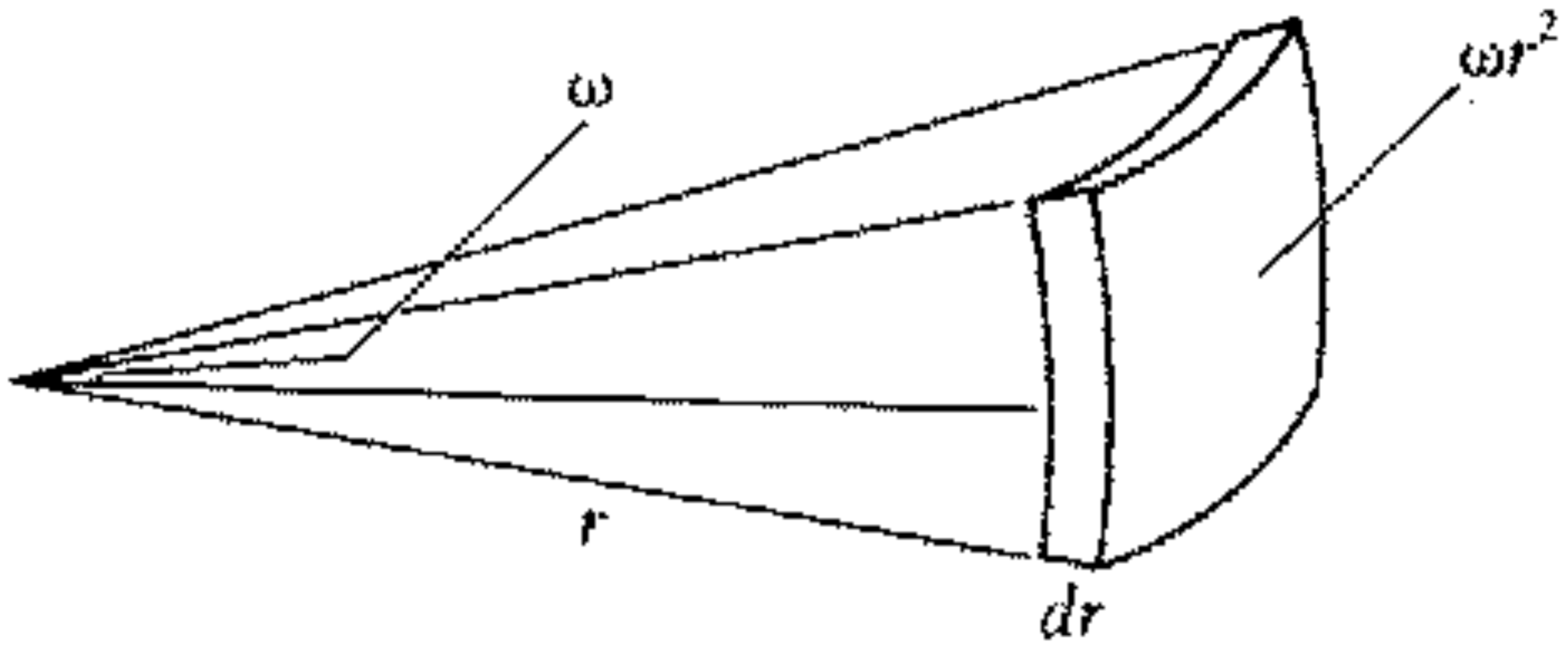


Imagine looking at stars in a patch of sky of solid angle ω and at a distance r.

The volume of space in the thin patch between r and r+dr is

$$dV = \omega r^2 dr$$

If the galaxy has a uniform density of stars (given by n), and we integrate over radius, we get the total number of stars between us and r:

$$N(r) = \omega n \int_0^r r^2 dr = \frac{1}{3}\omega n r^3$$

Now remember the relationship between absolute and apparent magnitude...

$$m - M = 5\log r - 5$$

...which we can turn around to solve for r...

$$r = 10^{[0.2(m-M)+1]}$$

...and plug into N(r) to get ***N(m), the number of stars brighter than some apparent magnitude m***:

$$N(m) = 10^{(0.6m+C)}$$

So for every magnitude fainter we go, we ought to see

$10^{0.6}$ = 4 times as many stars. We don't.

$$\log N(m) = 0.6m + C$$

But it gets worse. Let's look at how much light we'd be seeing from these stars.

Let's say the apparent brightness of an m=0 star is l_0. Then, using the definition of magnitudes, the light coming from a star of apparent magnitude m is:

$$l(m) = l_0 10^{-0.4m}$$

so the total amount of light coming from stars of magnitude m is:

