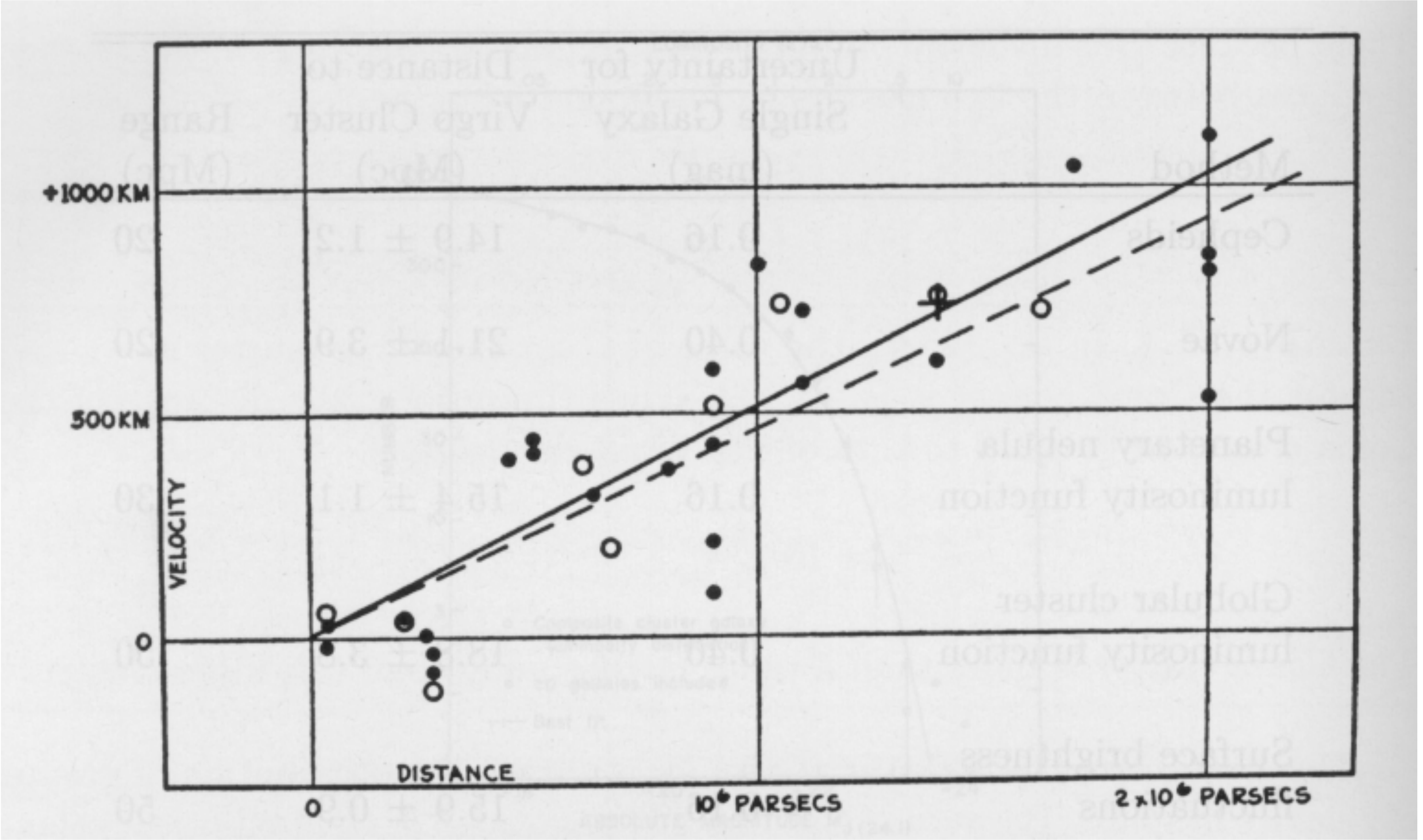
1917: Willem de Sitter used general relativity to describe an expanding universe. Einstein favored a static model which neither expanded nor contracted, by inserting an ad-hoc term he called the cosmological constant.

1929: Hubble measures the expansion of the Universe

We can rewrite this relationship as $v=H_0d$, where H_0 is the Hubble constant. Hubble's original derivation gave H_0 ~ 200 km/s/Mpc, but this is wrong because Hubble had the distances to the galaxies wrong. Modern measurements give a value more like H_0 = 72 km/s/Mpc (+/- a handful) (Because of historic uncertainty in the Hubble constant, in the past, astronomers often defined a parameter $h=H_0/100$, and write distance dependant results as (for example), $d=200h^{-1}$ Mpc.)



