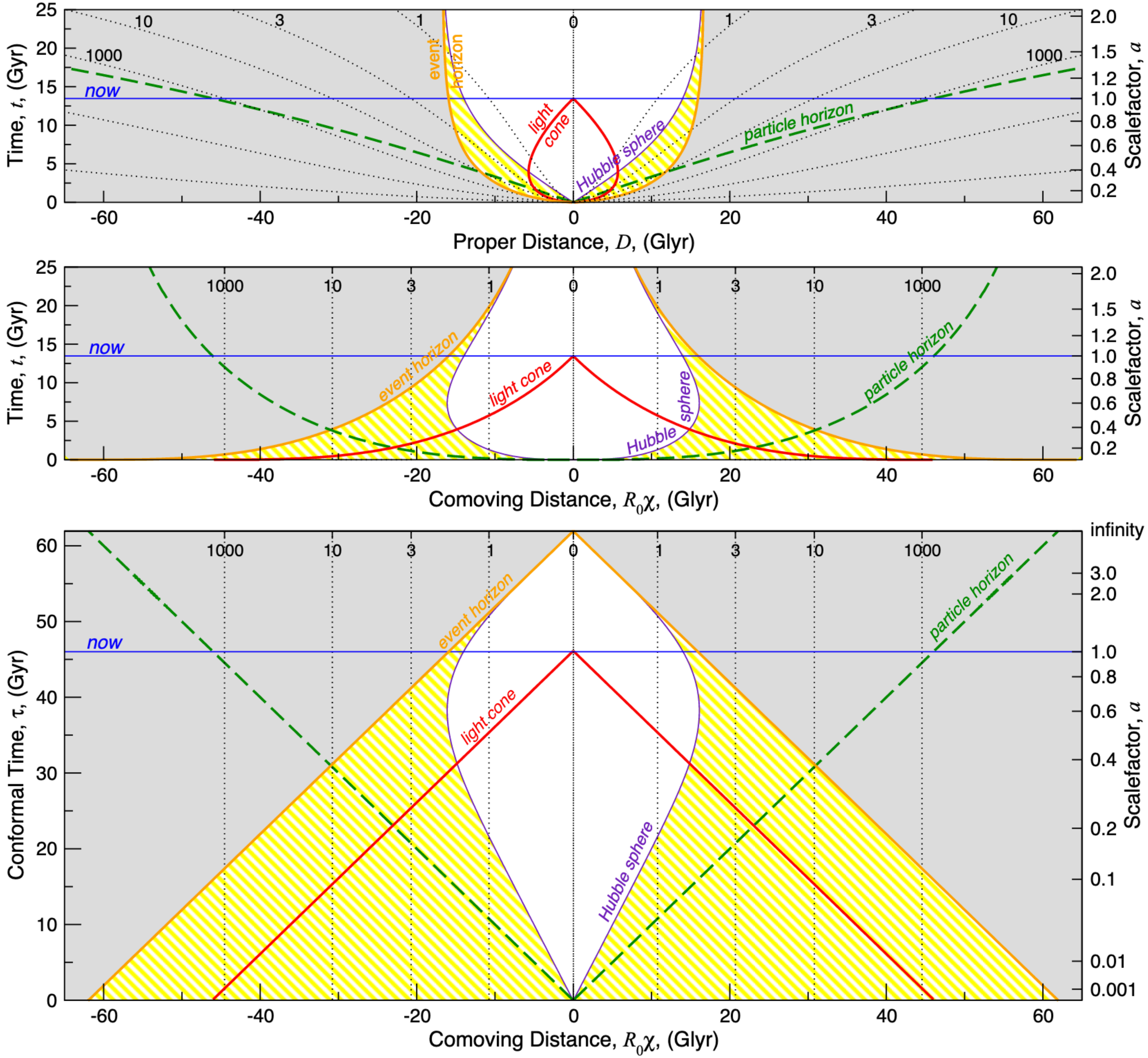
D_T = **"light travel time distance"**

$$D_A = \frac{D_C}{1+z}$$

$$D_L = (1+z)\ D_C = (1+z)^2\ D_A$$

$$SB = \frac{L}{D^2}\ \frac{1}{(1+z)^4}$$

$$D_L(z) = (1+z)\frac{c}{H_0}\int_0^z \frac{dz}{\sqrt{\Omega_M(1+z)^3 + \Omega_k(1+z)^2 + \Omega_\Lambda}}$$


Time, t, (Gyr)
Scalefactor, a
Proper Distance, D, (Glyr)
now
event horizon
light cone
Hubble sphere
particle horizon
Comoving Distance, $R_0\chi$, (Glyr)
Conformal Time, τ, (Gyr)
infinity
1000
10
3
1
0

The Redshift-Distance Test

As a function of redshift, the apparent magnitude of distant objects changes under different cosmologies, for two reasons:

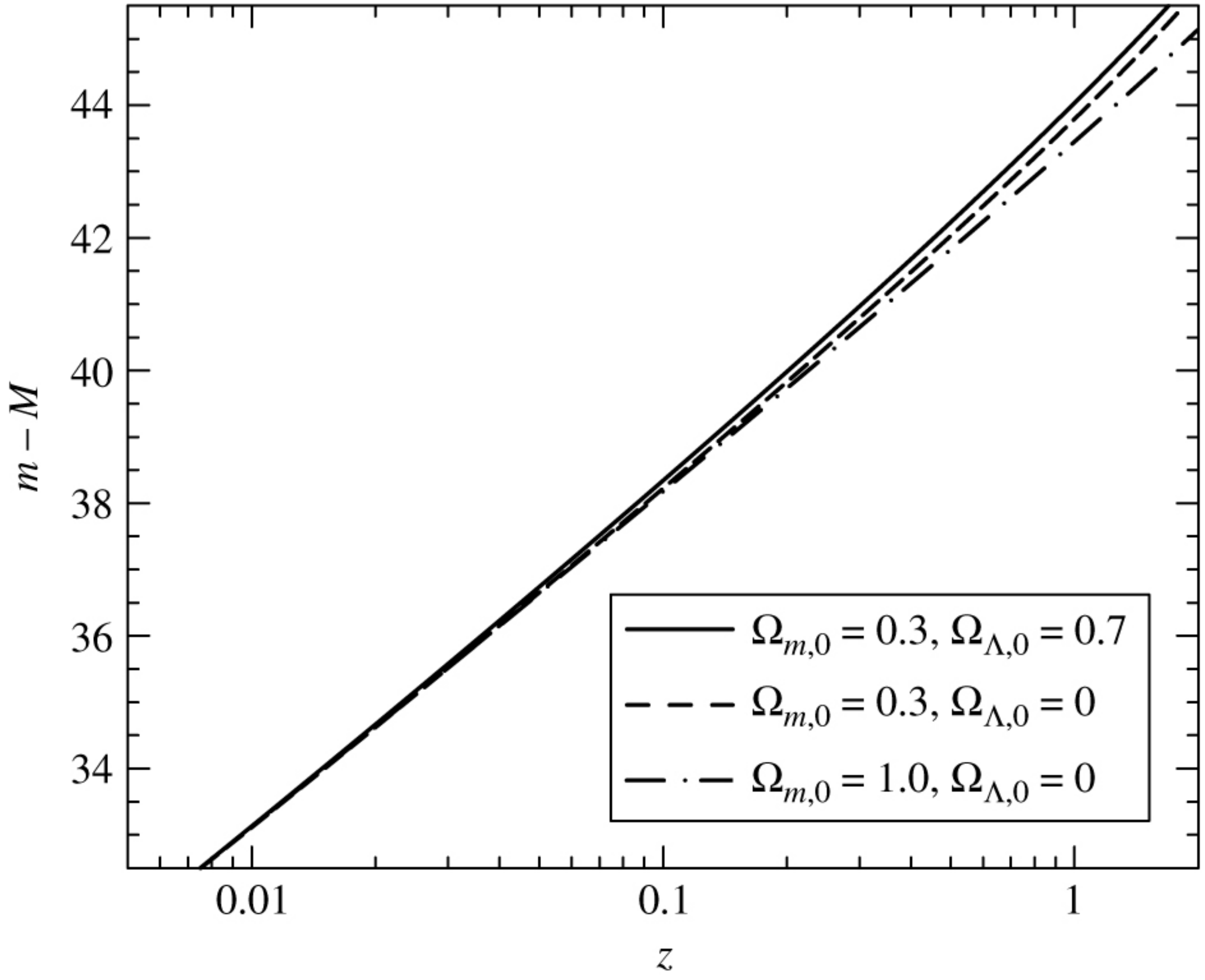
1. The shape of space determines how photons spread out as they move outwards (the classic $1/d^2$ effect)
2. The expansion history determines how the photons are redshifted.

This can be worked out under different cosmologies to derive a form akin to our regular magnitude-distance expression:

$$m - M = 5 \log d_L(z) - 5$$

where $d_L(z)$ is the luminosity distance, and depends on H_0, Ω_M, Ω_L, and k. We typically plot this using the distance modulus, not the distance, though:

If we had an object of fixed brightness -- a standard candle -- we could plot its apparent magnitude as a function of distance and work out the cosmology.



$$d_L(z) = (1+z)\frac{c}{H_0}\int_0^z \frac{dz}{\sqrt{\Omega_M(1+z)^3 + \Omega_k(1+z)^2 + \Omega_\Lambda}}$$

Remember Type Ia supernovae: the explosion of a ~ 1.4 Msun white dwarf. These are pretty good approximations to a standard candle, and they are extremely bright. That's exactly what we want to use for the redshift-distance test.

But are SN Ia's really standard candles?

The Astrophysical Journal, 826:56 (31pp), 2016 July 20

Table 5
Approximations for Distance Parameters

Host	SN	$m^0_{B,i} + 5a_B$	σ[a]	$\mu_{\rm Ceph}$[b]	σ	$M^0_{B,i}$	σ
				(mag)			
M101	2011fe	13.310	0.117	29.135	0.045	−19.389	0.125
N1015	2009ig	17.015	0.123	32.497	0.081	−19.047	0.147
N1309	2002fk	16.756	0.116	32.523	0.055	−19.331	0.128
N1365	2012fr	15.482	0.125	31.307	0.057	−19.390	0.137
N1448	2001el	15.765	0.116	31.311	0.045	−19.111	0.125
N2442	2015F	15.840	0.142	31.511	0.053	−19.236	0.152
N3021	1995al	16.527	0.117	32.498	0.090	−19.535	0.147
N3370	1994ae	16.476	0.115	32.072	0.049	−19.161	0.125
N3447	2012ht	16.265	0.124	31.908	0.043	−19.207	0.131
N3972	2011by	16.048	0.116	31.587	0.070	−19.103	0.136
N3982	1998aq	15.795	0.115	31.737	0.069	−19.507	0.134
N4038	2007sr	15.797	0.114	31.290	0.112	−19.058	0.160
N4424	2012cg	15.110	0.109	31.080	0.292	−19.534	0.311
N4536	1981B	15.177	0.124	30.906	0.053	−19.293	0.135
N4639	1990N	15.983	0.115	31.532	0.071	−19.113	0.135
N5584	2007af	16.265	0.115	31.786	0.046	−19.085	0.124
N5917	2005cf	16.572	0.115	32.263	0.102	−19.255	0.154
N7250	2013dy	15.867	0.115	31.499	0.078	−19.196	0.139
U9391	2003du	17.034	0.114	32.919	0.063	−19.449	0.130

Notes.
[a] For SALT-II, 0.1 mag added in quadrature to fitting error.
[b] Approximate, SN-independent Cepheid-based distances as described at the end of Section 3.

Which gives an average peak absolute magnitude of -19.26 +/- 0.16.

This uncertainty in peak mag includes the distance uncertainties to the galaxies, so the real dispersion in peak magnitude is even smaller, about 0.1 mags or so. That's a pretty good standard candle.

But there's a significant drawback to using Type Ia SNe. You gotta find them...