$$L(m) = l(m)N(m) = l_0 10^{-0.4m} 10^{0.6m+C} = C_2 10^{0.2m}$$

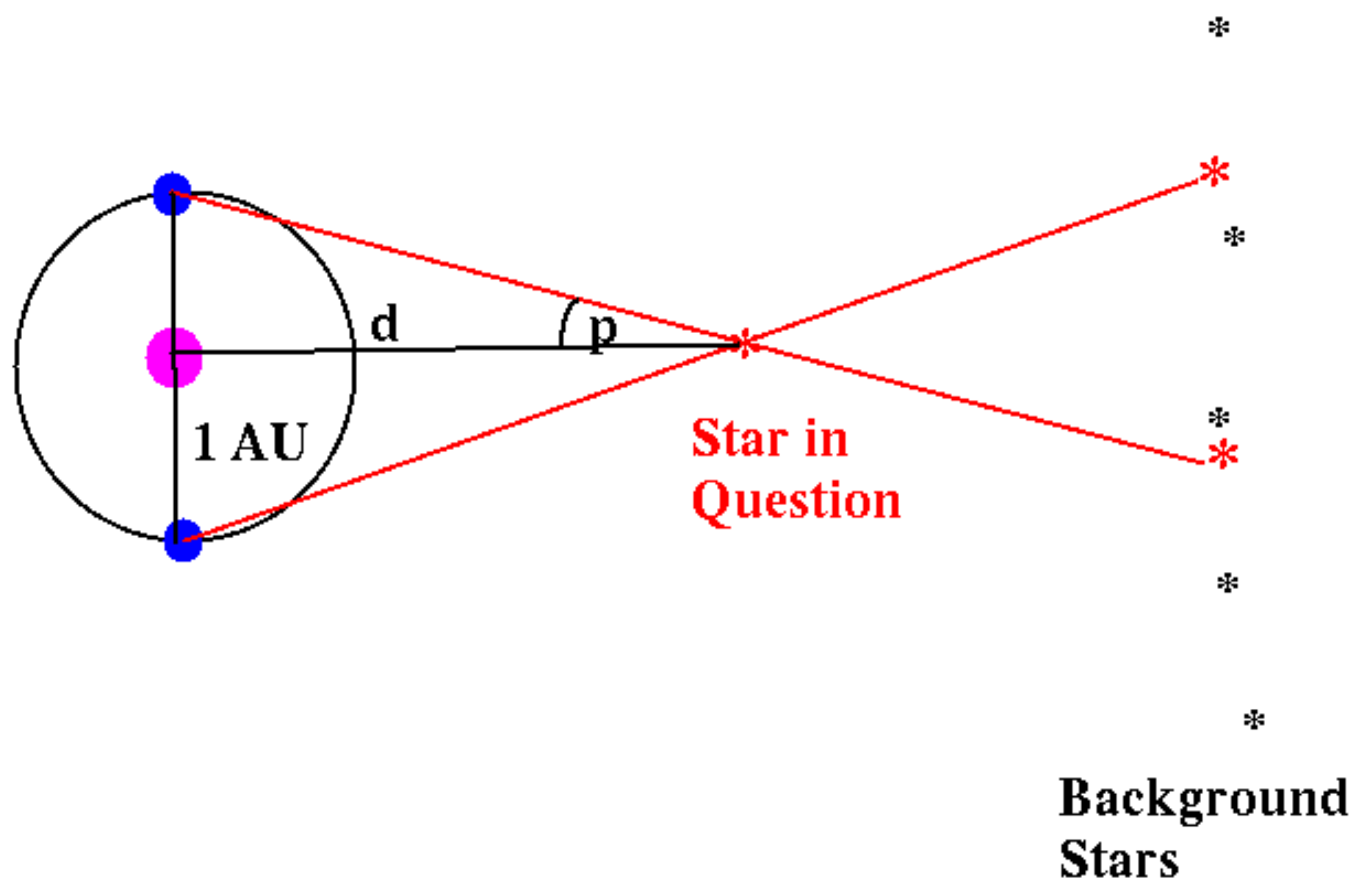
So the total amount of light coming from all stars brighter than apparent magnitude m is:

$$L_{tot}(m) = C_2 \int_{-\infty}^{m} 10^{0.2m} dm = K 10^{0.2m}$$

This diverges as m gets bigger: infinite brightness!

This problem is known as **Olber's paradox**. If the galaxy were infinite and homogeneous, the sky should be blazingly bright.

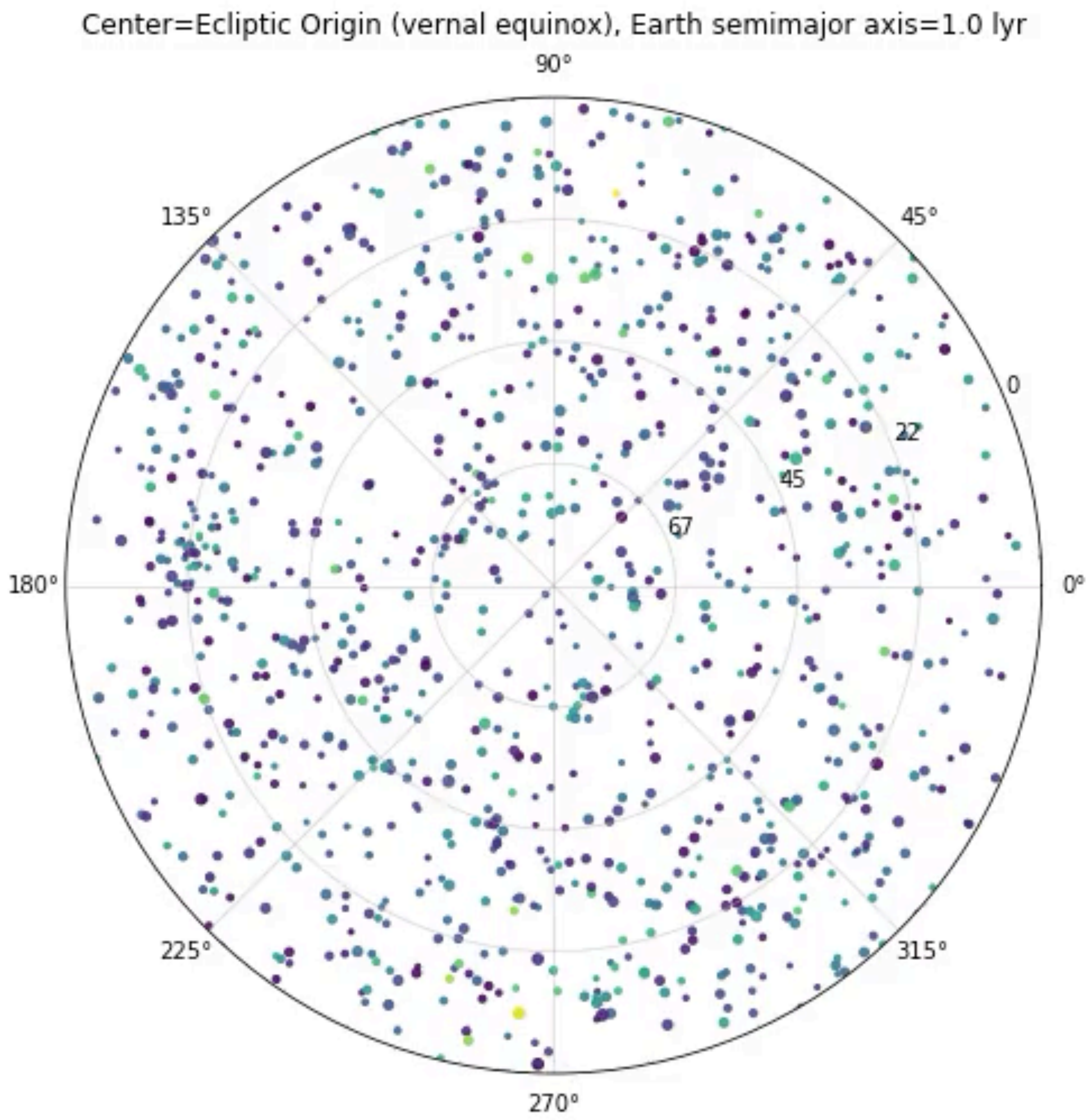
So what's the point of this failed exercise? It's not a failure! Turn the question around: fit star counts to different models of stellar distributions to derive the structure of the galaxy.



