Dwarf Galaxies, Dark Matter, Dynamical Friction and Modified Gravity
(Is there really such a thing as dark matter?)

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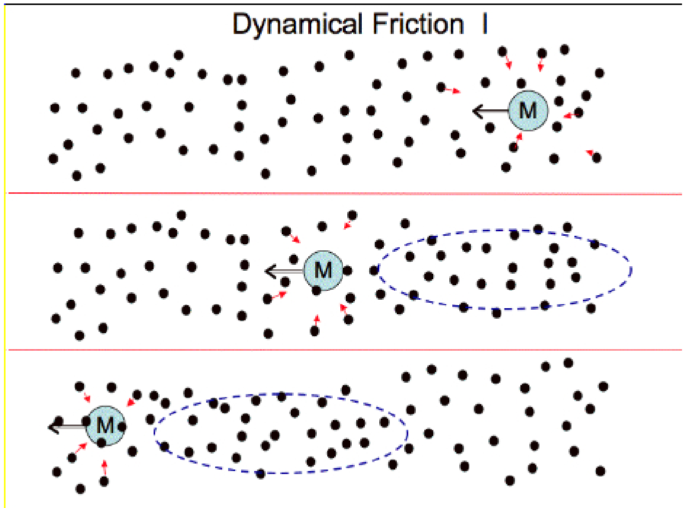
Dynamical Friction Intro....



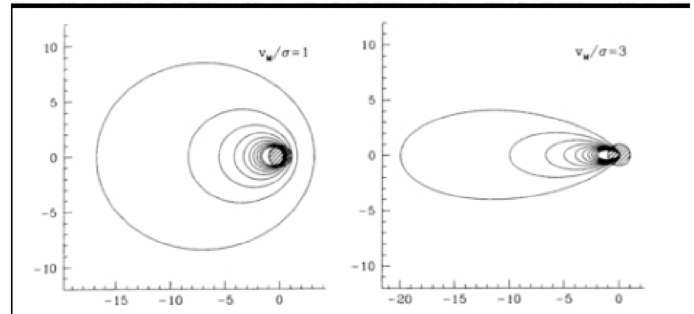


Figure 8.2 A mass M travels from left to right at speed v_M, through a homogeneous Maxwellian distribution of stars with one-dimensional dispersion σ. Deflection of the stars by the mass enhances the stellar density downstream, and the gravitational attraction of this wake on M leads to dynamical friction. The contours show lines of equal stellar density in a plane containing the mass M and the velocity vector $\mathbf{v}_M$; the velocities are $v_M = \sigma$ (left panel) and $v_M = 3\sigma$ (right panel). The fractional overdensities shown are $0.1, 0.2, \ldots, 0.9, 1$. The unit of length is chosen so that $GM/\sigma^2 = 1$. The shaded circle has unit radius and is centered at M. The overdensities are computed using equation (8.148), which is based on linear response theory; for a nonlinear treatment see Mulder (1983).

For a perturber of mass M moving at velocity V through a halo of much smaller particles of density ρ and Maxwellian isotropic velocity dispersion σ,

$$\boxed{F_{DF} = -4\pi ln\Lambda G^2 \rho M^2 \frac{A(X)}{V^2}}.$$

Where $\Lambda = b_{max} V^2/(GM)$ and $A(X) = erf(X) - 2Xe^{-X^2}/\pi^{1/2}$, where the variable $X = V/(\sqrt{2}\sigma)$. For stars moving at the circular equilibrium velocity of an isothermal dark matter halo, $X = 1$, $A(X) = 0.428$ and $\rho = V^2/(4\pi G R^2)$, R the orbital radius of the star in question, yielding:

$$F_{DF} = -0.428 ln\Lambda \frac{GM^2}{R^2}.$$

Dynamical Friction Intro....