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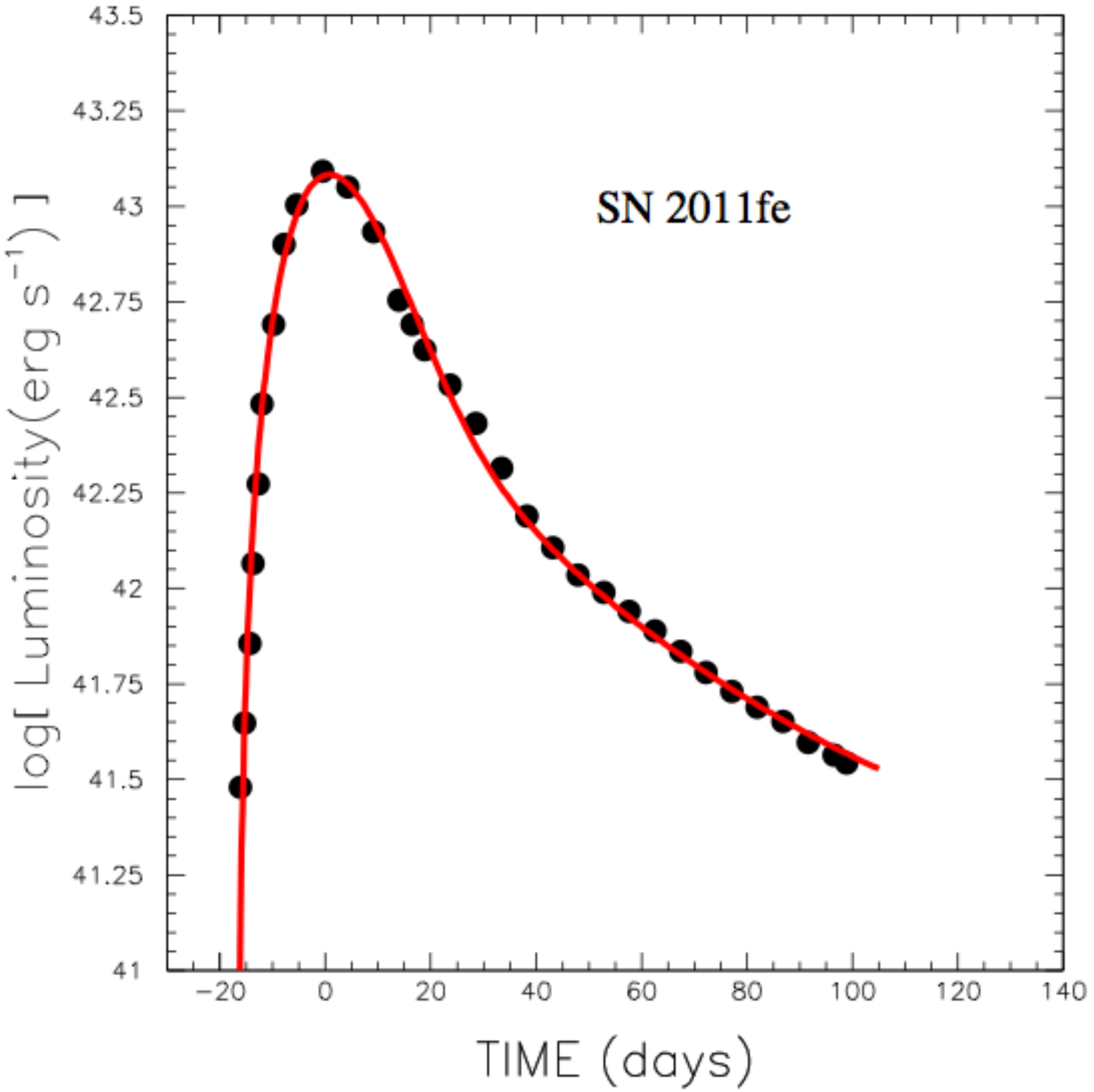
