surface brightness = flux per unit angular area on the sky

flux	**surface brightness**
magnitude	magnitude per square arcsecond
luminosity	luminosity per square parsec
$m = -2.5 \log f$	$\mu = -2.5 \log \frac{f}{\text{area}}$

BE CAREFUL: you can’t multiply a surface brightness by an area to get a magnitude!

$$\mu = -2.5 \log \frac{f}{\text{area}}$$

$$\mu = -2.5 \log f + 2.5 \log \text{area}$$

“

$$\mu = m + 2.5 \log \text{area}$$

Note that while the observed brightnesses and sizes of galaxies drop at larger distances, ***surface brightness does not change.*** Look at a patch of the galaxy of fixed *angular* size, and imagine moving the galaxy to larger distance.

$$f = \frac{L}{4\pi d^2}$$

$$\text{physical size} \sim \text{angular size} \times d$$

$$\text{area} \sim (\text{angular size} \times d)^2$$

Each star is fainter by r^{-2} but the total number of stars in the patch increases as r^2, so the flux coming from that patch = r^{-2} x r^2 = constant.

So we can measure the surface brightness of spiral galaxies and learn immediately the luminosity density -- the density of starlight in L_{sun}/pc^2 -- inside the galaxy without knowing the distance.

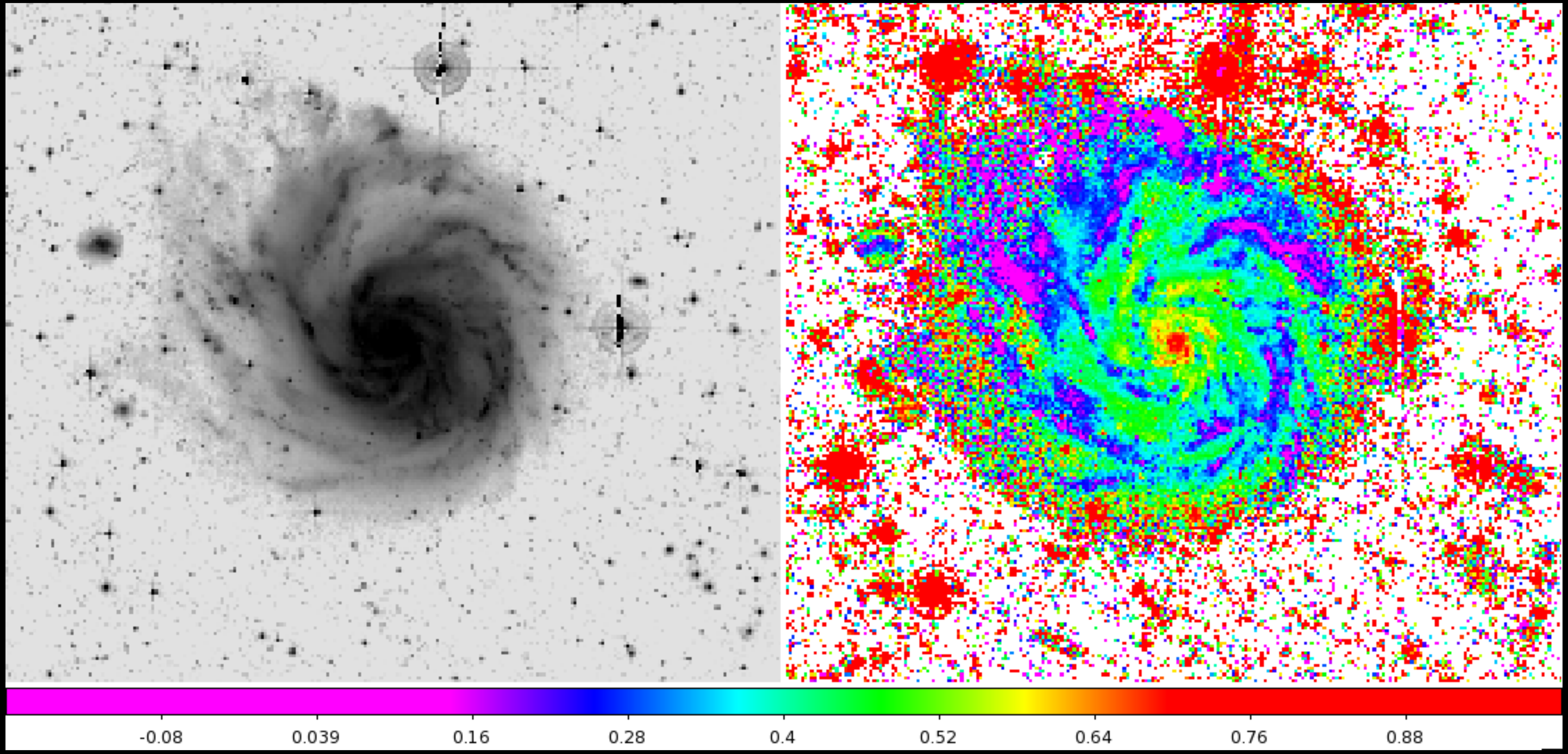
In the B filter, μ = 27 mag/arcsec^2 corresponds to 1 L_{sun}/pc^2.
A surface brightness of μ_B = 22 mag/arcsec^2 (5 mags brighter) corresponds to 100 L_{sun}/pc^2.