For M on a circular orbit the force is tangential and the torque is given by:

$$\frac{dL}{dt} = \frac{F_{DF}R}{M} = -0.428 ln\Lambda \frac{GM}{R}$$

Assuming circular orbits, tight spirals, $L = RV$ yields:

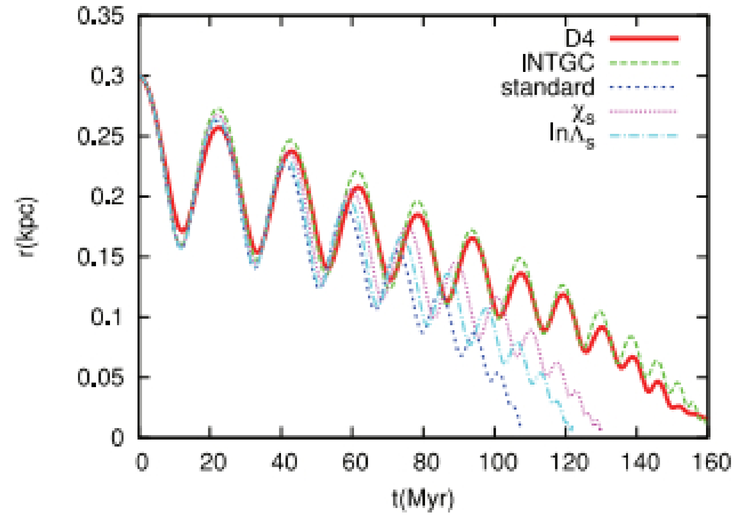
$$V\frac{dR}{dt} = -0.428 ln\Lambda \frac{GM}{R}$$

and hence,

$$\int_{R_i}^{0} RdR = 0.428 \frac{ln\Lambda GM}{V} \int_{\tau_{DF}}^{0} dt$$

$$\boxed{\tau_{DF} = \frac{1.17 R_i^2 V}{ln\Lambda GM}}$$

Dynamical Friction Intro....



For $ln\Lambda = 10$ and going to Astronomical units,

$$\tau_{DF} = 6.6 \times 10^6 \left(\frac{R}{kpc}\right)^2 \left(\frac{V}{250km/s}\right) \left(\frac{M_\odot}{M}\right) Gyr$$

Which is interesting for $10^6 M_\odot$ and above objects moving in the halo of MW type galaxies, but irrelevant for stars! But notice R^2V dependence...

Taking $ln\Lambda = 15$ and changing units....

$$\boxed{\tau_{DF} = 1.7 \left(\frac{R}{1kpc}\right)^2 \left(\frac{V}{km/s}\right) \left(\frac{10^4 M_\odot}{M}\right) Gyr}$$

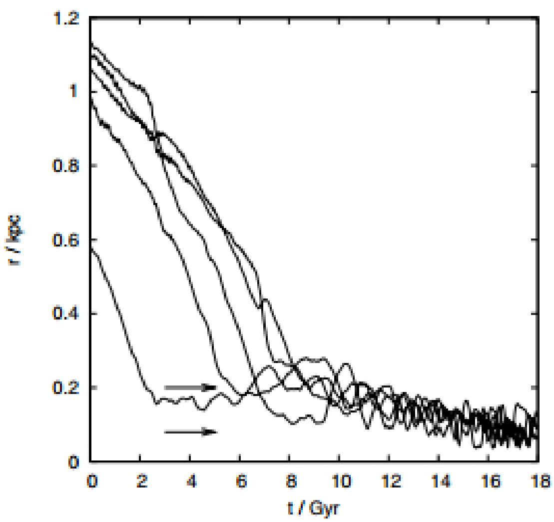
Which removes GCs in dSphs !

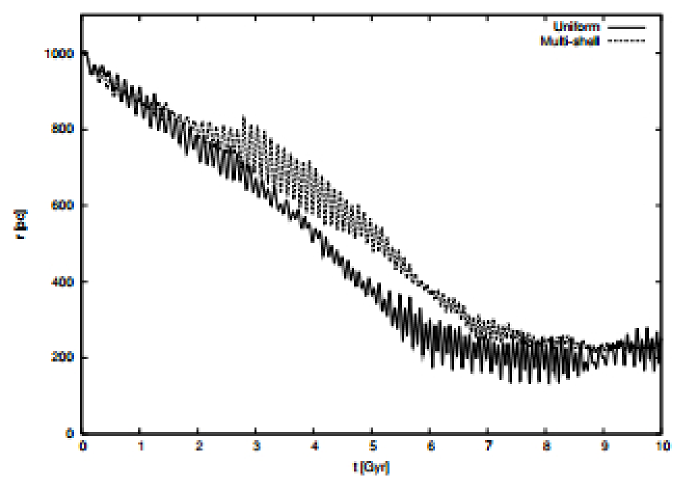
Dynamical Friction Intro....