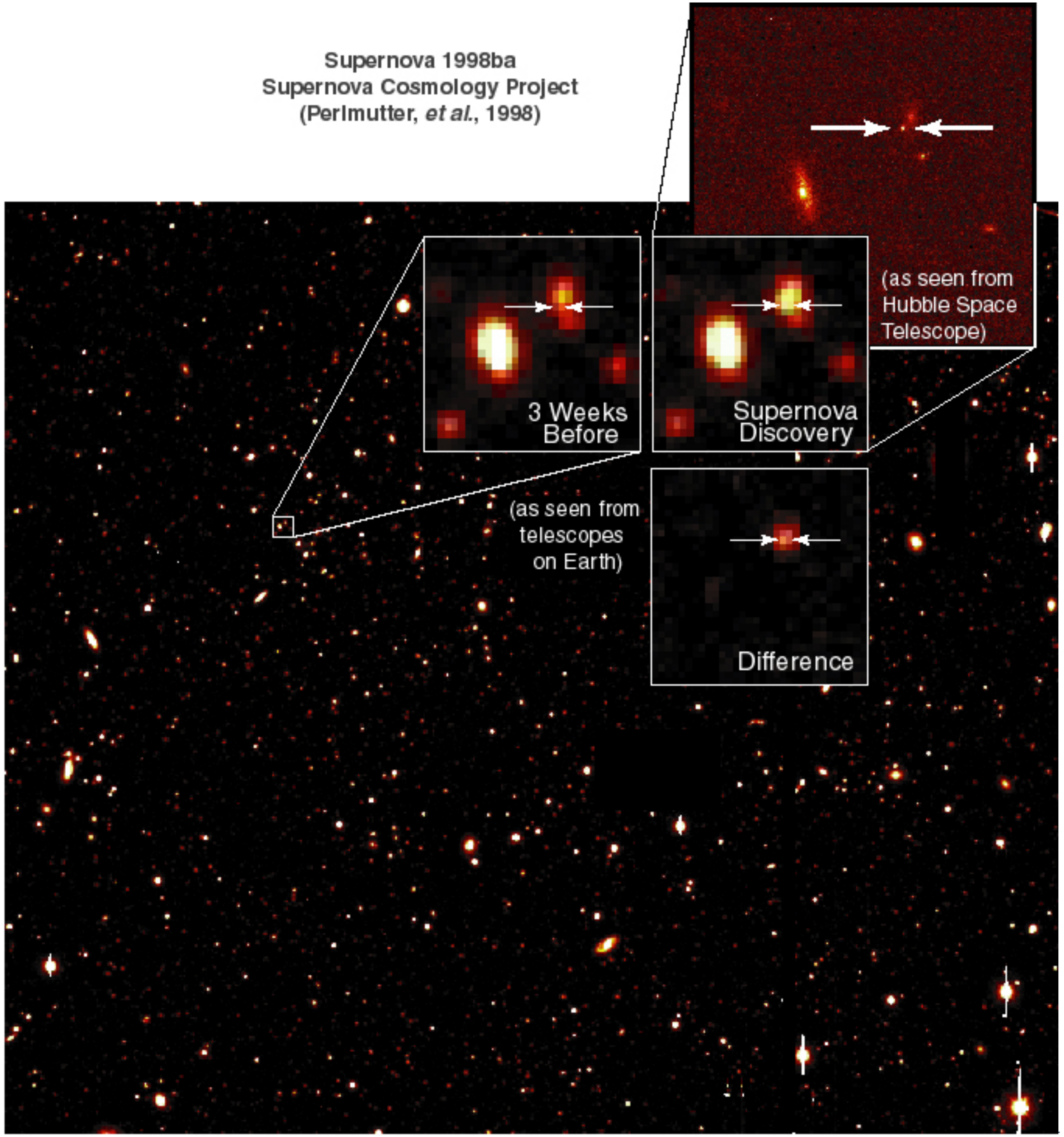
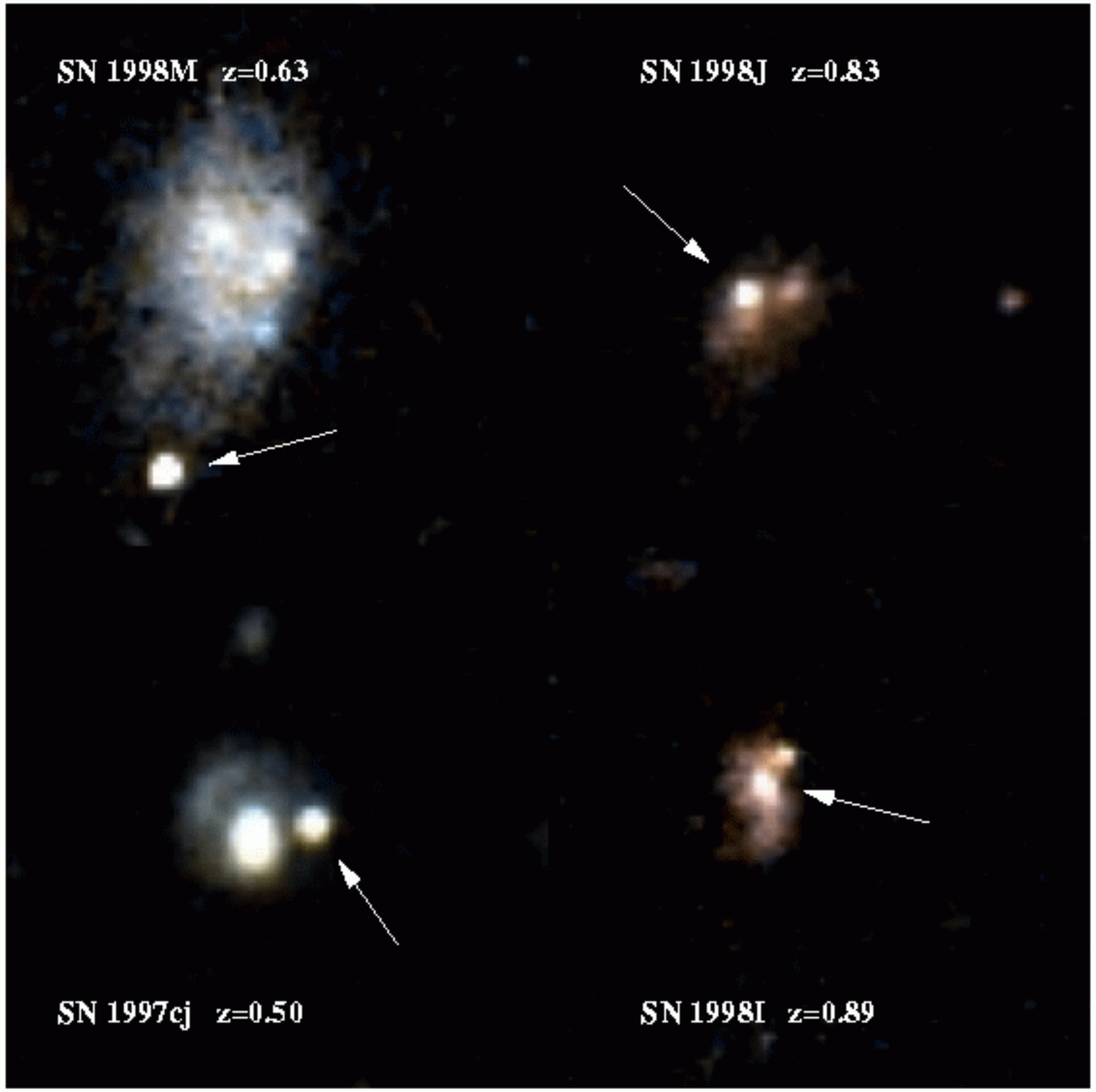