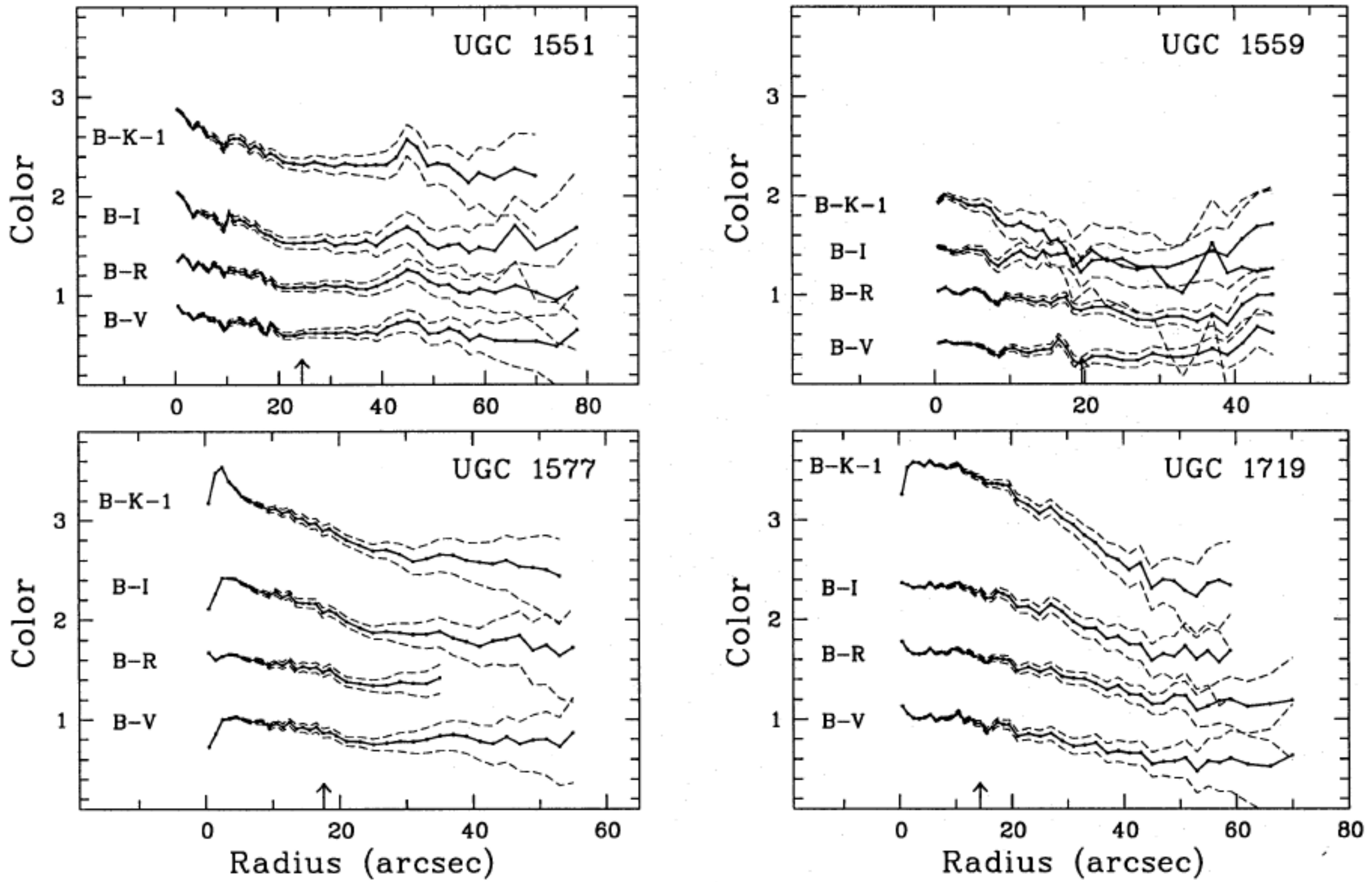
surface brightness of spiral disks described by exponential

$$I(r) = I_0 e^{-r/h}$$ in L/pc^2

in magnitudes/square arcsecond

$$\mu(r) = \mu_0 + 1.09(r/h)$$


-0.08
0.039
0.16
0.28
0.4
0.52
0.64
0.76
0.88


UGC 1551
UGC 1559
UGC 1577
UGC 1719
B−K−1
B−I
B−R
B−V
Color
1
2
3
0
20
40
60
80
Radius (arcsec)

Spiral galaxies typically show flat rotation curves.

The luminosity of a spiral galaxy correlates with its rotation velocity: the Tully-Fisher Relationship:

$L \sim V_{circ}^{\alpha}$

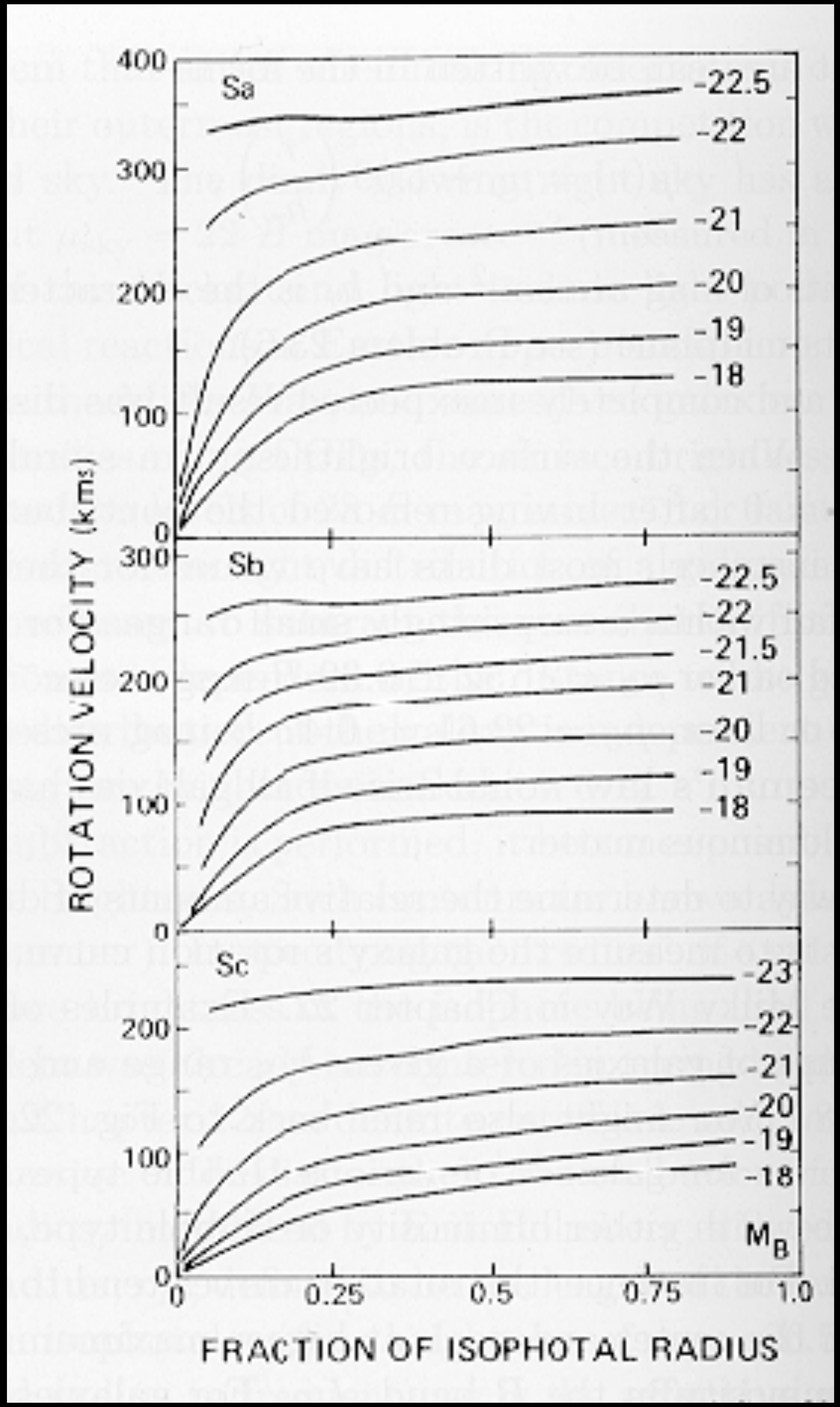
or

$\log L \sim \alpha \log V_{circ}$

or in magnitudes

$m \sim -2.5 \log L \sim -2.5\alpha \log V_{circ}$

Is there a physical basis for this?
Hint: what do L and V_{circ} trace in a galaxy?



start with circular velocity: $V^2 = \frac{GM}{R}$ **or** $M \sim RV^2$

we don't know the mass of a galaxy, but we know its luminosity, so let's make up a quantity called the total mass-to-light ratio (which includes everything: stars, gas, dark matter)

$$M = L\left(\frac{M}{L}\right)_{tot}$$

now remember that surface brightness is luminosity over area: $I = \frac{L}{\pi R^2}$ **or** $R \sim \sqrt{\frac{L}{I}}$