...unless you change the isothermal (or cuspy) profile for a cored one....

where you change from $\tau_{DFiso} = 1.7 \left(\frac{R}{1kpc}\right)^2 \left(\frac{V}{km/s}\right) \left(\frac{10^4 M_\odot}{M}\right) Gyr$ to an exponential decay with timescale:

$$\tau_{DFcore} = \frac{(V_c/kms^{-1})(r_c/kpc)^2}{3(M/10^5 M_\odot) ln\Lambda} Gyr$$





...as confirmed numerically by e.g. Goerdt et al. (2006) and S. Inoue (2009)

X. Hernandez & G. Gilmore (1998) MNRAS 297, 517 , Sanchez-Salcedo et al. (2006) MNRAS 370, 1829.

Decay of Wide Binaries in dSphs

Not only will the orbit of the binary about the galaxy decay, but in the presence of dark matter, each component of the binary itself will be subject to dynamical friction, leading to a tightening of the binary.

We can approach the problem by noting that the presence of each component of the binary will result in a local enhancement of dark matter.

As these enhancements are turning, the dark matter distribution acquires angular momentum, which must necessarily come from the tightening of the binary.

We begin by considering a single component of the binary, at rest with respect to a locally constant dark matter distribution of density ρ_0 and Gaussian velocity dispersion σ.

The presence of the star will result in a local perturbation to the dark matter distribution function $f(r, v) = f_0(v) + \epsilon f_1(r, v)$.

Decay of Wide Binaries in dSphs

The full distribution function will satisfy the Boltzmann equation:

$$\frac{\partial f}{\partial t} + v \cdot \vec{\nabla} f - \vec{\nabla}\Phi \cdot \vec{\nabla}_v f = 0.$$

Forcing a stationary solution and taking $\Phi_0 = 0$, $\Phi_1(r) = \frac{-GM}{r}$, we obtain to first order for the radial component:

$$v \frac{\partial f_1}{\partial r} = \frac{GM}{r^2} \frac{\partial f_0}{\partial v} = \frac{-v}{\sigma^2} \frac{GM}{r^2} f_0,$$

where we used $f_0(v) \propto \exp(-v^2/2\sigma^2)$, and which we can integrate over velocity space to yield:

$$\frac{d\rho_1}{dr} = \frac{-GM}{(\sigma r)^2} \rho_0.$$

The above equation yields the density perturbation induced upon the dark matter distribution by the single star at rest as:

$$\boxed{\rho_1(r) = \frac{GM}{r\sigma^2} \rho_0}.$$

Decay of Wide Binaries in dSphs

We can think of the response of the dark matter halo to the presence of a wide binary, where the orbital velocity is lower than σ, as being composed of two such enhancements, one centred upon each star.

As these have to be constantly reformed, we can calculate the angular momentum loss for each star in the binary star as:

$$\dot{L} = \frac{M_e V_o R_o}{\tau}, \qquad M_e = 2\pi \frac{GM}{\sigma^2} R_0^2 \rho_0,$$

where M_e is the mass of each enhancement, obtained by integrating ρ_1 out to R_o, and V_o and R_o are the binary orbital velocity and orbital radius, with $\tau = \alpha/\Omega$ a characteristic timescale over which the density enhancement is being replenished, with α a dimensionless constant expected to be of order unity.

