

Dwarf Galaxy Dispersion Profile Calculations Using a Simplified MOND External Field Effect

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Outline:

- Review our model of dSph's in the isolated MOND case
- Review our results for dispersion profiles for MW dSph's in the isolated MOND case
- Extensions of our model to include the External Field Effect
- Some results that we have generated in the last few months.

Introductions: How did I get here?

Disclaimer: *I am not now -- nor have I ever been -- a galactic astronomer.*

Until about 4 yrs, ago I was a planetary dynamics guy,
then these two undergrads showed up in my office:

Ben Amend
(BS)



**Josh
Schussler**
(MS)



**Tristan
Clark**
(BS `14)



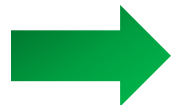
**Mike
Zito**
(BS)



**Ben
Blankartz**
(BS `15 MS `17)



**Matt
Walentosky**
(MS `16)



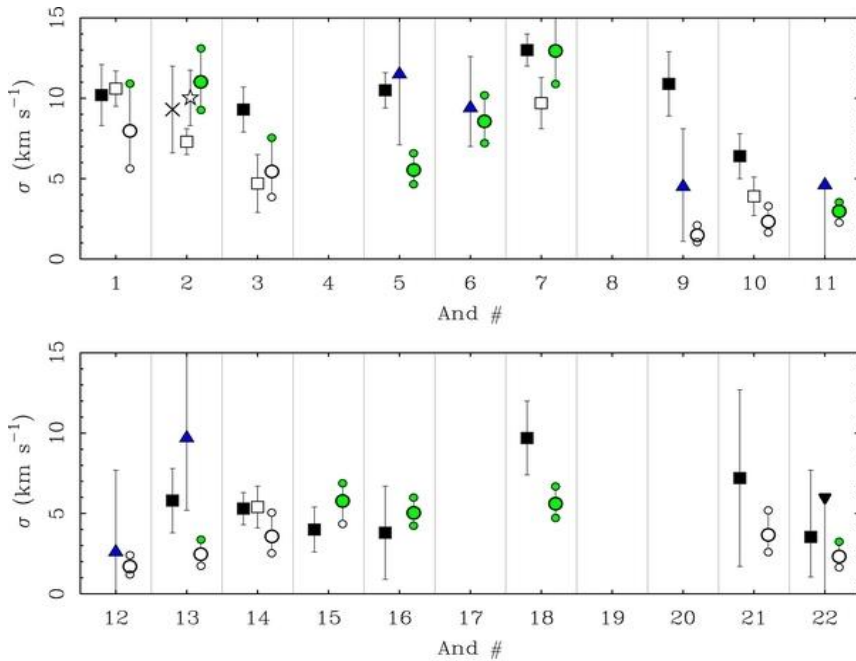
**Alex
Staron**
(BS `17)



**Justin
Messinger**
(BS `17)



What Did We Want To Try To Do?