equate the two expressions for mass

$$RV^2 \sim L\left(\frac{M}{L}\right)_{tot}$$

substitute in for R

$$\sqrt{\frac{L}{I}}V^2 \sim L\left(\frac{M}{L}\right)_{tot}$$

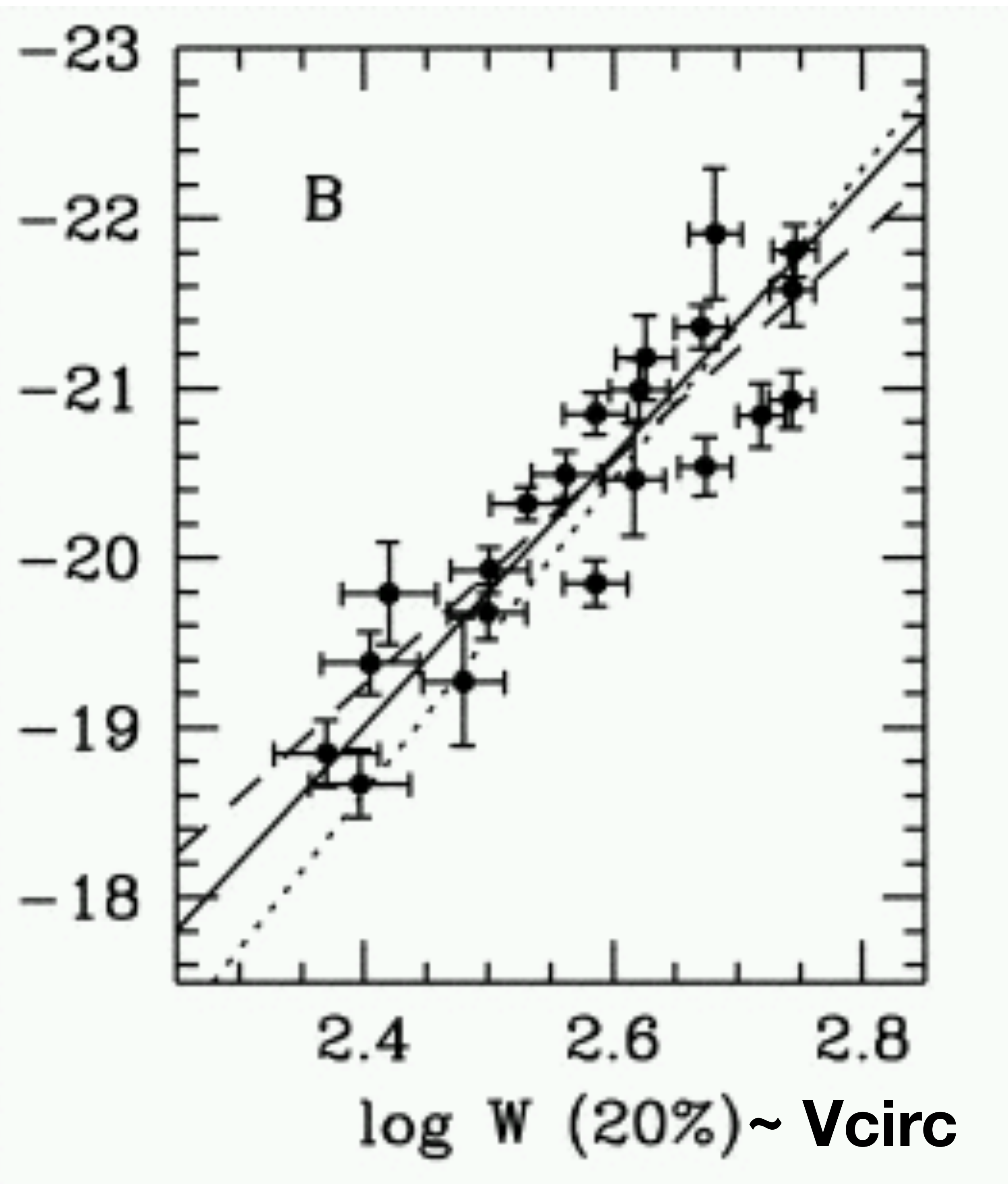
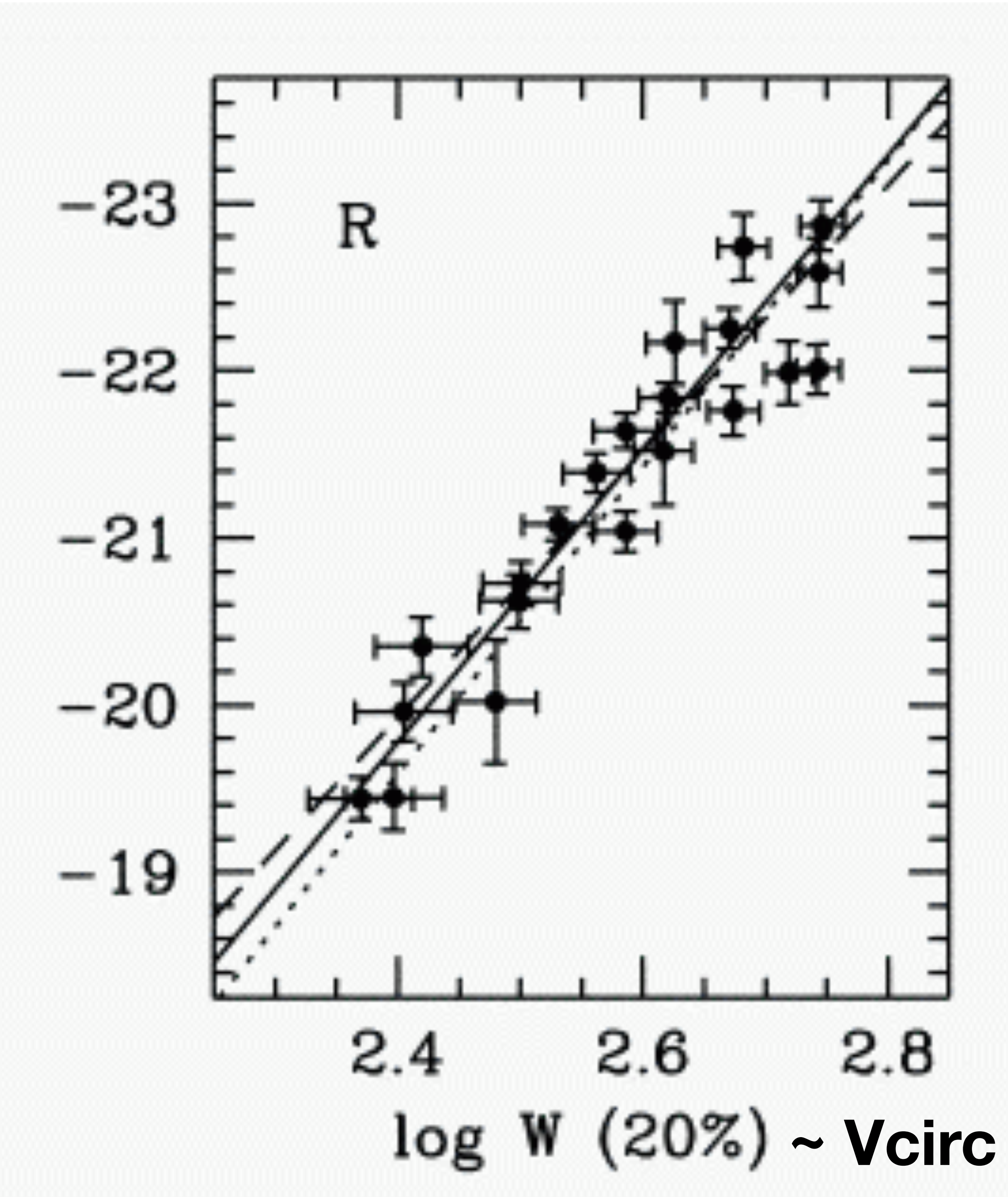
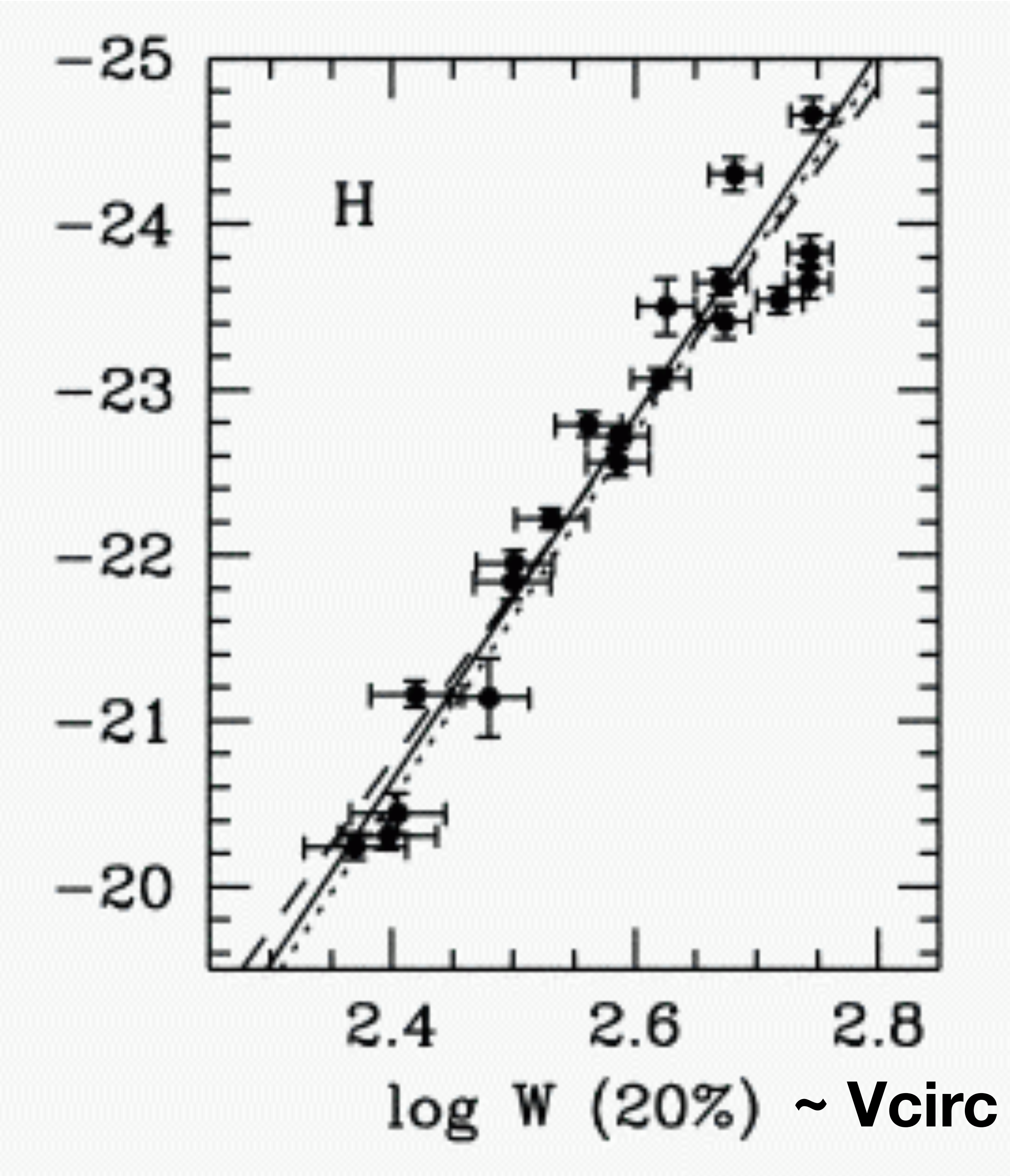
solve for L