$$d = \frac{1}{p''}$$

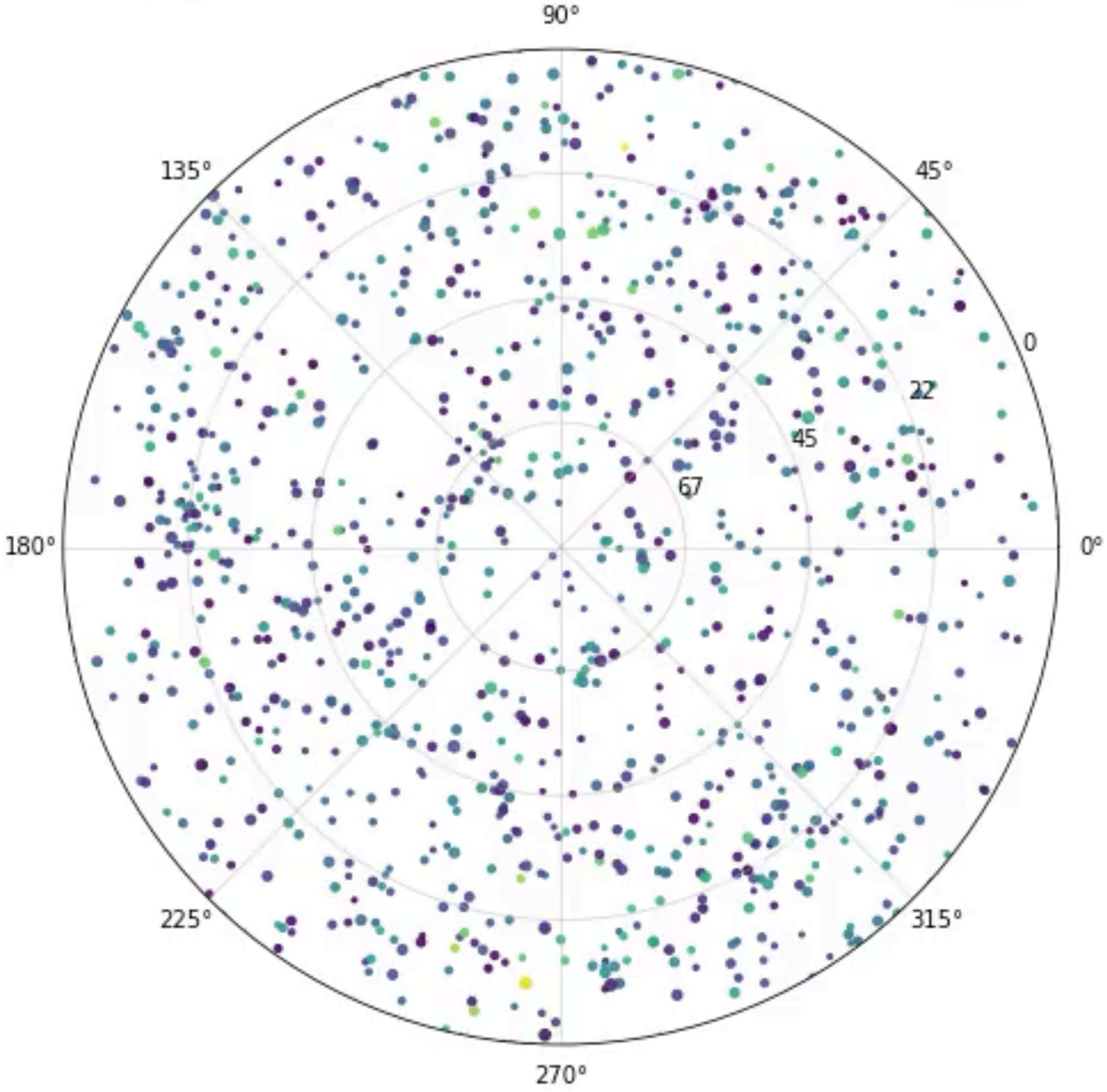
1 parsec = distance for one arcsecond of parallax