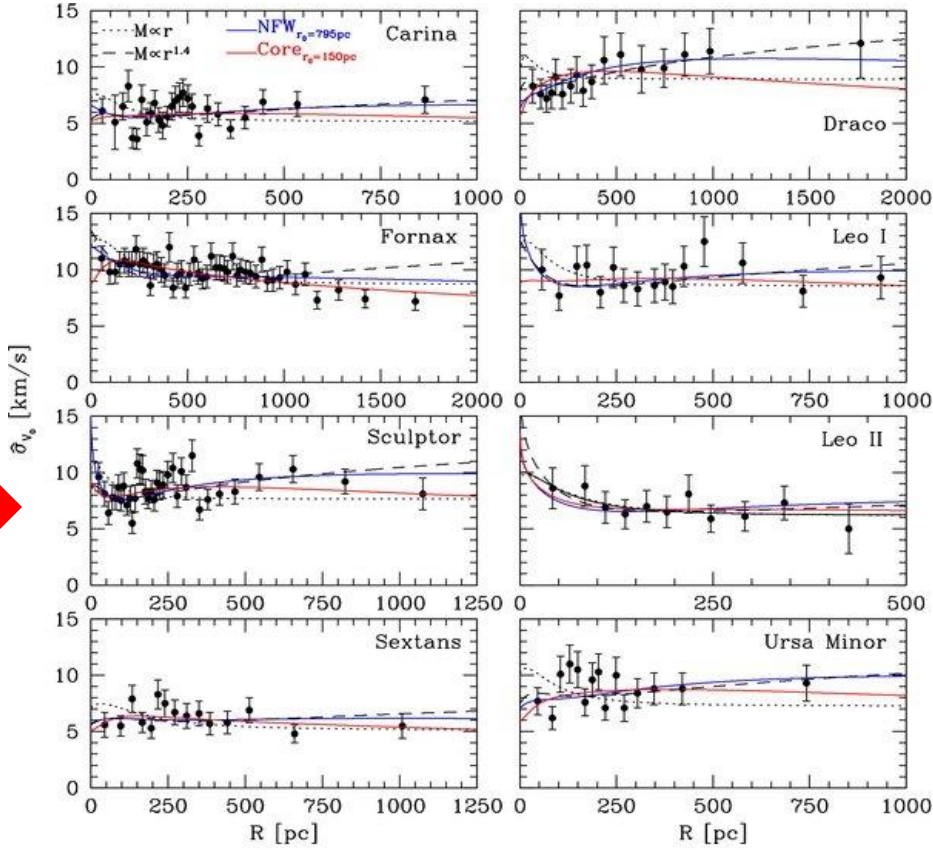


We looked at the bulk dispersion calculations for Andromeda dSph's in McGaugh & Milgrom (2013).

A former student of mine, Louis Strigari, recommended that we take a look at the observed dispersion profiles of MW dSph's in Walker et al. (2009).



We thought: *Can we put together a model that incorporates MOND that can calculate these things?*



Our Model of a dSph

(keep it as simple as possible)*

- Model consists of a spherically symmetric gravitating baryon cloud of mass M with half-mass radius $r_{1/2}$ and 10,000 stars that feel only the gravity of the cloud.

- The distribution of mass in the cloud is given by a Hernquist profile:

$$\rho(r) = \frac{M a_s}{2\pi r} \frac{1}{(r + a_s)^3} ; a_s = \text{scalelength}$$
$$r_{1/2} = (1 + \sqrt{2})a_s$$

- This gives the interior mass for a star at r :

$$M_{\text{int}}^h(r) = M \frac{r^2}{(r + a_s)^2}$$

- To prevent unphysical accelerations for $r \rightarrow 0$, we impose a constant density core with a radius that is some fraction of the scalelength, $r_c = \alpha a_s$. Matching the densities at $r = r_c$ gives the core mass and core interior mass:

$$M^c = \frac{2\alpha^2}{3(\alpha + 1)^3} M \quad M_{\text{int}}^c = \frac{M^c}{r_c^3} r^3 \quad \text{For } \alpha = 0.1, \text{ this gives } M^c \sim 0.005M$$

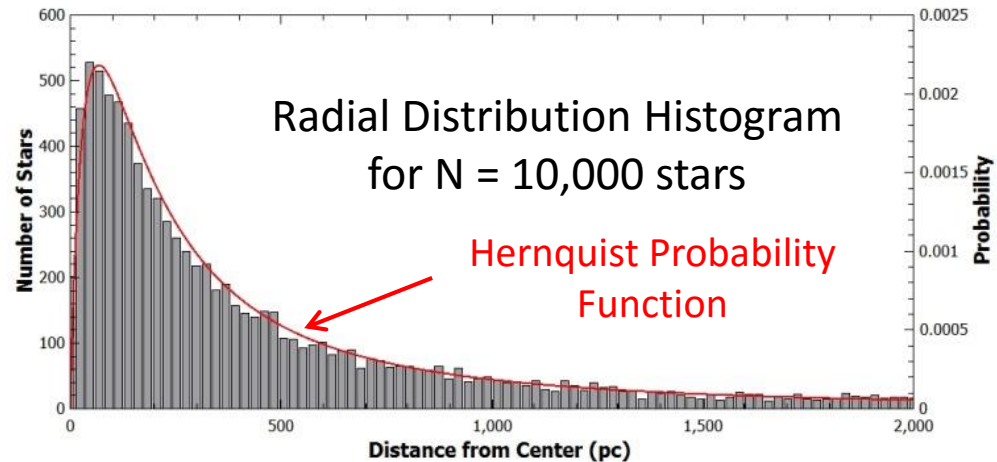


* *“For most problems: you can get 90% of the answer with 10% of the effort”*

Dr. Thomas M. York, Penn State (circa 1975)

Our Model of a dSph: Initial Conditions

- The **initial radial coordinate** of each star is chosen consistent with the Hernquist probability function. The **angular coordinates** are assigned randomly to give spherical symmetry.