The Cosmological Redshift

Remember the redshift:

$$z = \frac{\lambda_{\text{obs}} - \lambda_{\text{em}}}{\lambda_{\text{em}}} = \frac{\lambda_{\text{obs}}}{\lambda_{\text{em}}} - 1$$

Using normal Doppler equation (v<<c), v=cz, and d=cz/H_0.

But it is important to realize here that the cosmological redshift is not really due to galaxies moving through space at high velocities. Galaxies are actually "embedded" in space and space itself is expanding. The light emitted from the galaxies is "stretched" as it makes its way towards us, and we see it shifted to longer wavelengths.

Measuring distances in an expanding universe:

The distance between two objects can be defined as $d = R(t) \times r$

- d = **proper distance**
- R = **dimensionless scale factor**
- r = **comoving coordinate**

For two galaxies r only changes due to peculiar motion of the galaxies (ie due to gravity in a cluster, etc). The change in proper distance due to the expansion of space is entirely contained in R(t). We define R(now) = 1.

Note that on cosmological scales, this makes the distance between two objects an ambiguous concept. There are different ways of defining distances, which give different answers.

Go back to the redshift. What does that tell us about the expansion?

$$1 + z = \frac{\lambda_{\text{obs}}}{\lambda_{\text{em}}} = \frac{R_{\text{obs}}}{R_{\text{em}}} = \frac{1}{R_{\text{em}}}$$

So if we observe a quasar at a redshift of z=3, at the time the light left the quasar, the universe was 1/4 its current size.

As z gets large, R gets small. At some point, R=0! When did that happen?

Let's see. Space is expanding in such a way that $d=v/H_0$, then to get to a distance d moving at velocity v would take time $t_0=d/v=1/H_0$. So the inverse of the Hubble constant is a (rough) measure of the age. This is called the Hubble time.

If H_0 = 72 (km/s)/Mpc, this is ~ 72 (pc/Myr)/Mpc, which is 72×10^{-6} Mpc/Myr/Mpc = 7.2×10^{-5} Myr^{-1}.
Then $t_0 = 1/H_0 = 1.39\times10^{4}$ Myr = 13.9 billion years.

There's a pretty big assumption built into this -- what is it?

We'll illustrate expansion dynamics using Newtonian dynamics. Happily, we will derive the same dynamical equations that come out of general relativity for a relativistic cosmology, with a few terms redefined.

Start with a test particle on the surface of an expanding sphere of radius R. Its equation of motion starts with F=ma and works out to be:

$$\ddot{R} = -\frac{4\pi}{3}G\rho R$$

Since density is proportional to R^{-3}, and we define "now" with a 0 subscript, and R_0=1, we have

$$\rho = \rho_0 R^{-3}$$

Which we can insert into the equation of motion to get

$$\ddot{R} = -\frac{4\pi}{3}\frac{G\rho_0}{R^2}$$

Note that if ρ_0 is nonzero, the Universe must be expanding or contracting. It cannot be static.

Equation of motion: $\ddot{R} = -\frac{4\pi}{3}\frac{G\rho_0}{R^2}$

Multiply both side by Ṙ to get $\dot{R}\ddot{R} + \frac{4\pi}{3}\frac{G\rho_0}{R^2}\dot{R} = 0$

And remember that $\frac{d(\dot{R^2})}{dt} = 2\dot{R}\ddot{R}$

$$\frac{1}{2}\frac{d(\dot{R^2})}{dt} + \frac{4\pi G\rho_0}{3}\frac{1}{R^2}\frac{dR}{dt} = 0$$

Now, another remembrance: $\frac{1}{R^2}\frac{dR}{dt} = -\frac{d(1/R)}{dt}$

$$\frac{d}{dt}\left[\dot{R}^2 - \frac{\left(8\pi G\rho_0/3\right)}{R}\right] = 0$$

$\dot{R}^2 - \frac{\left(8\pi G\rho_0/3\right)}{R} = -k$ where -k is a constant of integration.

Replacing ρ_0 with ρ*R^3, and then dividing by R^2, we finally get $\left(\frac{\dot{R}}{R}\right)^2 - \frac{8}{3}\pi G\rho = -\frac{k}{R^2}$

$$\left(\frac{\dot{R}}{R}\right)^2 - \frac{8}{3}\pi G\rho = -\frac{k}{R^2}$$

What does this mean?

If **k=0**, then $\dot{R}$ is always positive, and the expansion continues at an ever slowing pace (since rho is dropping). This is called a **critical or flat universe**.

If **k>0**, $\dot{R}$ is initially positive, but will reach a point where it goes to zero. Expansion stops, gravity wins, and the universe then starts to collapse. This is a **closed universe**.

If **k<0**, $\dot{R}$ is always positive, and never goes to zero -- expansion always continues. This is an **open universe**.