$$L \sim \frac{V^4}{I\,(M/L)^2_{tot}}$$

Tully-Fisher works if surface brightness times *total* (not stellar) mass-to-light-ratio squared is constant.
In other words, the stars and the dark matter are somehow linked. HUH???

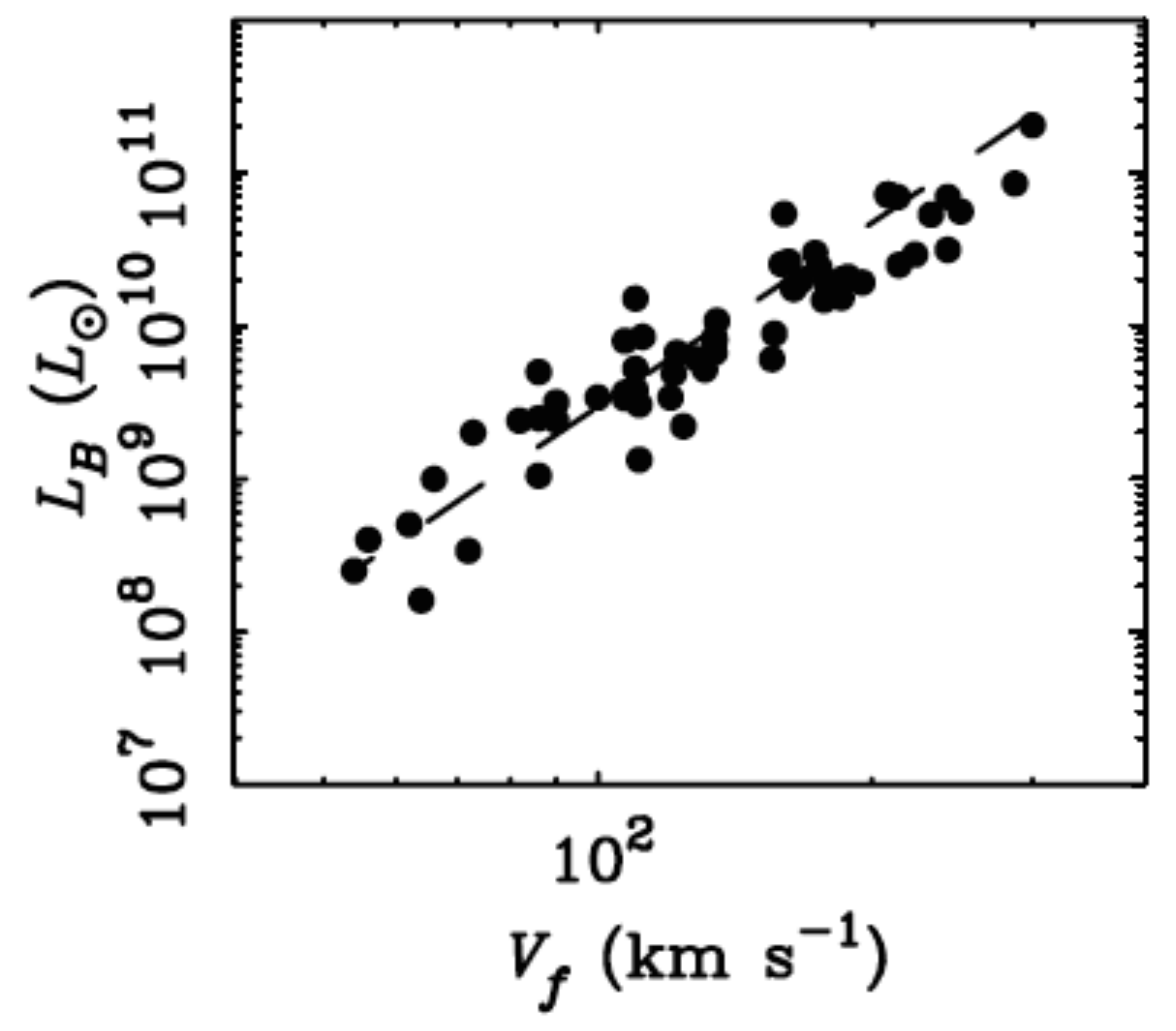
$$m \sim -2.5\log L \sim -2.5\alpha\log V_{circ}$$