The rate of loss of angular momentum for each component of the binary becomes:

$$\dot{L} = \left(\frac{4\pi}{\alpha}\right) \left(\frac{GM}{\sigma}\right)^2 \rho_0 R_0.$$

Decay of Wide Binaries in dSphs

Assuming that the decay of the binary describes a tight spiral, we take the orbit as circular throughout the evolution. This allows to obtain the temporal evolution of the binary radius by deriving the angular momentum of each star in the binary $L_b = (2GM^3R_0)^{1/2}$ with respect to time, and equating it to the previous equation giving:

$$\dot{R}_0 = \left(\frac{2^{5/2}\pi}{\alpha}\right)\left(\frac{G^{3/2}M^{1/2}\rho_0}{\sigma^2}\right)R_0^{3/2},$$

yielding:

$$\tau_{1/2} = \left(\frac{\alpha}{2^{3/2}\pi}\right)\frac{\sigma^2}{\rho_0 M^{1/2}G^{3/2}R_0}.$$

Where R_0 is the original binary orbital radius and we have introduced $\tau_{1/2}$ as the time for R_0 to go down by a factor of two, which for $1M_\odot$ stars and $\alpha = 1$ gives in astronomical units:

$$\boxed{\frac{\tau_{1/2}}{10Gyr} = \frac{(\sigma/5kms^{-1})^2}{(\rho/M_\odot pc^{-3})(R_0/pc)^{1/2}}}$$

Decay of Wide Binaries in dSphs

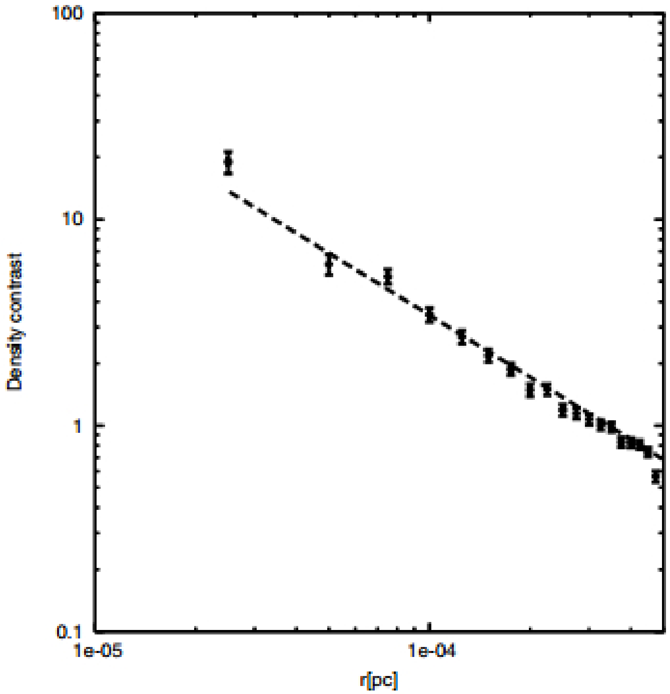


Figure 1. Dark matter density profile around a single star as a function of distance from it, after a steady state has been achieved, with error bars showing sampling errors. The straight line is the analytical prediction of equation (5).