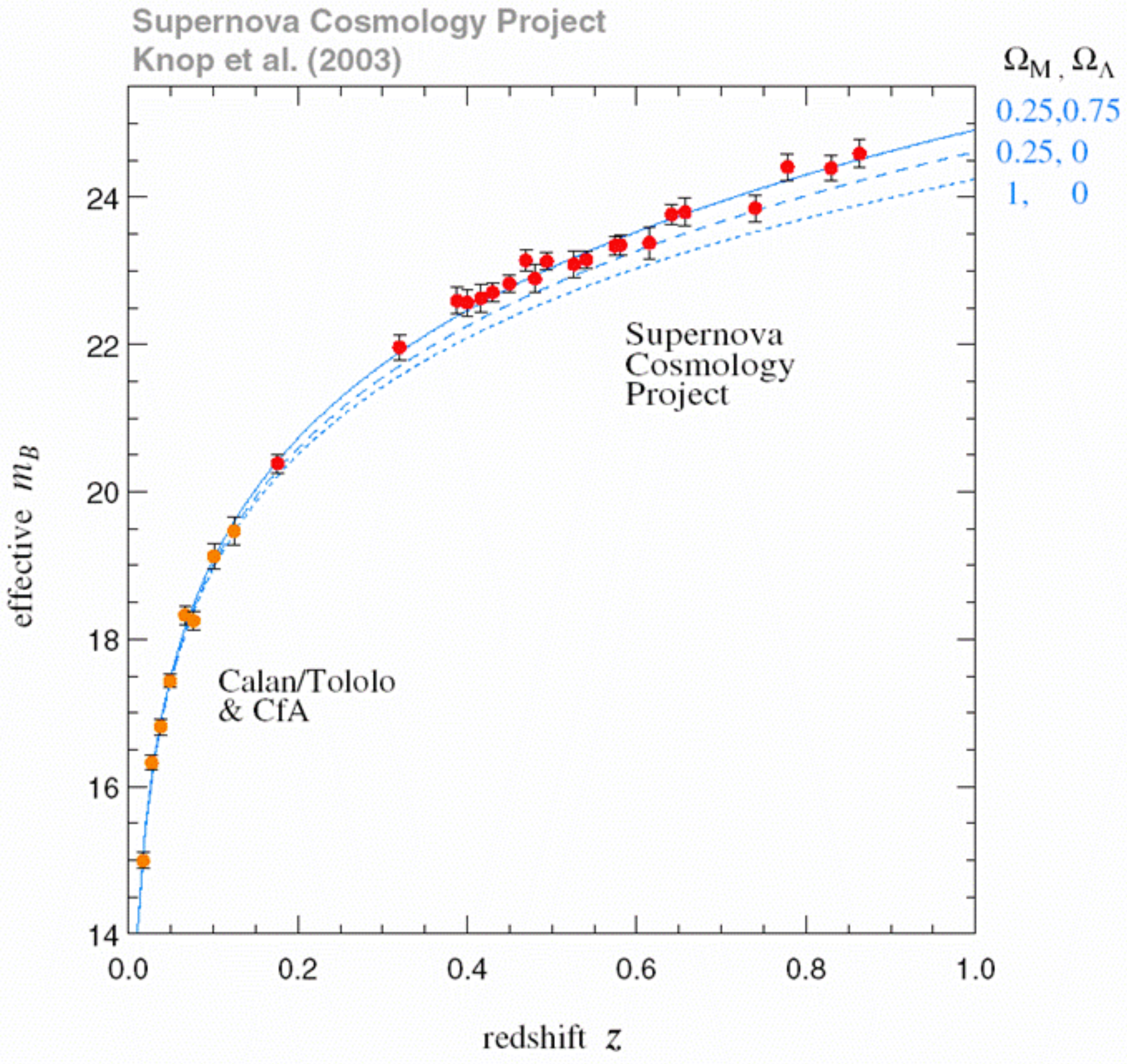
How would you find enough supernovae to determine cosmological parameters?