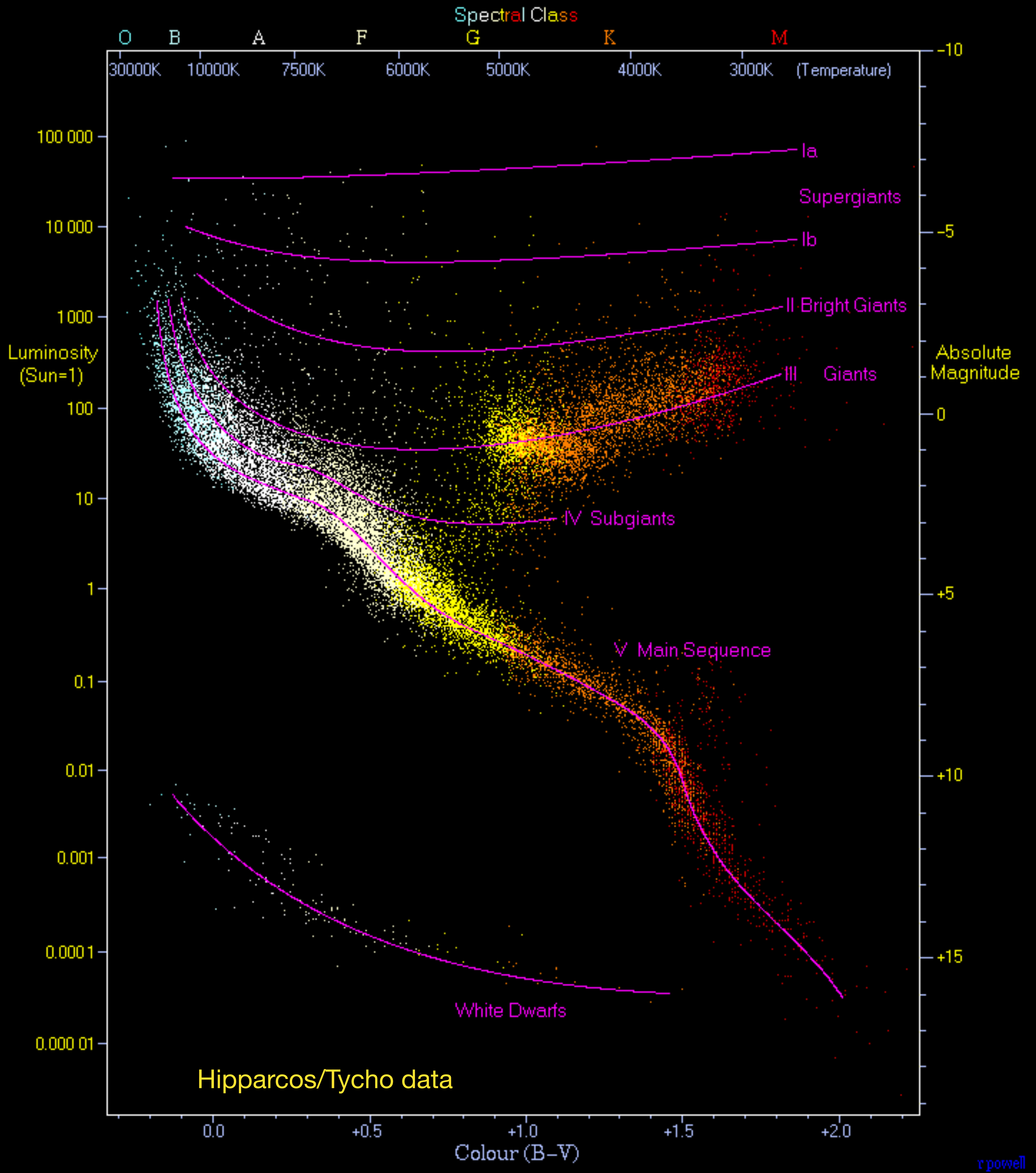
1 pc = 206,265 AU = 3.26 ly



	Hipparcos	**Gaia**
Dates	1989-1993	2014-2019
Limiting magnitude	~12	~ 20
Parallax precision	milliarcsec	20 microarcsec (@ 15 mag) 200 microarcsec (@ 20 mag)
Distance	< 1 kpc	10 kpc
N(stars)	~ 10^5	~ 2×10^7 (1% accuracy) ~ 2×10^8 (10% accuracy)
Velocities	no	yes -- radial and space motions!

Center=Ecliptic Origin (vernal equinox), $a_{\oplus}$=1.0 lyr, t_{PM}=0.00 kyr





Know the spectral type,
know the absolute magnitude,
know the distance…

$m - M = 5 \log d - 5 + A$

What's the problem
(or rather, problem<u>s</u>)
with this, though?

…and how to get around it?

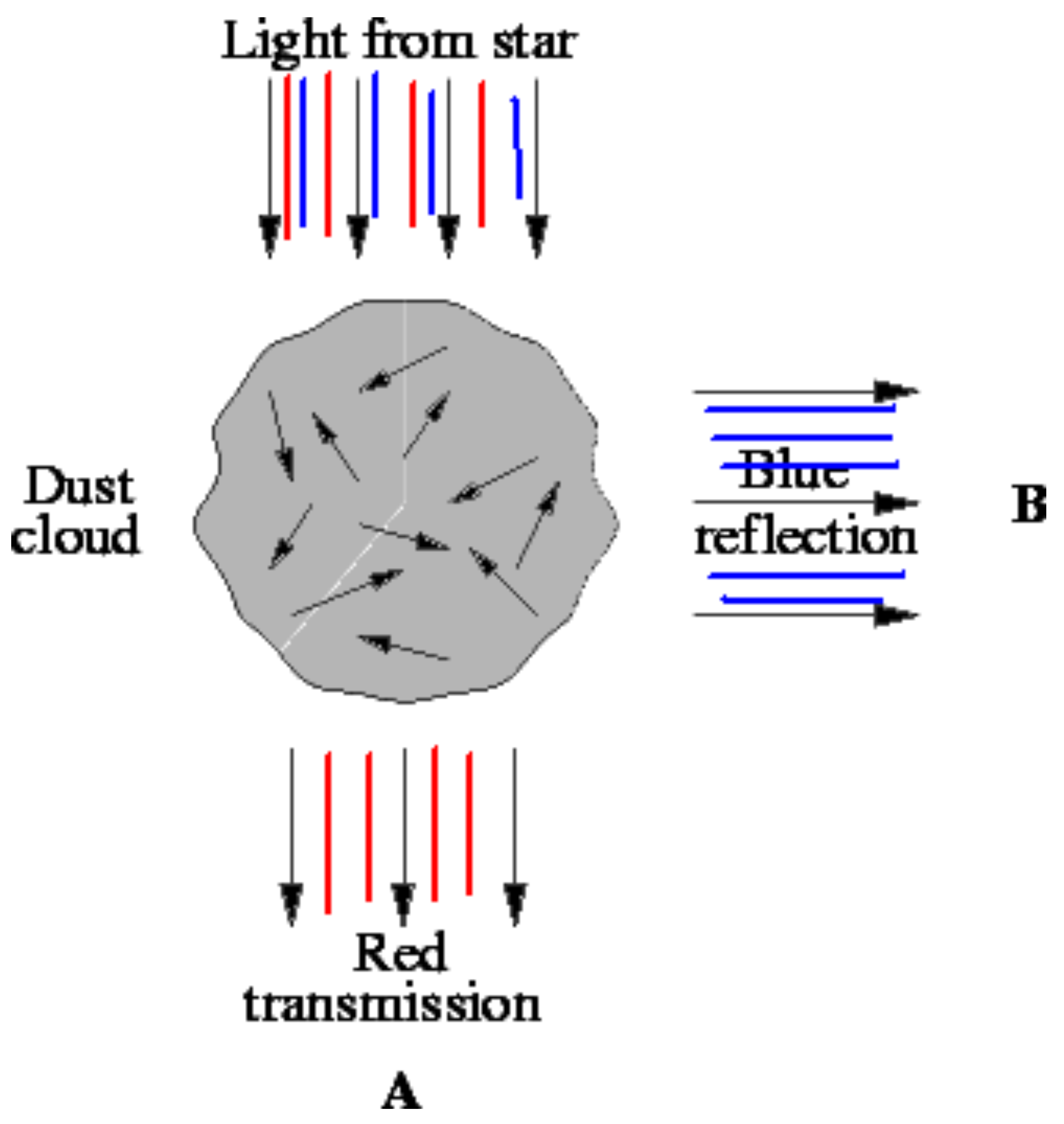
When we look in different directions of the sky, we often see dark "holes" in the distributions of stars. These are not gaps where there are no stars, but instead are **interstellar dust clouds**.