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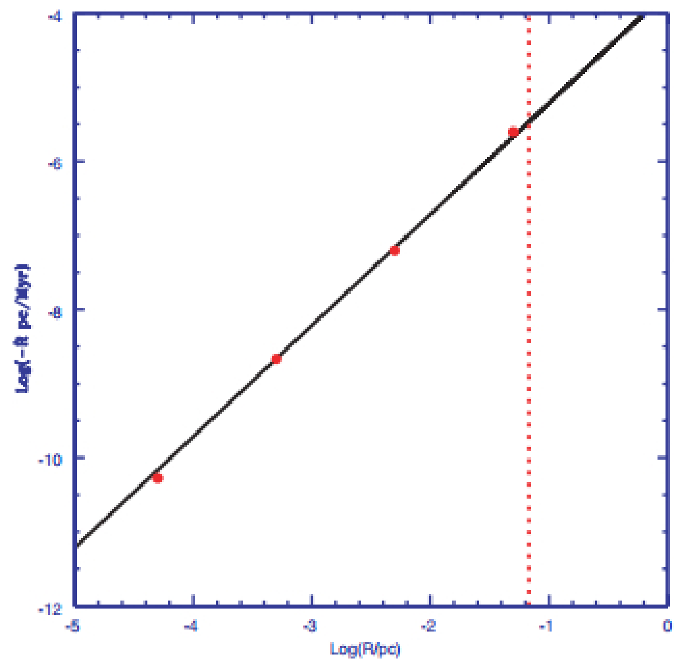
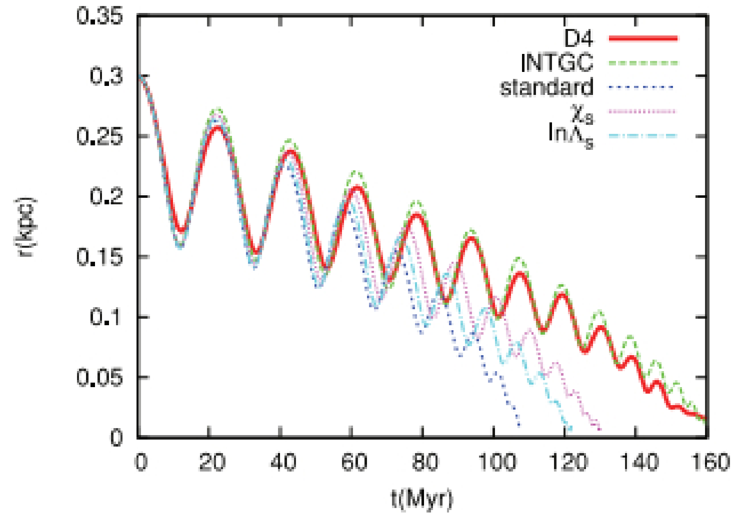


The previous dark matter density enhancement and resulting decay rate were confirmed numerically for $\boxed{\alpha = 1.07}$

Binaries wider than 1pc should not exist in dSph dark halos, cored or not!

X. Hernandez et al. (2008) MNRAS 387, 1727

Dynamical Shrivelling of Ultra Faint dSphs



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Taking $ln\Lambda = 15$ and changing units....

$$\boxed{\tau_{DF} = 1.7 \left(\frac{R}{10pc}\right)^2 \left(\frac{V}{km/s}\right) \left(\frac{M_\odot}{M}\right) Gyr}$$

Which suggests τ_{DF} for ultra faint dSphs might get interesting...

Dynamical Friction in Pressure Supported Systems

The loss of potential energy for the star in moving through a distance dx is now $dw = F_{DF} dx$. Dividing by dt we obtain,

$$\frac{dw}{dt} = F_{DF}\frac{dx}{dt} = VF_{DF} = -V0.428 ln\Lambda \frac{GM^2}{R^2}.$$

Assuming still circular orbits in an isothermal halo characterised by a logarithmic potential, the loss of potential energy for the star when its orbital radius changes from $R+dR$ to R will be given by $dw = MV^2 ln(1+dR/R)$, which for tightly wound orbits and $dR << R$ reduces to $dw = MV^2 dR/R$ and hence,

$$\frac{dw}{dR} = \frac{MV^2}{R}.$$

Since $dR/dt = (dR/dw)(dw/dt)$, we can now write the evolution equation for the orbital radius as:

$$\boxed{\frac{dR}{dt} = -0.428 ln\Lambda \frac{GM}{RV}}.$$

The above is exactly the same evolution equation which results from starting from tracing the loss of angular momentum for the star assuming slowly inspiraling circular equilibrium orbits, and validates the approach introduced of tracing rather, the evolution of the potential energy of the star being followed.