- During a simulation, $r_{1/2}$ can be calculated to adjust the scalelength, or the baryon distribution can be held constant.
- For **initial velocities**, we have tried three different scenarios:
 1. **Circular** with orbital planes randomly oriented in three dimensions
 2. **Radial** (i.e. back and forth through the origin) with random initial speeds
 3. **Randomized**, i.e. each velocity component is randomly assigned between $\pm V_c$We chose #3 as it reproduces a pure randomized pressure supported system.
- Since each star only interacts with the baryon cloud, the choice of **stellar mass** is not important; however, we can select a distribution of masses consistent with an empirical Initial Mass Function.

Our Model of a dSph: Integration Technique

- We have many years experience working with the **Hermite Individual Timestep Scheme (HITS)** that was developed by Aarseth and Makino (1992).
- In HITS, each body has its own time and timestep so that objects with large accelerations can be integrated with small timesteps, while objects with small accelerations can use larger timesteps. We have used this feature to study motion in planetary systems that include both satellites and rotation.
- HITS requires analytical expressions for the acceleration and its time derivative for each body. For the spherically symmetric baryon cloud and Newtonian gravity:

$$\mathbf{g}_n = -\frac{GM_{\text{int}}}{r^3}\mathbf{r}$$

Hernquist:

$$\dot{\mathbf{g}}_n = -\frac{GM_{\text{int}}^h}{r^3} \left\{ \mathbf{v} - \left[3 - \frac{2a_s}{(r + a_s)} \right] \frac{(\mathbf{r} \cdot \mathbf{v})\mathbf{r}}{r^2} \right\}$$

Core:

$$\dot{\mathbf{g}}_n = -\frac{GM_{\text{int}}^c}{r^3}\mathbf{v}$$

- HITS then integrates in time to obtain positions to 5th order and velocities to 4th order

Our Model of a dSph: Including MOND

(keep it as simple as possible)

- In its original form, Milgrom's MOND can be considered as a modification of Newtonian gravity of the form:

$$\mathbf{a} \mu \left(\frac{a}{a_0} \right) = \mathbf{g}_n$$

where \mathbf{a} is the actual acceleration vector, \mathbf{g}_n is the Newtonian, a_0 is the characteristic MOND acceleration (we use $a_0 = 1.2 \times 10^{-10} \text{ m/s}^2$), and μ is a matching function that gives the correct limiting values.

- Remembering to *keep it as simple as possible*, for the matching function, we chose the simple function:

$$\mu(x) = \frac{x}{1 + x}$$

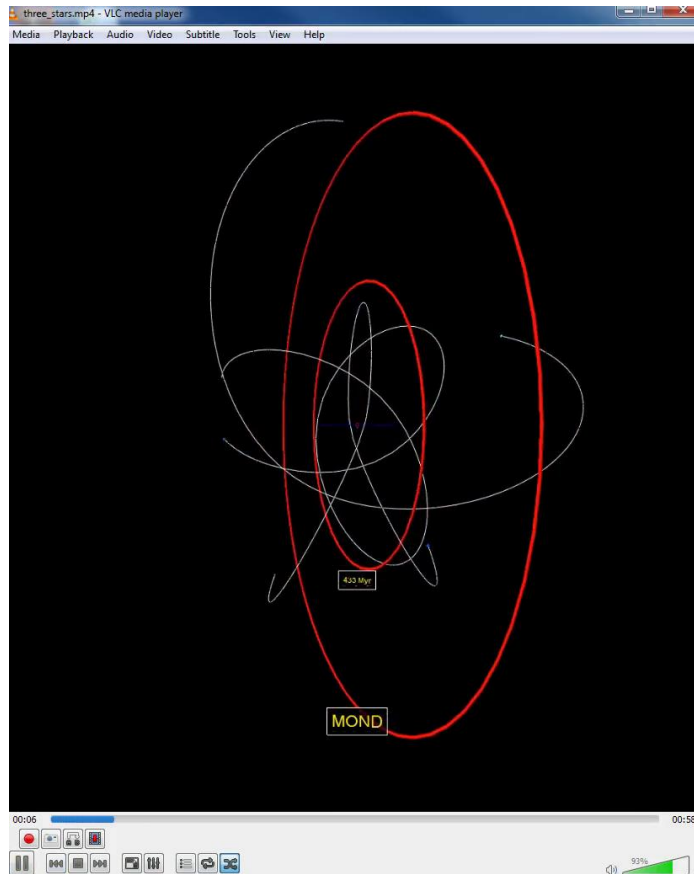
- This gives the MOND acceleration and its time derivative:

$$\mathbf{a} = \frac{\mathbf{g}_n}{2} \left[1 + \sqrt{1 + 4 \left(\frac{a_0}{g_n} \right)} \right]$$
$$\dot{\mathbf{a}} = \frac{1}{2} \left\{ \dot{\mathbf{g}}_n \left[1 + \sqrt{1 + 4 \left(\frac{a_0}{g_n} \right)} \right] - \frac{2\mathbf{g}_n}{\sqrt{1 + 4 \left(\frac{a_0}{g_n} \right)}} \left(\frac{\dot{\mathbf{g}}_n \cdot \mathbf{g}_n}{g_n^2} \right) \left(\frac{a_0}{g_n} \right) \right\}$$

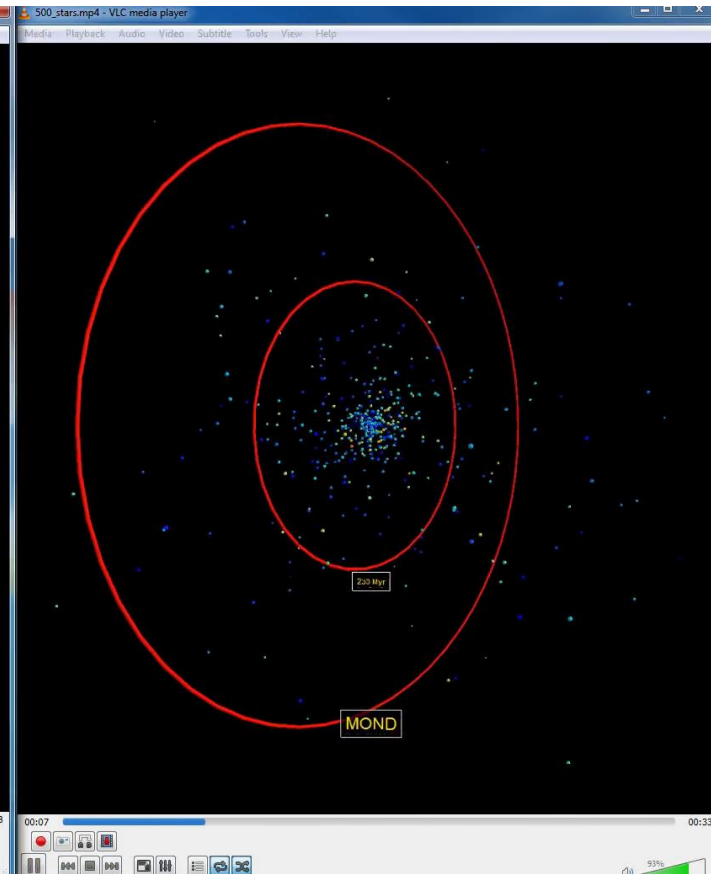
- These can be fed into HITs to solve for the motion of the stars.

Animations of the Motion of Stars in our Model

Three Stars with traced orbits



500 Stars



Red circles are 2 kpc and 4 kpc in diameter; Clock reads in My's, but it's hard to see

Our Model of a dSph: Calculating Dispersions

- Each simulation generates snapshots of position/velocity data for all stars at selected time intervals.
- At each time, we calculate the **bulk line-of-sight velocity dispersion** as the standard deviation of one the velocity components:

$$\sigma_x = \sqrt{\langle v_x^2 \rangle - \langle v_x \rangle^2}$$

This generates the dispersion as a function of time, so the final step is a time average.

- A **dispersion profile** is generated at each snapshot by putting an equal number of stars into concentric annular bins and calculating the l.o.s dispersion for that bin.
- For a simulation with 10,000 stars, we use 100 bins, so we have ~ 100 stars/bin.
- To obtain the final dispersion profile, we take a time average over the time-dependent profiles.