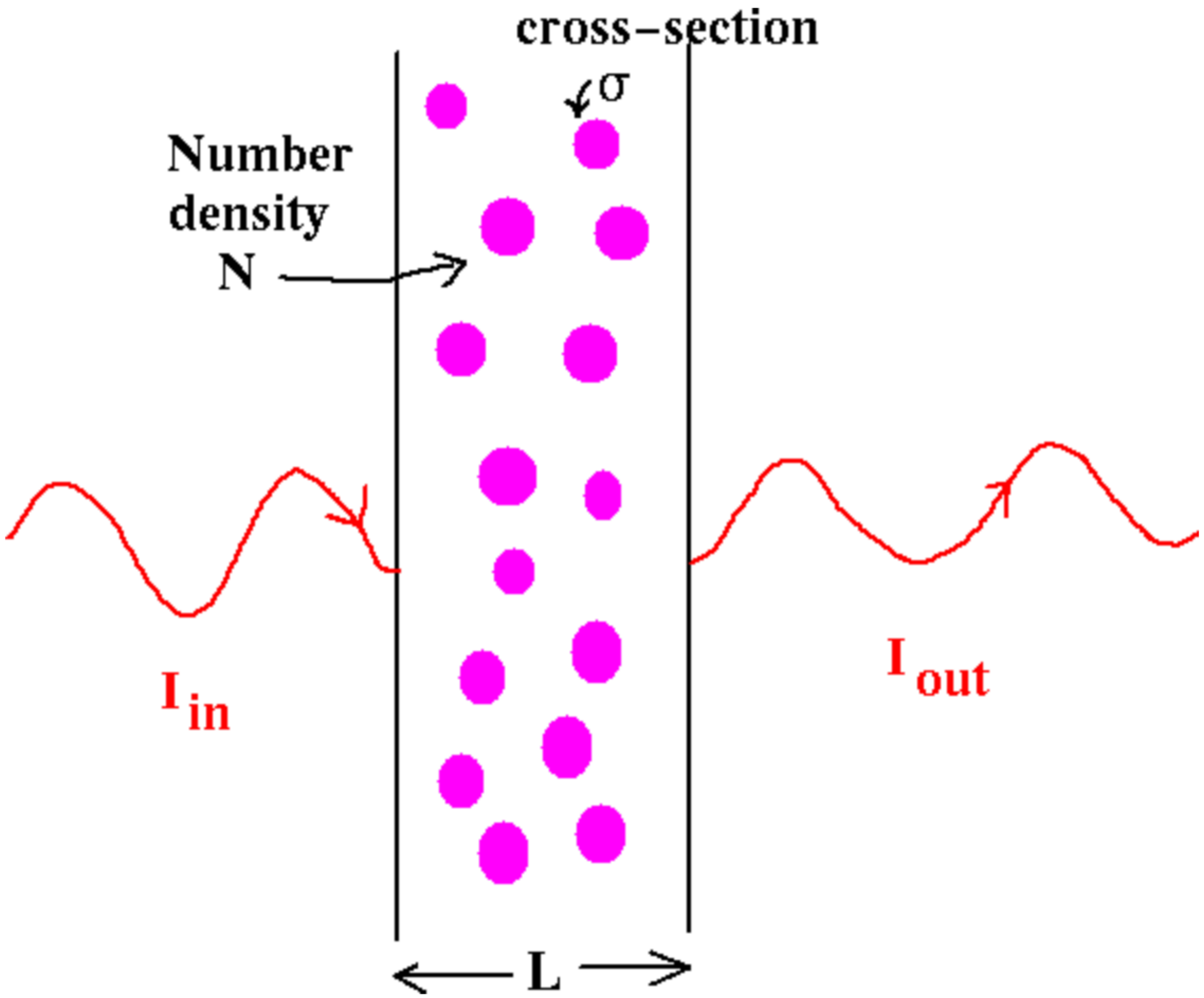
Dust doesn't have to come just in thick clouds, it can also be spread **diffusely** throughout space.

What does dust do to star light? Several things:

- it **absorbs** light
- it **reddens** light
- it **scatters** light

Question: ***What does this do to a star's observed position on the H-R diagram?***