Dynamical Friction in Pressure Supported Systems

If we now return to stars supported by velocity dispersion in dSphs, changing V for σ_*, and assuming initially that the velocity dispersion of the stars equals that of the dark matter particles, $X = 1/\sqrt{2}$, and $A(X) = 0.2$. Going back to Chandrasekhar's, the dynamical frictional force on the sample star is now:

$$F_{DF} = -2.5 ln \Lambda \rho \left(\frac{GM}{\sigma_*} \right)^2 .$$

As previously, $dw/dt = \sigma_* F_{DF}$, and hence the total rate of loss of potential energy for all the N stars in the dSph galaxy will be:

$$\frac{dW}{dt} = -2.5 \frac{ln \Lambda N \rho (GM)^2}{\sigma_*}$$

For a star of mass M at a radial distance R within the constant density core of a dark matter halo of density ρ, as inferred in general for dSphs (e.g. Goerdt et al. 2006), in moving a radial distance dR , the change in potential energy is given by:

$$\boxed{\frac{dW}{dR} = -\frac{4\pi}{3} GM\rho R.}$$

Dynamical Friction in Pressure Supported Systems

Evaluating the above at the half-light radius, r_h, and using $dW/dt = (dW/dr_h)(dr_h/dt)$ allows us to write for the full N stars:

$$\frac{dW}{dt} = 4.2 G M N \rho r_h \frac{dr_h}{dt},$$

from which we can solve for the decay rate of the half light radius,

$$\boxed{\frac{dr_h}{dt} = -0.6 \frac{ln\Lambda G M}{\sigma_* r_h}}$$

It is reassuring of the development presented that the above equation agrees with the classical expression for the decay rate of the orbital radius of a particle inspiraling within a dark matter halo along quasi-circular orbits, exactly in all the physical dependencies, and to within a factor of order unity in the numerical coefficient, a minor difference due to the slightly distinct physical conditions of the pressure supported system being treated here.

Dynamical Friction in Pressure Supported Systems

Integrating the above equation and defining τ_{DF} as the time taken for the stellar half-light radius to be reduced by a factor of two gives:

$$\tau_{DF} = 0.63 \frac{\sigma_* r_h^2}{ln\Lambda GM},$$

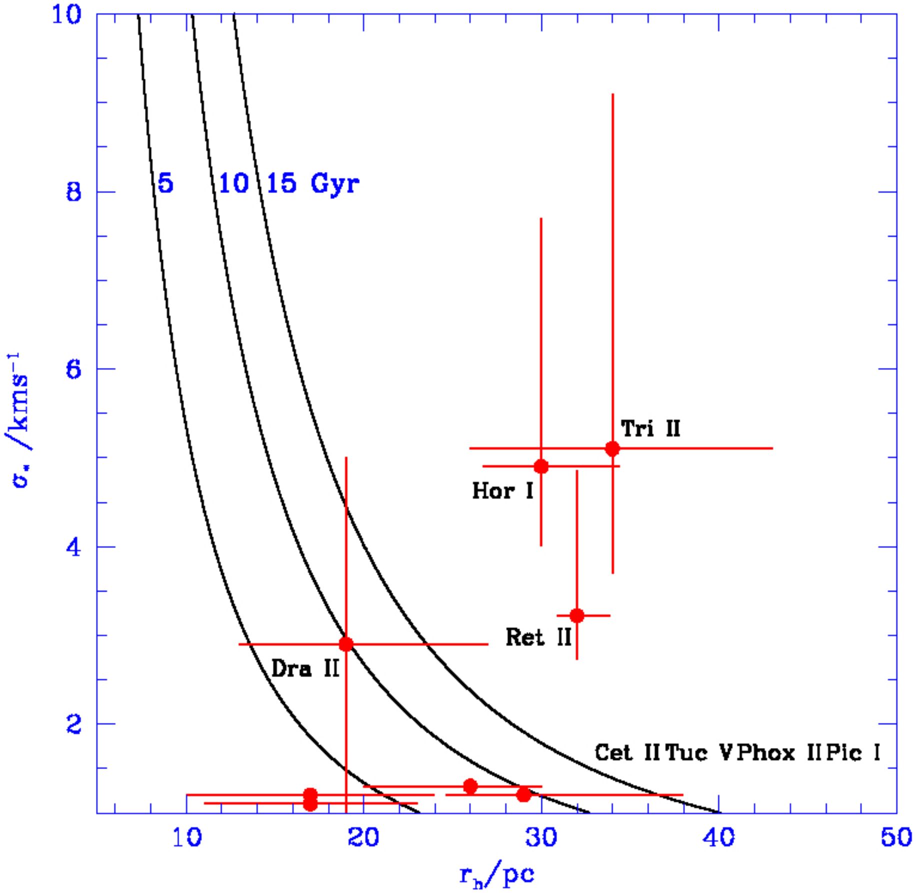
which in astronomical units for $1M_\odot$ stars, yields:

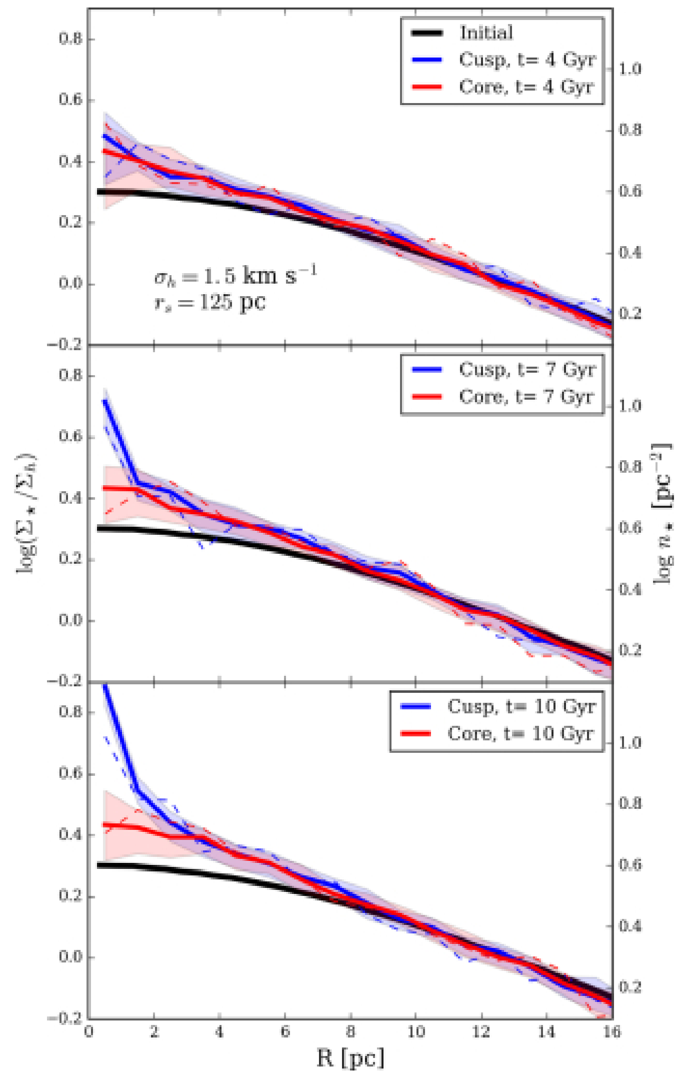
$$\tau_{DF} = \frac{0.14}{ln\Lambda} (\sigma_*/kms^{-1})(r_h/pc)^2 Gyr.$$

Again, the same physical scalings and order of magnitude than what results from following the loss angular momentum for a particle in a circular orbit within an isothermal halo. For a value of $ln\Lambda = 15$, corresponding to typical values for ultra faint dSphs, the above equation yields:

$$\boxed{\tau_{DF} = 0.93(r_h/10pc)^2(\sigma/kms^{-1})Gyr.}$$

Are observed ultra faint dSphs consistent? $\boxed{\tau_{DF} = 0.93(r_h/10pc)^2(\sigma/kms^{-1})Gyr}$





...interesting as the ages of the systems are $\geq 10Gyr$

As recently confirmed for cuspy ΛCDM dark matter halos by S. Inoue (2017)

Having a Cored DM halo in the ultra faint dSphs becomes $\boxed{\text{Crucial!}}$

X. Hernandez (2016) MNRAS 462, 2734, S. Inoue (2017) MNRAS, 467, 4491

Conclusions

Dynamical friction results in a consistency check on any dark matter explanations for the observed kinematics of dwarf galaxies, providing independent constraints on survival of GCs, wide binaries, and at the ultra faint end, survival of the galaxies themselves.