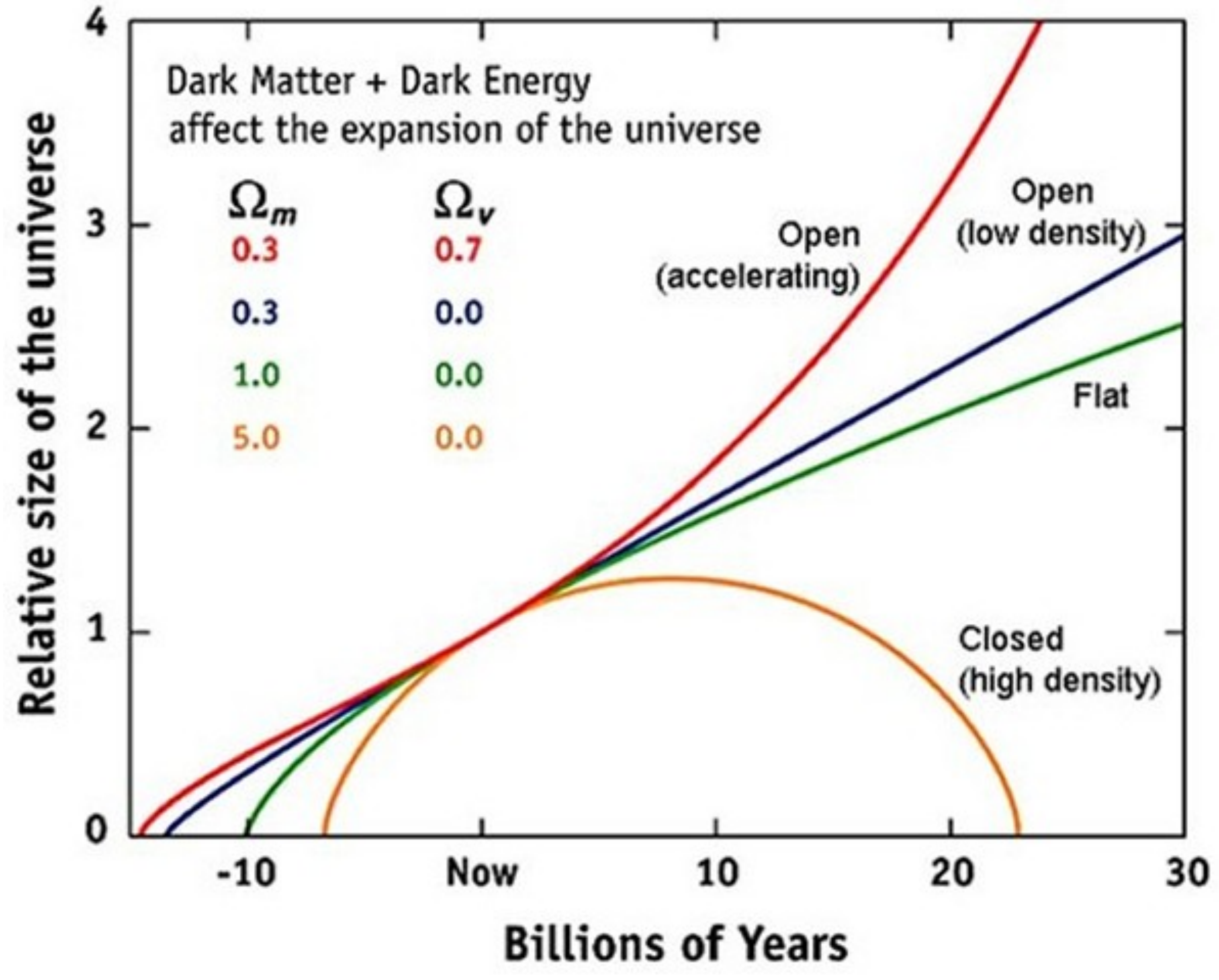
Using supernovae to study cosmology

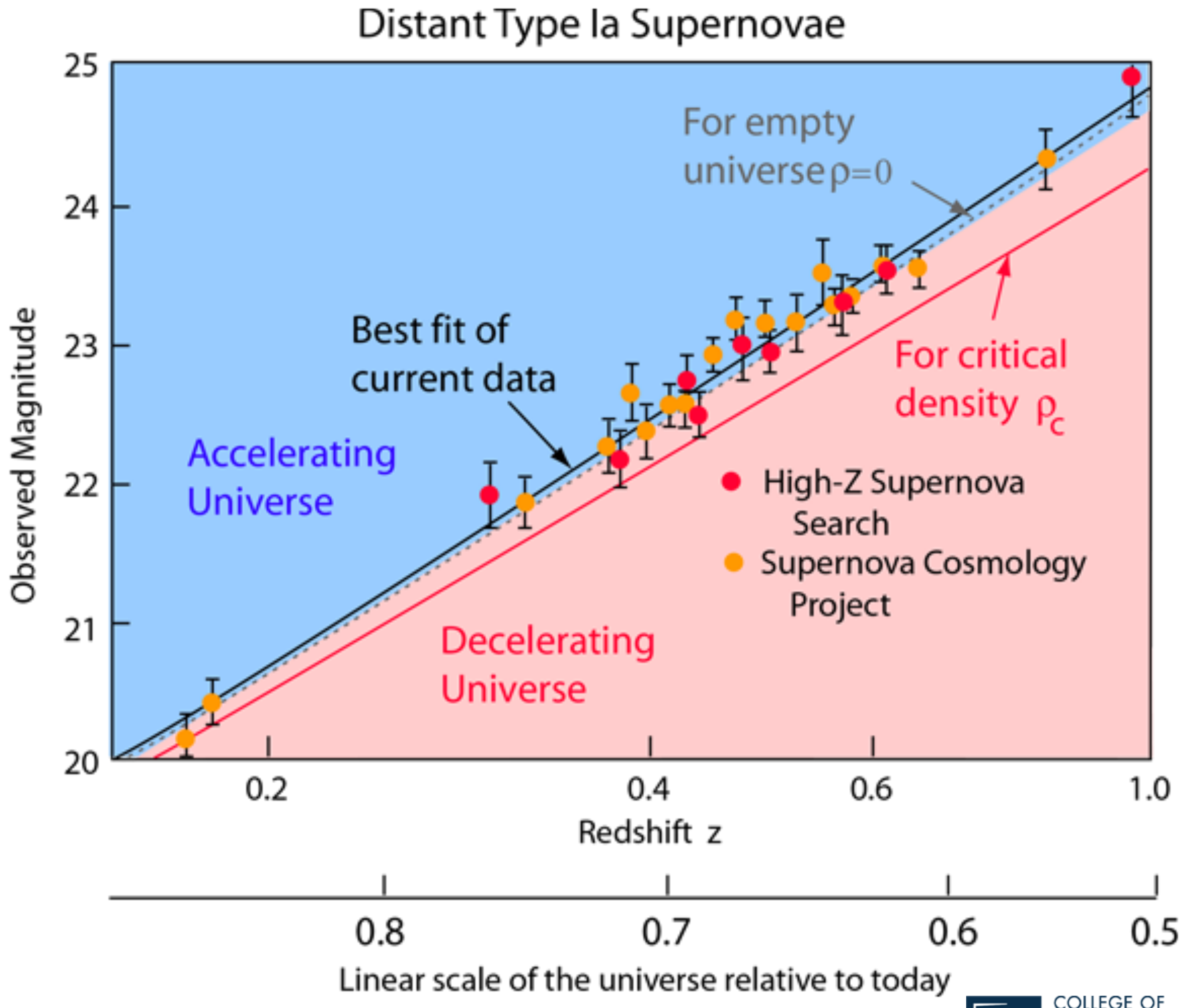
- Take a BIG picture of the sky.
- Come back next month and take the same picture.
- Compare the two. *Differences?*
- If you find a possible supernova, take a spectrum of it and make sure it is a Type Ia SNe.
- Also take a spectrum of the galaxy it lives in, to find its redshift.
- Watch the supernova as it fades, so we can get its peak apparent magnitude. This is important -- you probably didn't catch it when it was at its peak, so we need to fit it to a standard light curve to derive its peak magnitude.
- Keep doing this so you have a big sample of high redshift supernovae. Then compare those supernovae to ones at lower redshift.