So we have

$$I_{out} = I_{in} e^{-\tau}$$

where

$$\tau = N \sigma L$$

Rewrite as

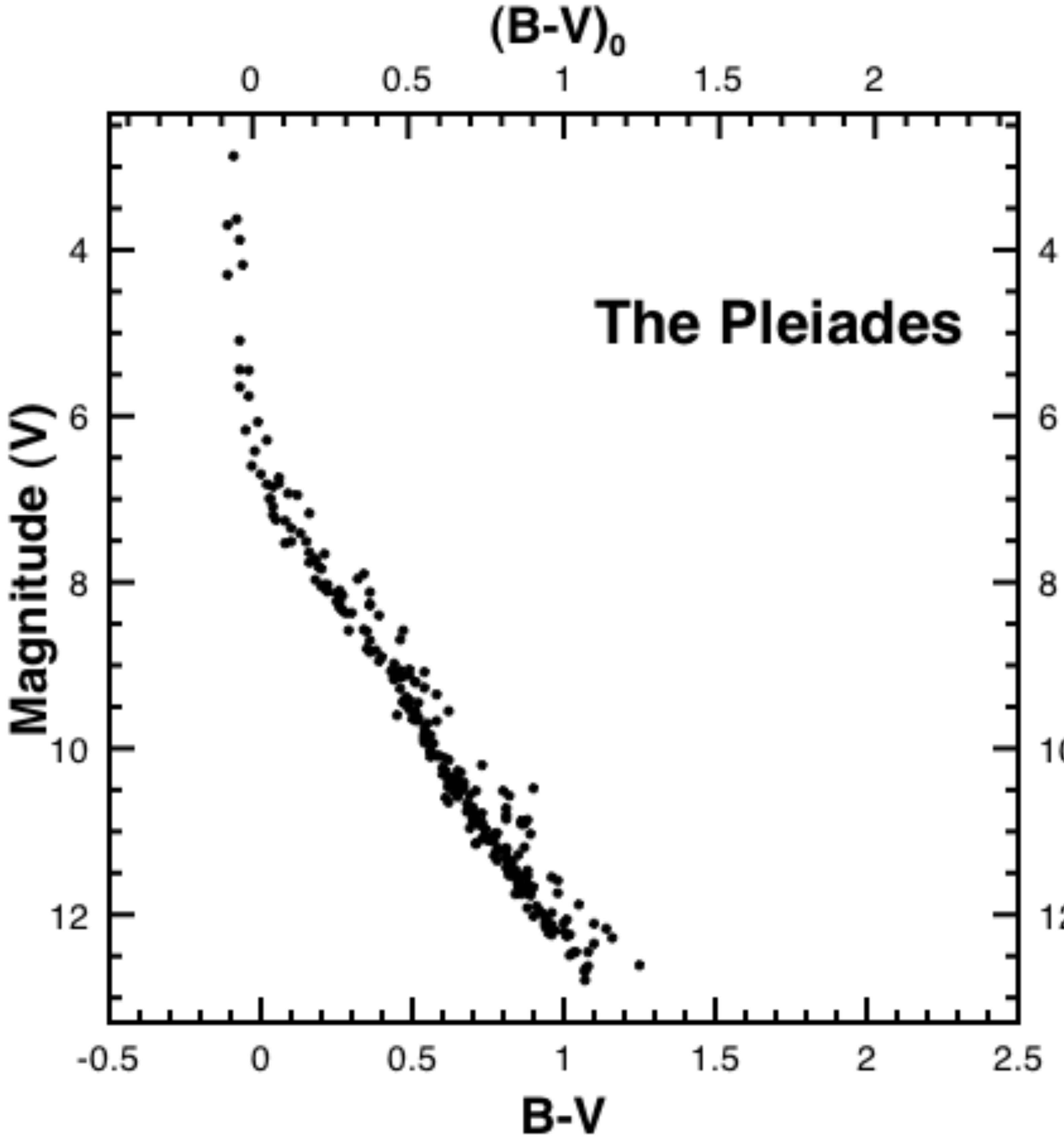
$$I_{out}/I_{in} = e^{-\tau}$$

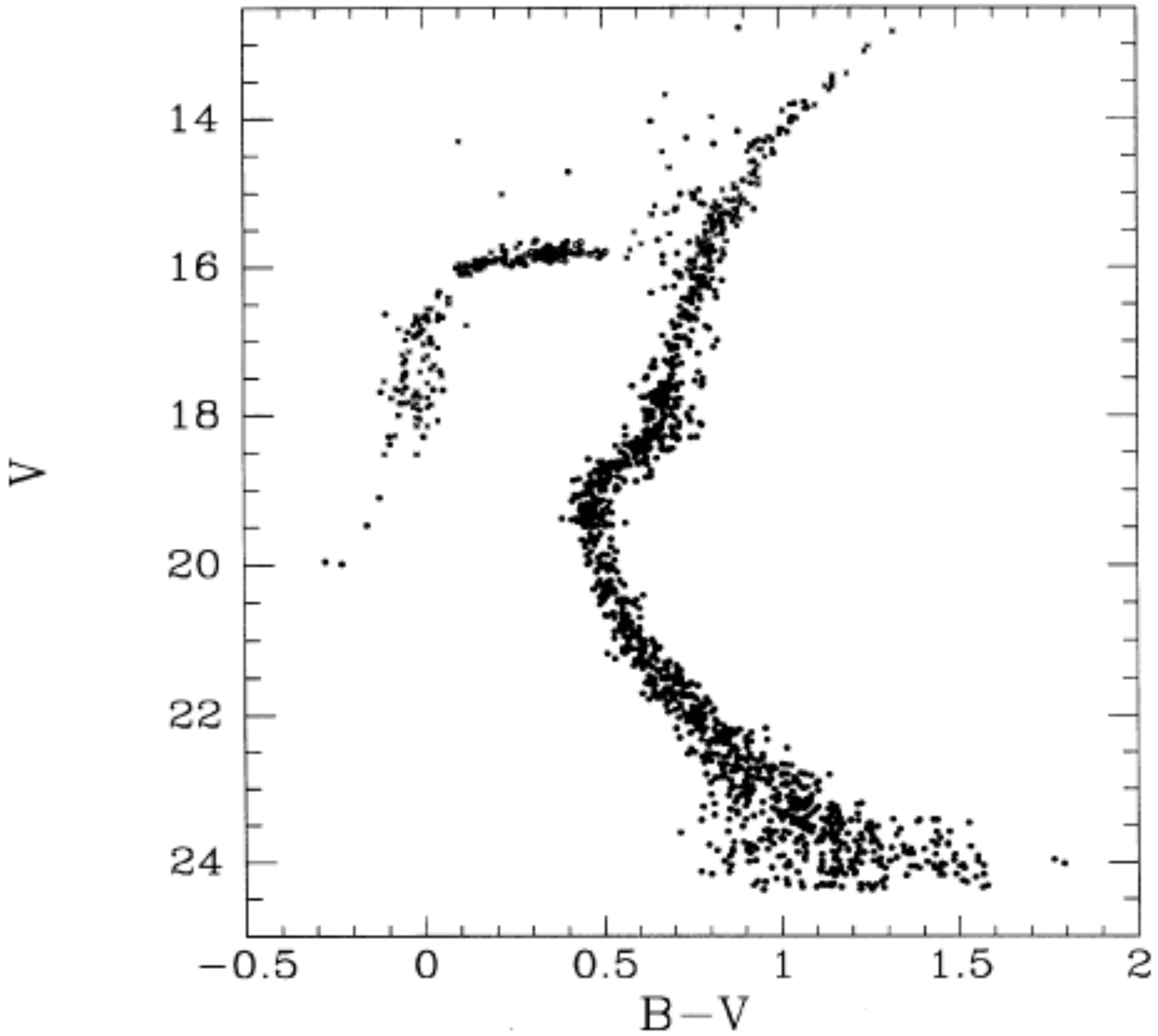
or, converting to magnitudes

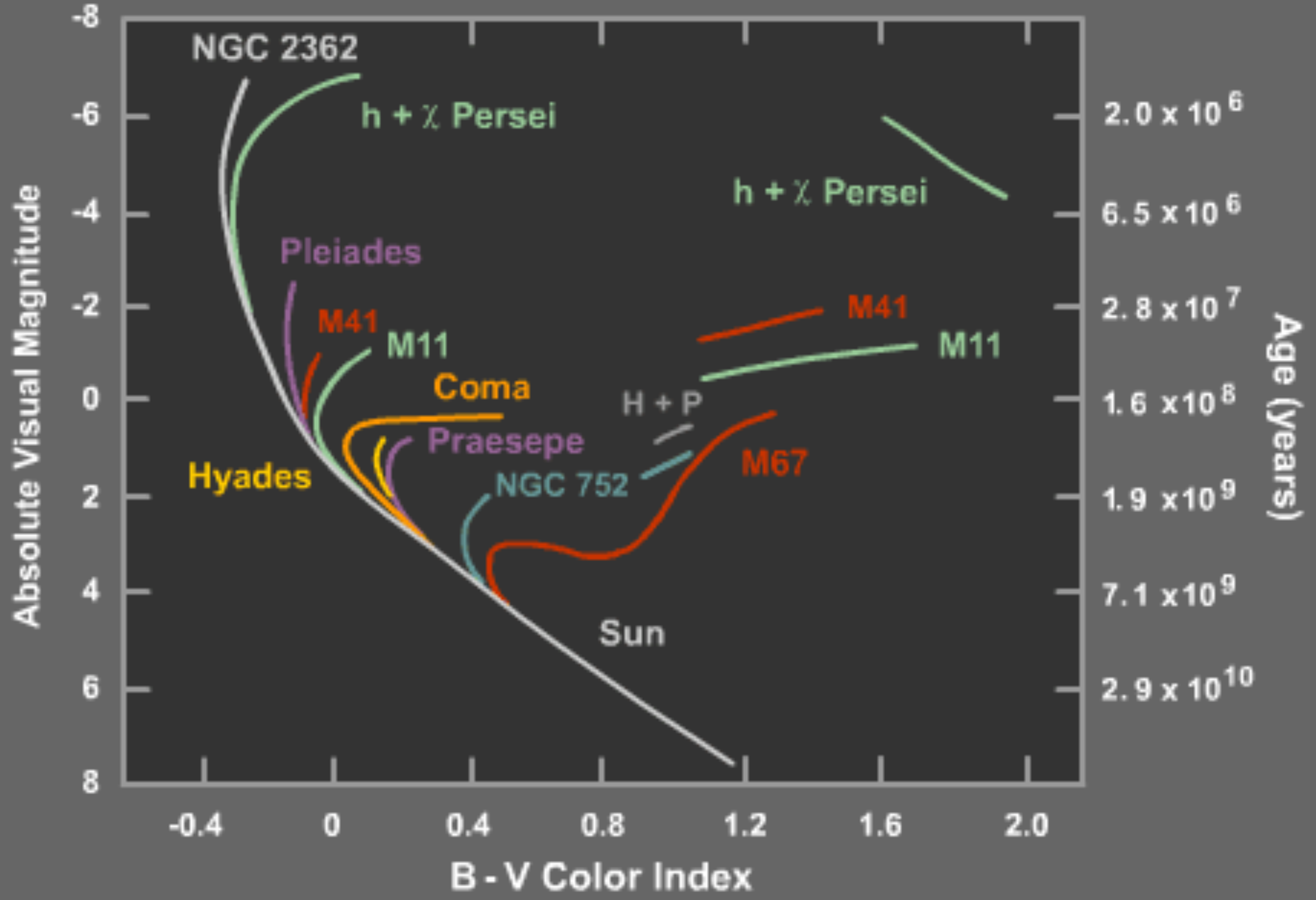
$$\begin{aligned} m_{out} - m_{in} &= -2.5 \log e^{-\tau} \\ &= -2.5(-\tau) \log e \\ &= 1.086\tau \end{aligned}$$

If we define **extinction** as $A=1.086\tau$, we can then correct the observed magnitudes: $m = m_{obs} - A$. (Why is it minus A?)

If we didn't correct the magnitudes, what would this do to our derived distances?