absolute magnitude

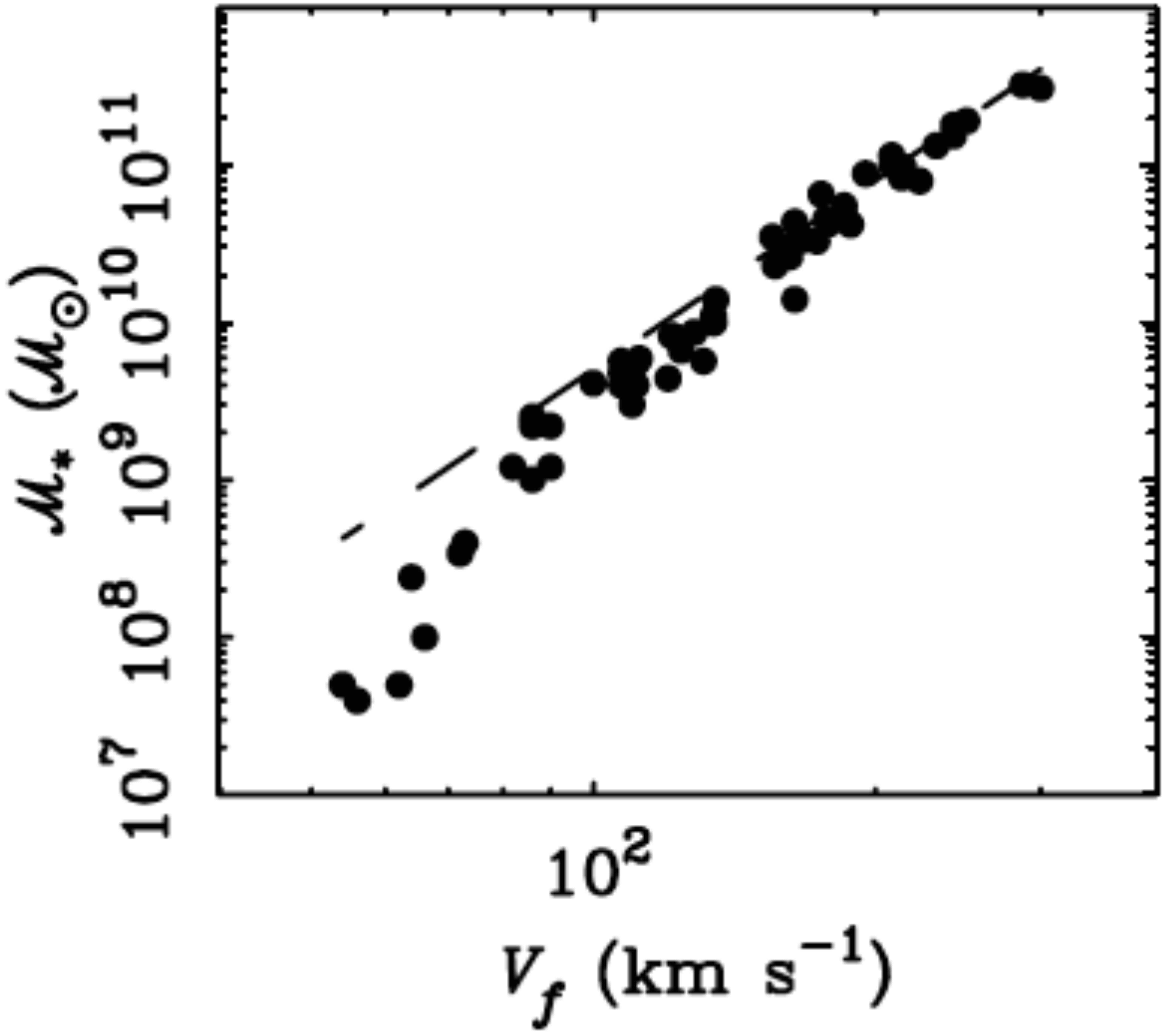
B (Blue) Tully Fisher	R (Red) Tully Fisher	H (Infrared) Tully Fisher
		
• slope: -8.0 • alpha: 3.2 • scatter: 0.25 mag	• slope: -8.8 • alpha: 3.5 • scatter: 0.25 mag	• slope: -11.0 • alpha: 4.4 • scatter: 0.19 mag

Question: Why would the relationship change depending on what wavelength you look at?

Let's look at a different version of the classic Tully-Fisher relationship: Blue luminosity versus circular speed for a sample of spiral galaxies.
We see the linear relationship, with a decent bit of scatter.