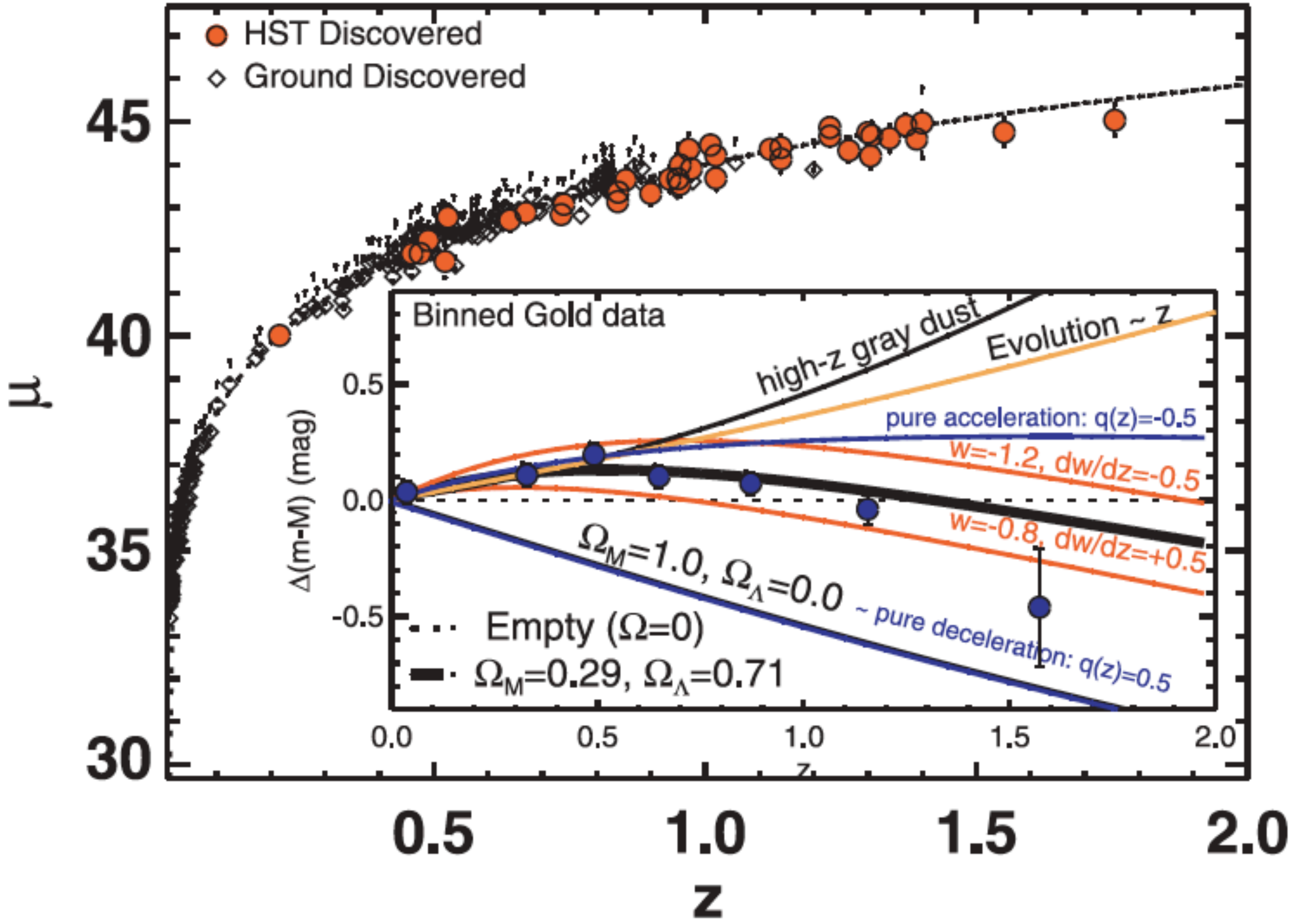

SN 1998M z=0.63
SN 1998J z=0.83
SN 1997cj z=0.50
SN 1998I z=0.89

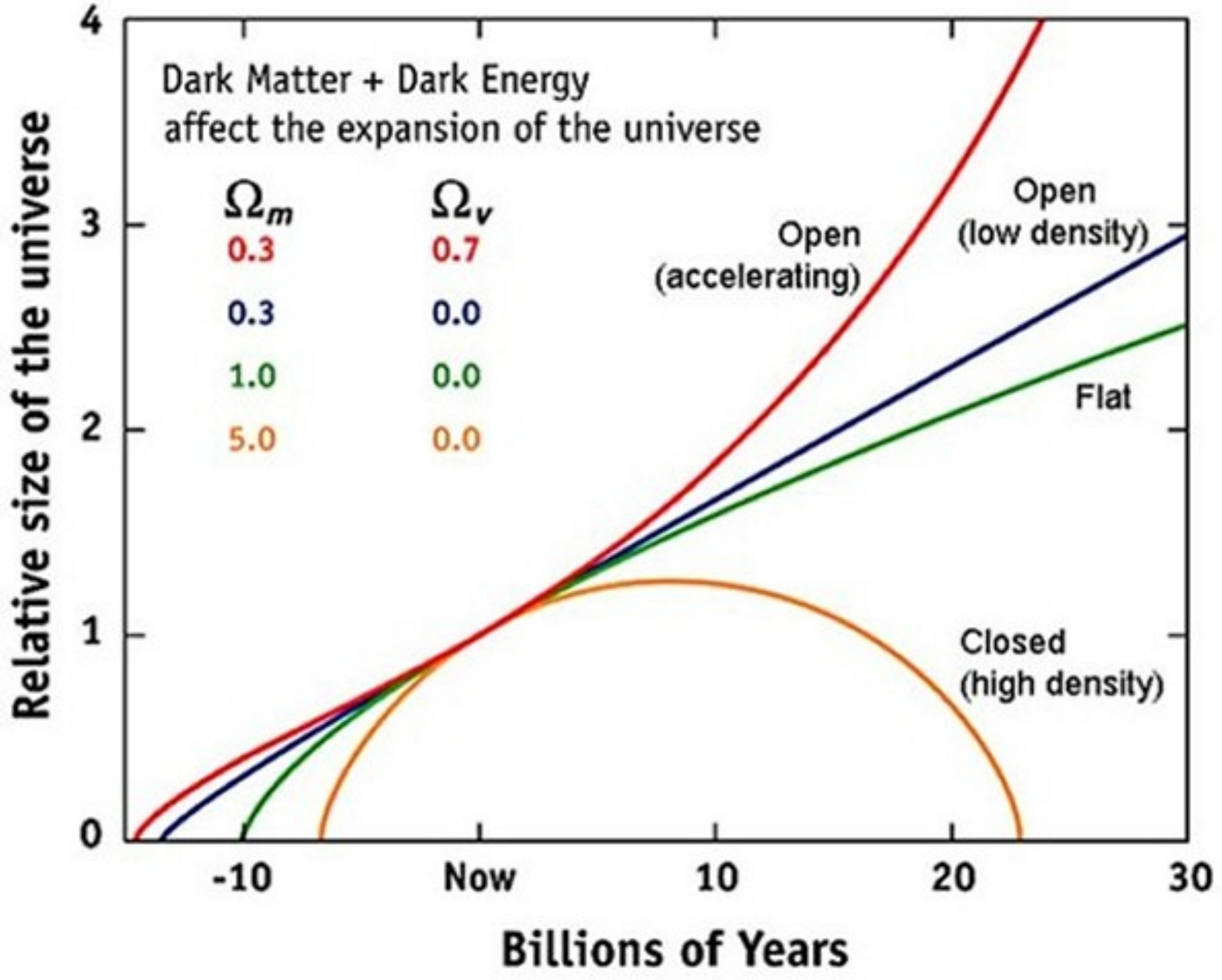

Supernova Cosmology Project
Knop et al. (2003)
Ω_M, Ω_Λ
0.25,0.75
0.25, 0
1, 0
Supernova Cosmology Project
Calan/Tololo & CfA
effective m_B
redshift z


Dark Matter + Dark Energy affect the expansion of the universe
Ω_m Ω_v
0.3 0.7
0.3 0.0
1.0 0.0
5.0 0.0
Open (accelerating)
Open (low density)
Flat
Closed (high density)
Relative size of the universe
Billions of Years


Distant Type Ia Supernovae
For empty universe ρ=0
Best fit of current data
Accelerating Universe
For critical density ρ_c
High-Z Supernova Search
Supernova Cosmology Project
Decelerating Universe
Observed Magnitude
Redshift z
Linear scale of the universe relative to today

More recent data: HST discovered supernovae, extending to higher redshift ([Riess et al 2007](#)). In this plot μ=m-M, the distance modulus. Curvature in the data is inconsistent with models that use dust or evolution to explain faintness of high-z SNe; instead it is indicative of the "jerk" in the expansion history when lambda began to dominate and the universe went from decelerating to accelerating.