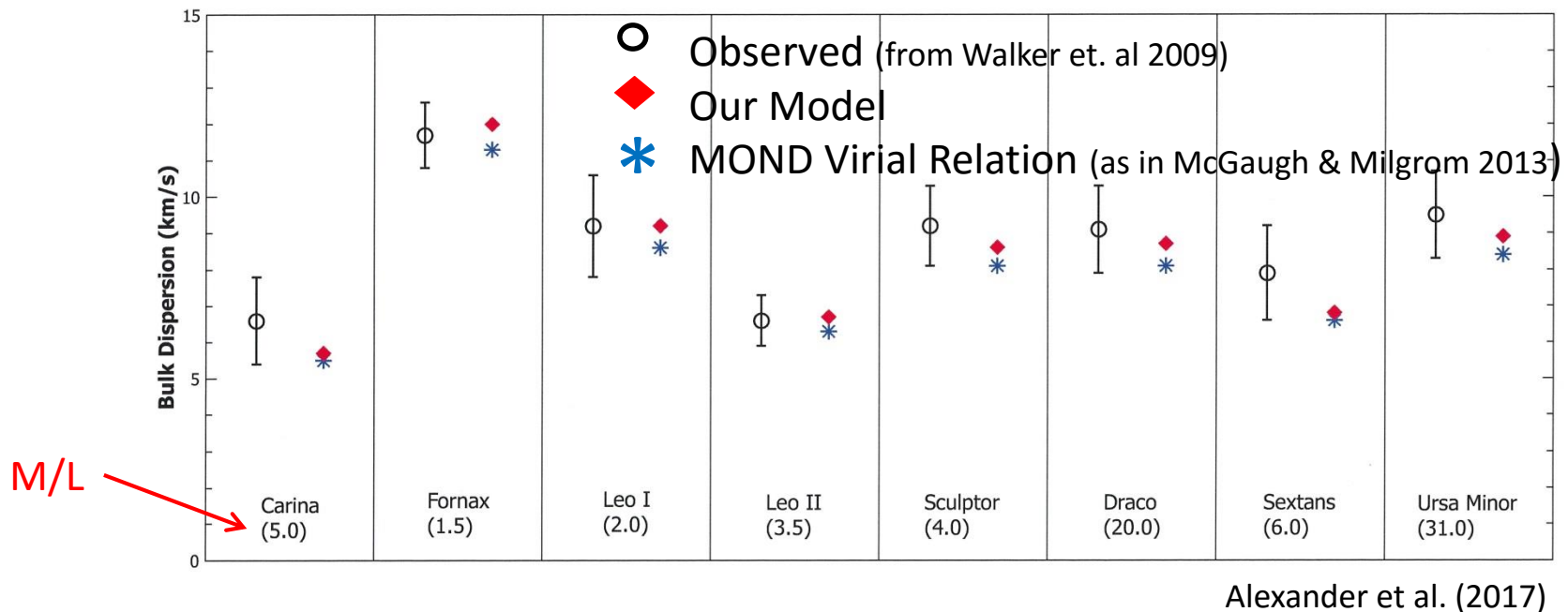


Simulations of MW dSph's: Bulk Dispersions

- We ran our simulations in the MOND case for the 8 MW dSph's shown here.
- We varied the mass to light ratio, M/L, until our calculated bulk dispersion was within the error bars of the observations.
- We get reasonable M/L values for all but two of the galaxies.