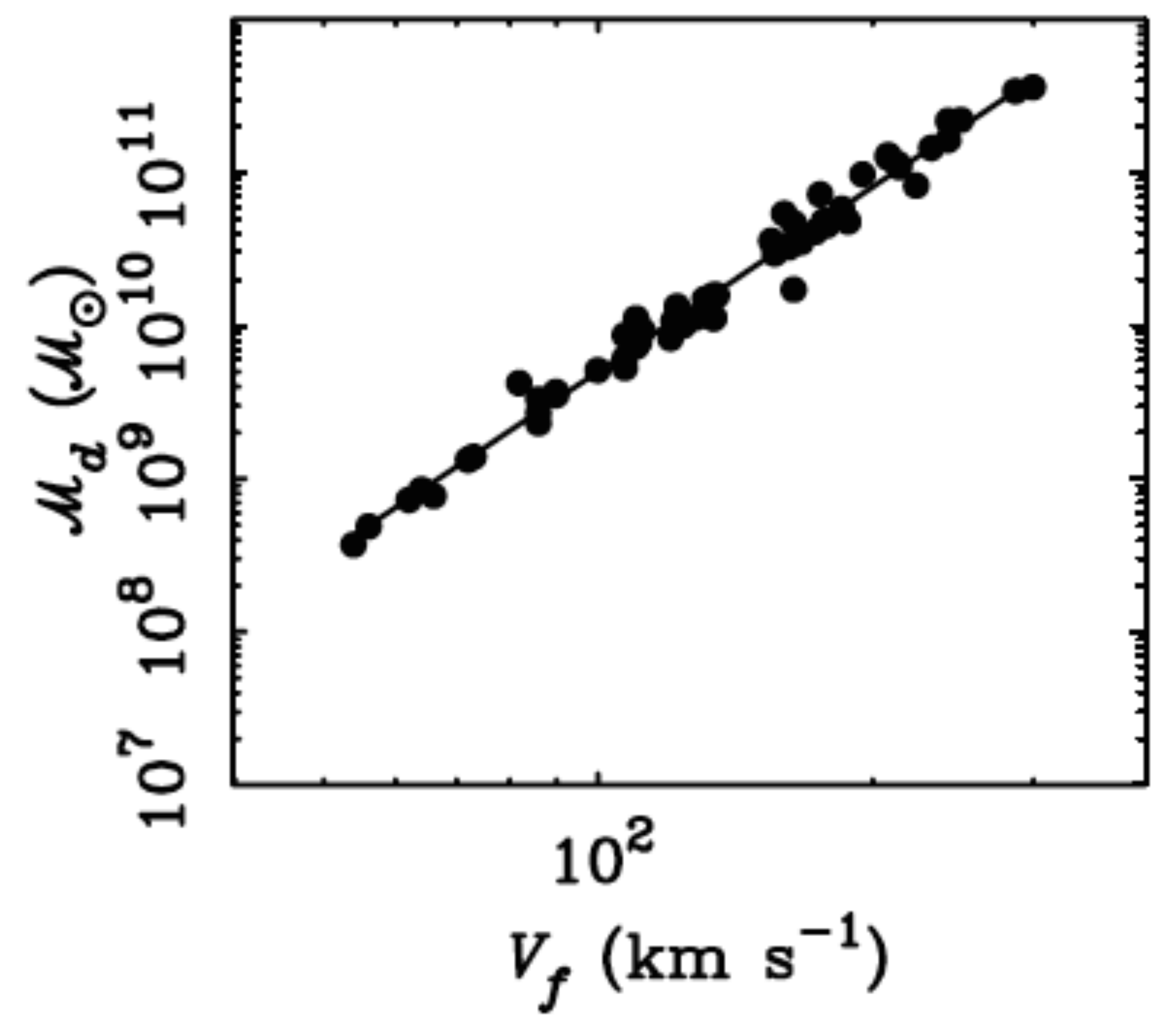


Now let's use the colors and luminosities of the galaxies, along with stellar population models, to work out the mass of all the stars in each galaxy. If we plot that on a TF-like diagram, the scatter is much less for massive galaxies, but the low mass galaxies don't fit. But there's more stuff than just stars in a galaxy -- we haven't accounted for *gas*. Low mass spirals are preferentially more gas-rich, so we are missing a lot of their mass.



If we define the total baryonic mass of the galaxy by adding both stars and gas together, we get any extremely tight relationship over orders of magnitude in mass!

There is a basic, fundamental relationship between the amount of normal (baryonic) mass in a spiral galaxy and the speed at which they rotate.