Dark Matter + Dark Energy
affect the expansion of the universe
Ω_m
Ω_v
0.3 0.7
0.3 0.0
1.0 0.0
5.0 0.0
Open
(accelerating)
Open
(low density)
Flat
Closed
(high density)
Relative size of the universe
0
1
2
3
4
-10
Now
10
20
30
Billions of Years

Constraints from Supernovae:

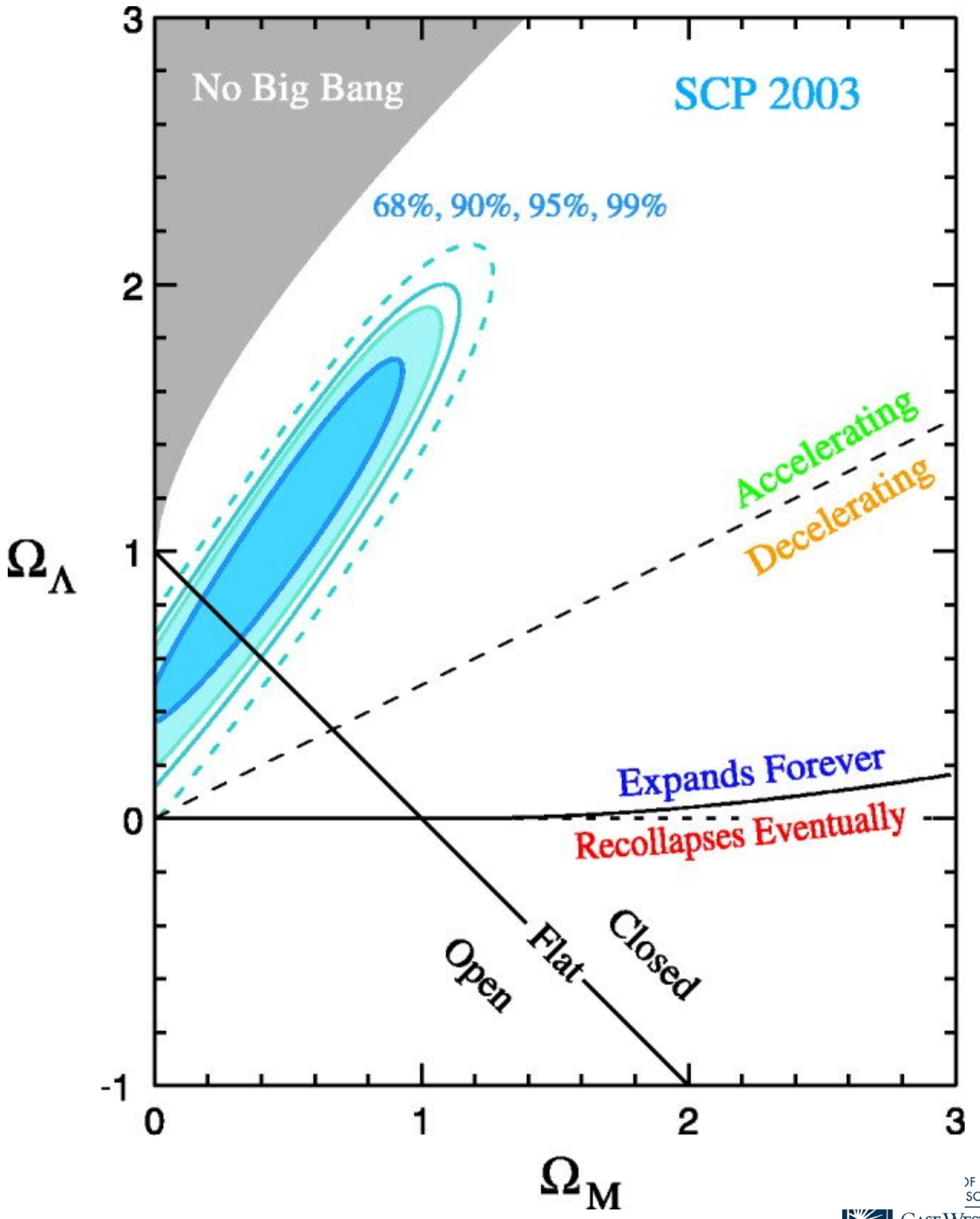
Remember, they tell us that the universe is accelerating in its expansion. So what does that mean for the universe's cosmological parameters?

Think about plotting a "plane" of all possible Ω_M, Ω_Λ values. On this plane we can also plot regions that show

- **shape of space**
- **accelerating / decelerating universe**
- **final outcome (expands forever or recollapses)**

So what values for lambda do we get from the supernovae?
Acceleration means Ω_Λ is "beating" Ω_M.
Things to notice.

- **An accelerating Universe is older. (ie the expansion rate was slower in the past, so the universe took longer to grow to its present size.**
- **Even universes which expand forever can be spatially flat or closed, and universes which collapse may yet be spatially open.**
- **As time goes by, Λ wins. $F_\Lambda \sim R$, $F_{gravity} \sim R^{-2}$. If the cosmological constant exists, it was end up dominating the expansion.**



A "union" plot-- multiple datasets, multiple methods

CMB = microwave background, sensitive to shape of space
SNe = supernovae, sensitive to R(t), the rate of expansion of the universe
BAO = tracing large scale structure of galaxies, sensitive to matter density parameter.

Working together, they suggest we live in a universe with:

- $\Omega_M \sim 0.25$
- $\Omega_\Lambda \sim 0.75$
- Other observations give $H_0 \sim$ 69-72 km/s/Mpc
- Results in $t_0 \sim$ 13.8 Gyr