The various GC systems of local dSphs imply that only $\boxed{\text{cored dark matter halos}}$ are consistent with observations, a density profile which can be hard to explain at the smallest galactic scales in terms of originally cuspy halos and feedback scenarios. (e.g. A. Di Cintio)

The existence of binary stars with separations larger than $\boxed{\text{1pc}}$ is inconsistent with the presence of a dark matter halo in dSphs. Are there any?
$\tau_{DF} = (\sigma/5kms^{-1})^2(\rho/M_\odot pc^{-3})^{-1}(R_0/pc)^{-1/2}10Gyr.$

Dark matter dominated dSphs will be subject to dynamical friction on their individual stars, over a timescale of $\tau_{DF} = 0.14(\sigma_*/kms^{-1})(r_h/pc)^2(ln\Lambda)^{-1}Gyr$. This dynamical friction shrivelling will become relevant for sizes of the order of the smallest recently detected ultra faint dSphs, with a theoretical lower stability limit of $\boxed{\approx 19pc}$. Any future detections of even slightly smaller ultra faint dSphs would be incompatible with a particle dark matter hypothesis and cuspy dark matter halos.

Under an alternative MONDian gravity scenario, no such consistency lower limits appear, as in the absence of dark matter, no dynamical friction ensues. Also, observed velocity dispersion values for ultra faint dSphs $\boxed{\text{agree with MOND}}$ predictions of equilibrium velocities, for the measured stellar content of the systems in question.

What happens after τ_{DF}?

After $3\tau_{DF}$, the dark matter mass within the typical stellar radius will have decreased by a factor of $8^3 = 512$, bringing the mass to light ratio to within stellar values, i.e., the structure will no longer be dark matter dominated, and will appear more like a globular cluster than a dSph. The stellar potential energy will then be given by:

$$W = -0.4 \frac{G(MN)^2}{r_h},$$

e.g. Binney & Tremaine eq. (4-80b). Differentiation w.r.t. r_h of the above, and proceeding as in the previous dark matter dominated case, yields:

$$\frac{dr_h}{dt} = -6.25 \frac{ln\Lambda G\rho r_h^2}{N\sigma_*}$$

as the corresponding equation. Similarly, integrating and defining again τ_{DF*} as the time required for r_h to go down by a factor of two, leads to:

$$\boxed{\tau_{DF*} = \frac{0.16N\sigma_*}{ln\Lambda G\rho r_h}}$$

$\boxed{\text{What happens after } \tau_{DF}\text{?}}$

Which in astrophysical units reads:

$$\boxed{\tau_{DF*} = \frac{40N_3}{ln\Lambda}(\sigma_*/kms^{-1})(\rho/M_\odot pc^{-3})^{-1}(r_h/pc)^{-1}Gyr}$$

where N_3 is the total number of stars in units of thousands. Again, for $ln\Lambda = 15$ we obtain, $\tau_{DF*} = 0.27N_3(r_h/10pc)^{-1}(\rho/M_\odot pc^{-3})^{-1}(\sigma_*/kms^{=1})$. By comparing to the corresponding expression for the dark matter dominated phase, we see a similar scale for $N_3 = 1$ and $\rho = 1M_\odot pc^{-3}$, parameters typical for ultra faint dSphs.

The change in the potential energy from dark matter dominated to a self gravitating population of stars, changes the radial dependence on the DF timescales from R^2 to R^{-1}, such that the process is now self-limiting and the evolution timescales now grow as the radius is reduced. As r_h goes down, the process stops and a tight stellar cluster, rather than a dark matter dominated dSph results.

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When the potential of the stars themselves begins to dominate over that of the dark matter, the evolution reaches a phase described by the above equations. Ultra faint dSphs with τ_{DF} shorter than their ages will be $\boxed{\text{unstable structures}}$ under a particle dark matter hypothesis.