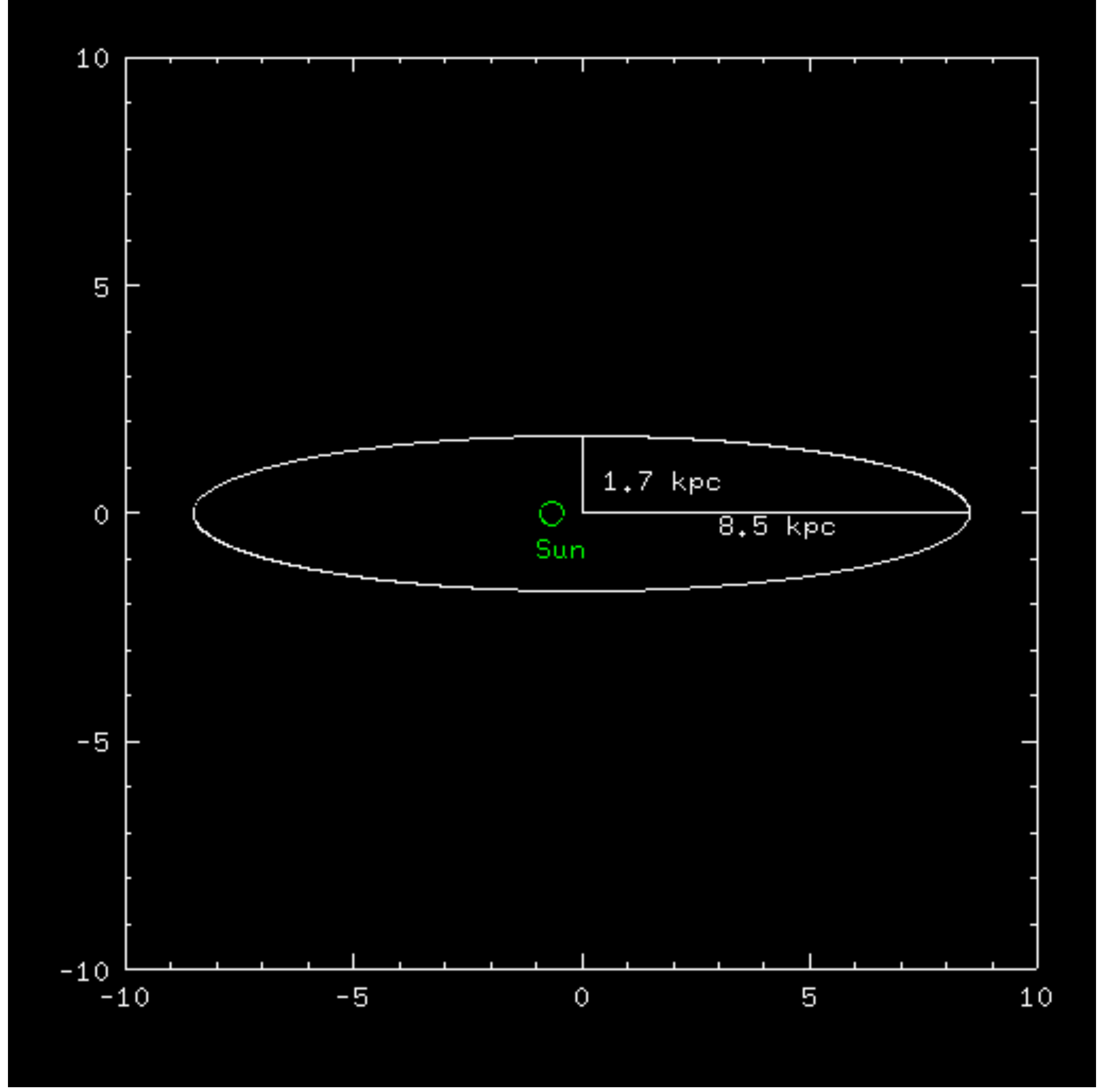

(B-V)$_0$
0
0.5
1
1.5
2
The Pleiades
Magnitude (V)
4
6
8
10
12
-0.5
0
0.5
1
1.5
2
2.5
B-V


V
14
16
18
20
22
24
−0.5
0
0.5
1
1.5
2
B−V

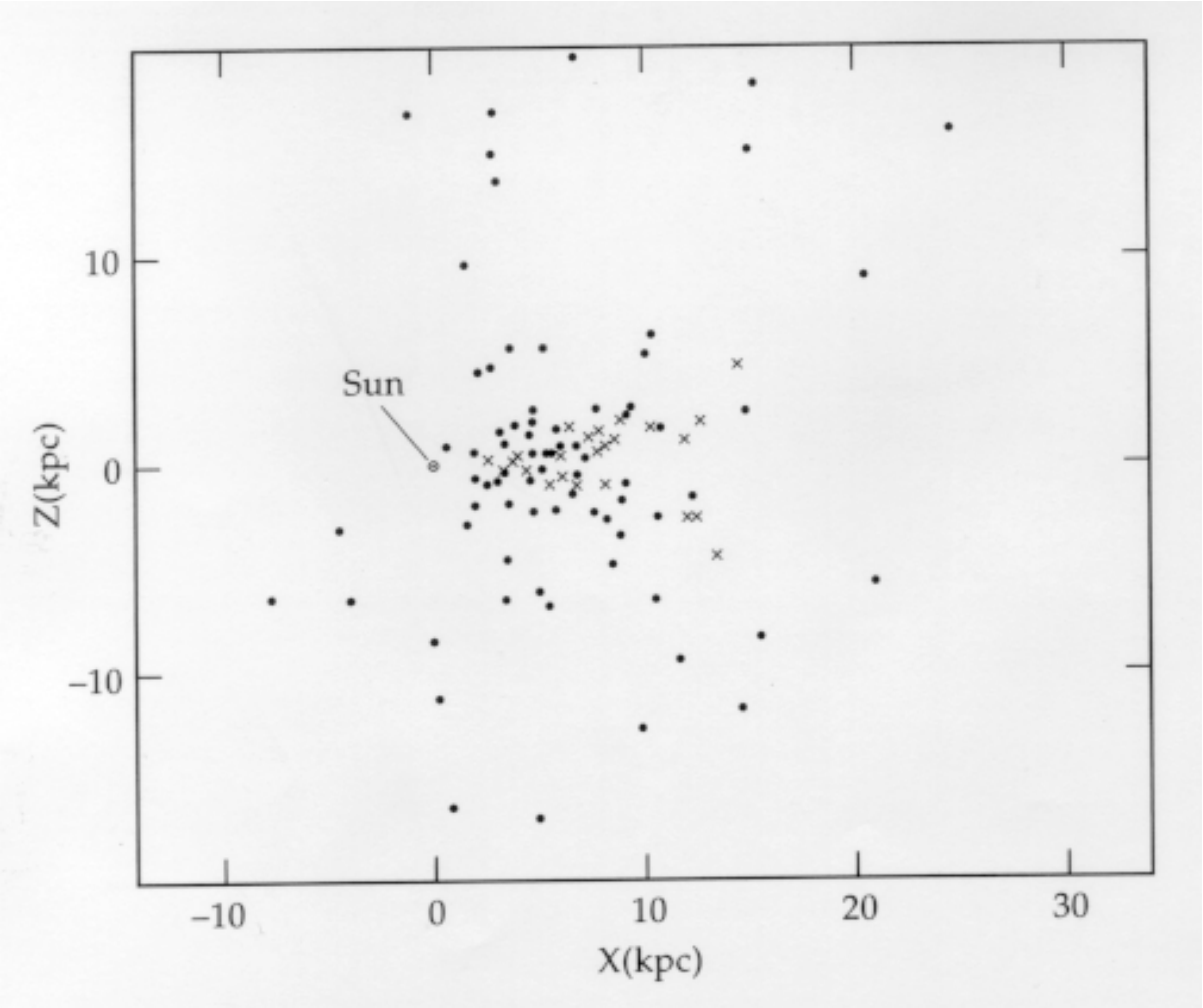


HR Diagrams for Various Open Clusters

- early 1900s: **Jacobus Kapteyn** uses quantitative star counts to measure the size of the Galaxy.
  - ellipsoidal
  - ~ 10 kpc in size
  - Sun near center



- ~1920: **Harlow Shapley** uses RR Lyrae variable stars to get distances to globular clusters.
  - Galaxy is ~ 100 kpc in size
  - Sun is ~ 15 kpc away from the center of the GC distribution.



Who's right? Actually, both were wrong. Their observations were compromised by the effects of interstellar dust, the presence of which was unknown at the time.

- **Kapteyn's problem:** because dust make stars fainter, Kapteyn couldn't see the already faint stars at large distances, so he thought the Galaxy was "running out of stars" at about 10 kpc away.
- **Shapley's problem:** since he was observing clusters of stars (instead of individual stars) he could see them further away. But since dust made them fainter, he thought they were even further away than they actually were.

Of the two, ***Shapley was much closer to the truth***. When the effects of dust were realized and corrected for, studies of galactic structure entered the modern era.