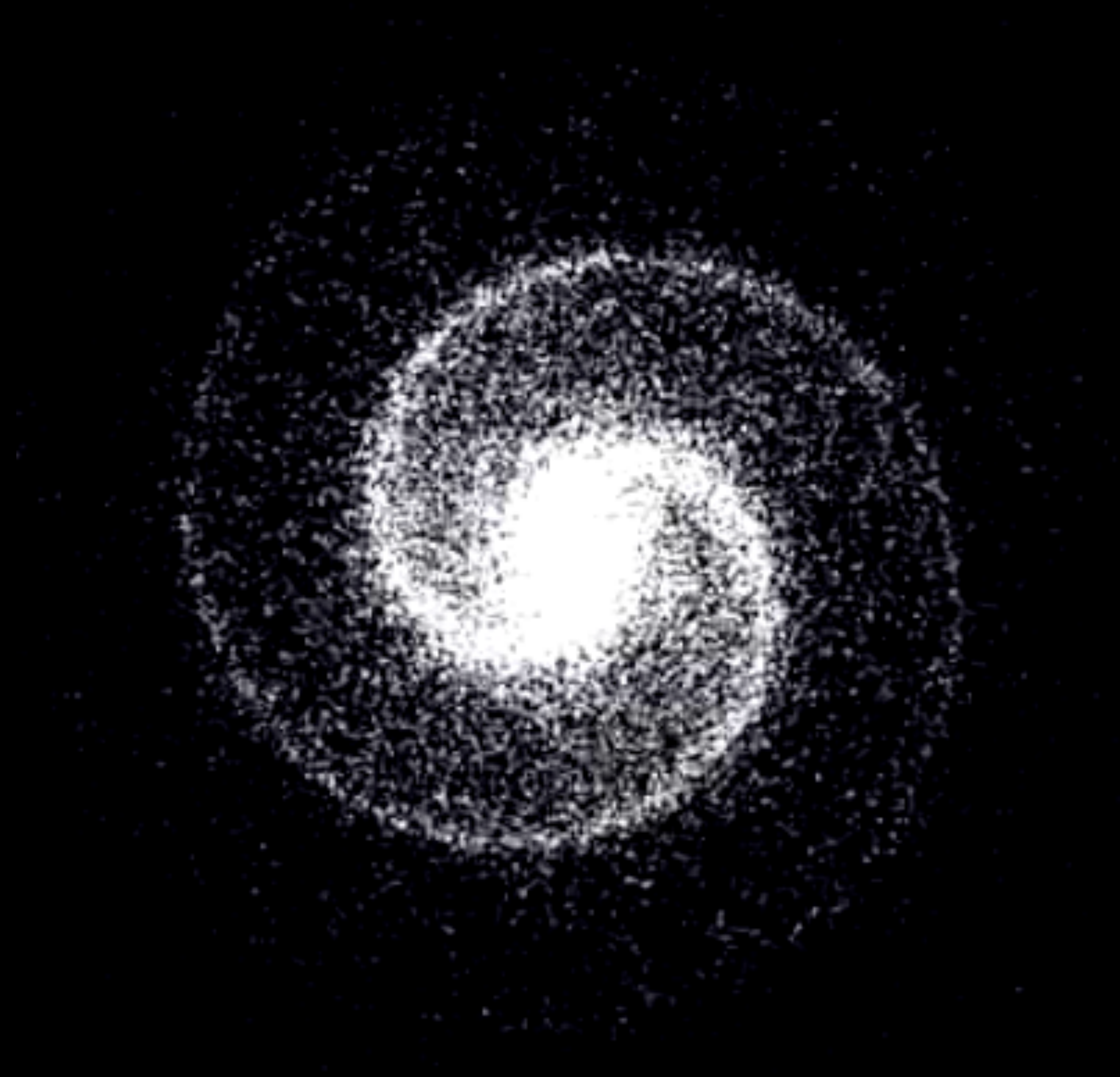


Spiral arms come in different "flavors":
~10% grand-design (two well-defined spiral arms)
~60% multiple-arm
~30% flocculent spirals (no well-defined arms at all)

Spiral arms are sites of strong star formation we see dust, HII regions, blue stars, lots of gas. In fact, spiral arms are much more prominent in blue light than in red.

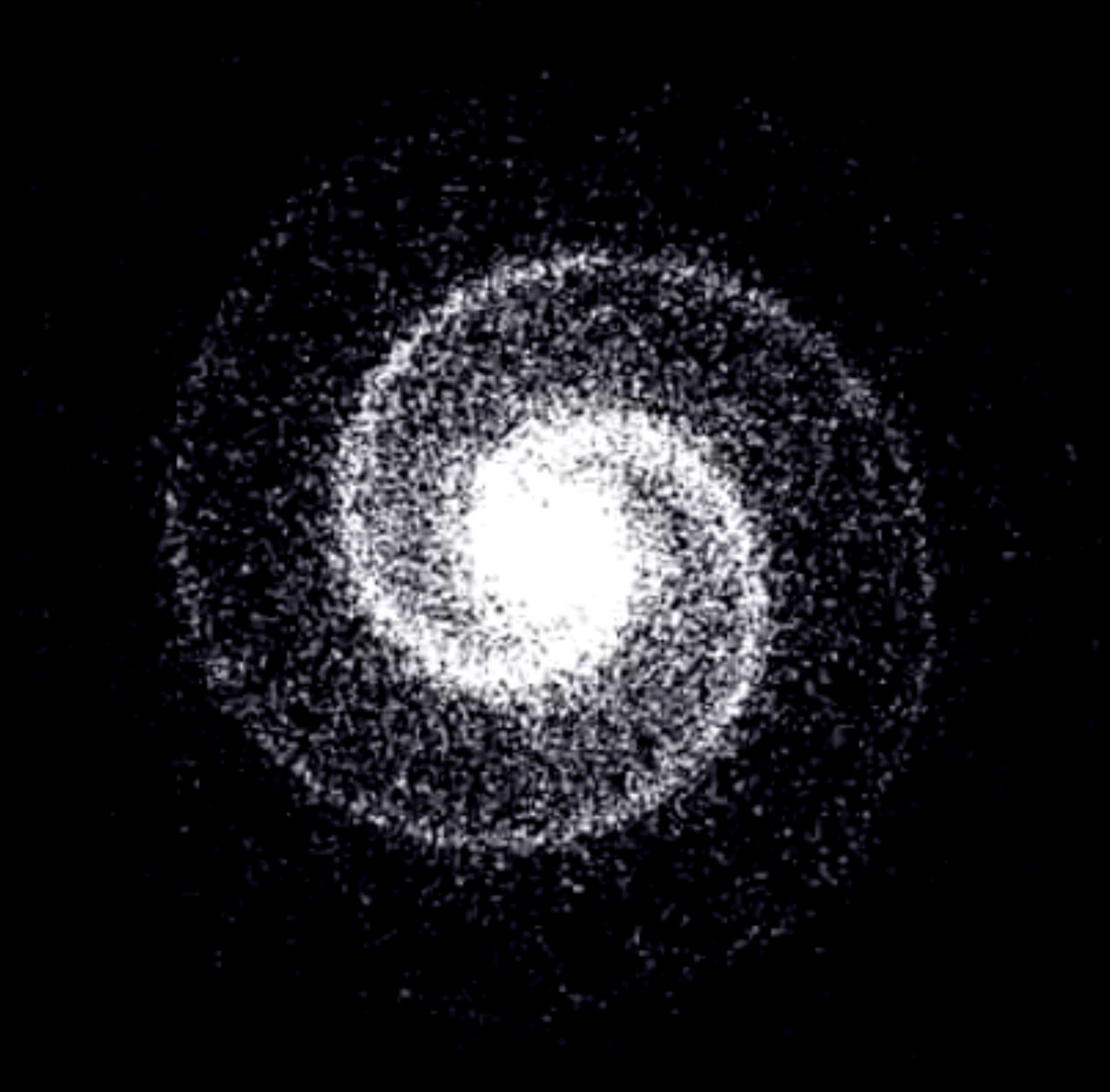
But what are they? How do they form? How long do they last?

solid body



angular speed = constant
linear speed increases with radius

wind up



angular speed increases with radius
linear speed ~ constant