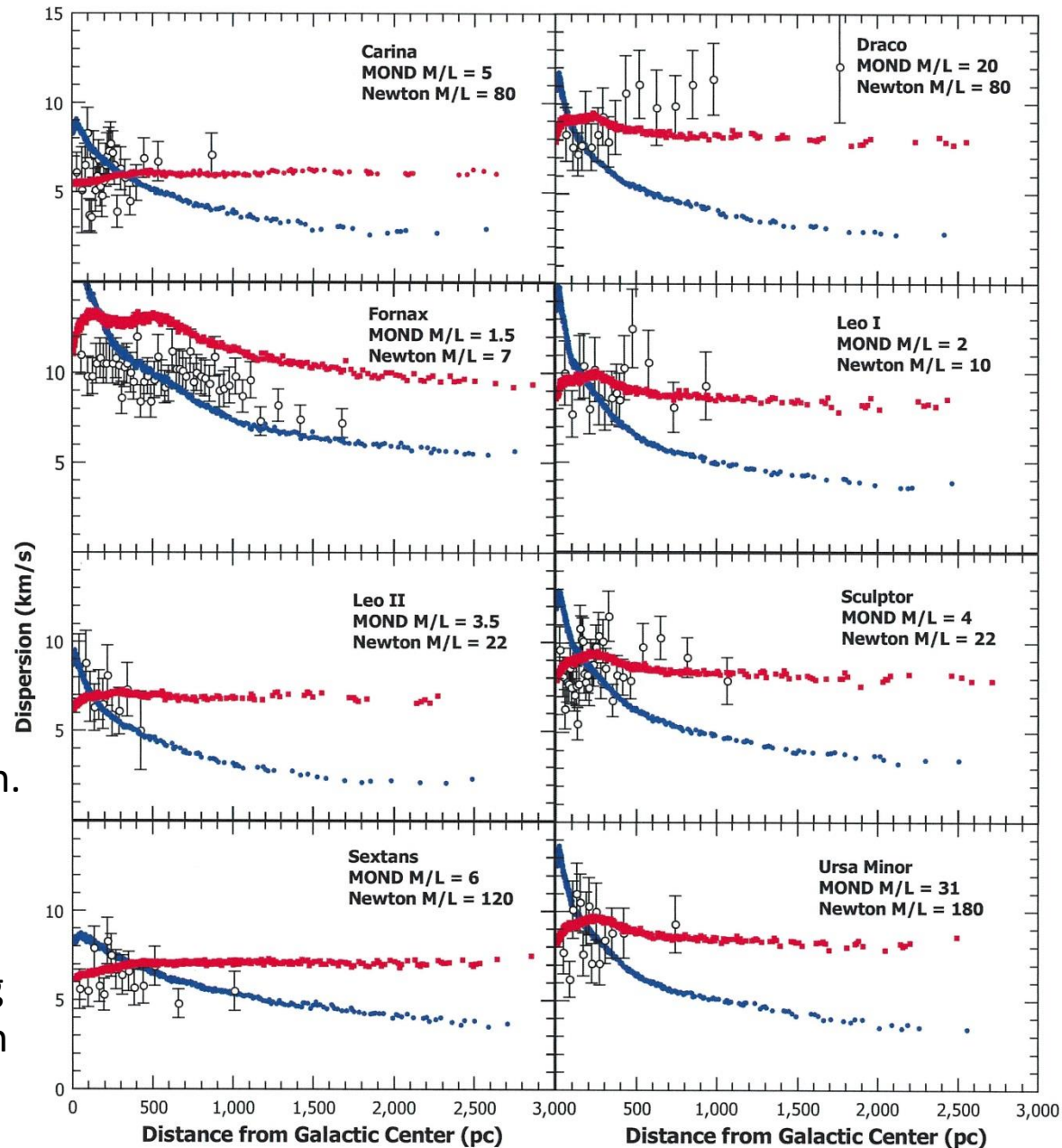
Table 1. Observed Properties of MW dSph's. Milky Way distances are from [McConnachie \(2012\)](#); 3D half-mass radii and luminosities are from [Walker et al. \(2010\)](#).

Galaxy	r_{MW} (kpc)	3D $r_{1/2}$ (pc)	L_V (L_\odot)	σ_{obs} ($km\ s^{-1}$)
Carina	107	321 ± 29	$2.4 \pm 1.0 \times 10^5$	6.6 ± 1.2
Fornax	149	891 ± 45	$1.4 \pm 0.4 \times 10^7$	11.7 ± 0.9
Leo I	258	328 ± 25	$3.4 \pm 1.1 \times 10^6$	9.2 ± 1.4
Leo II	236	201 ± 23	$5.9 \pm 1.8 \times 10^5$	6.6 ± 0.7
Sculptor	86	347 ± 52	$1.4 \pm 0.6 \times 10^6$	9.2 ± 1.1
Draco	76	261 ± 16	$2.7 \pm 0.4 \times 10^5$	9.1 ± 1.2
Sextans	89	909 ± 23	$4.1 \pm 1.9 \times 10^5$	7.9 ± 1.3
Ursa Minor	78	373 ± 20	$2.0 \pm 0.9 \times 10^5$	9.5 ± 1.2



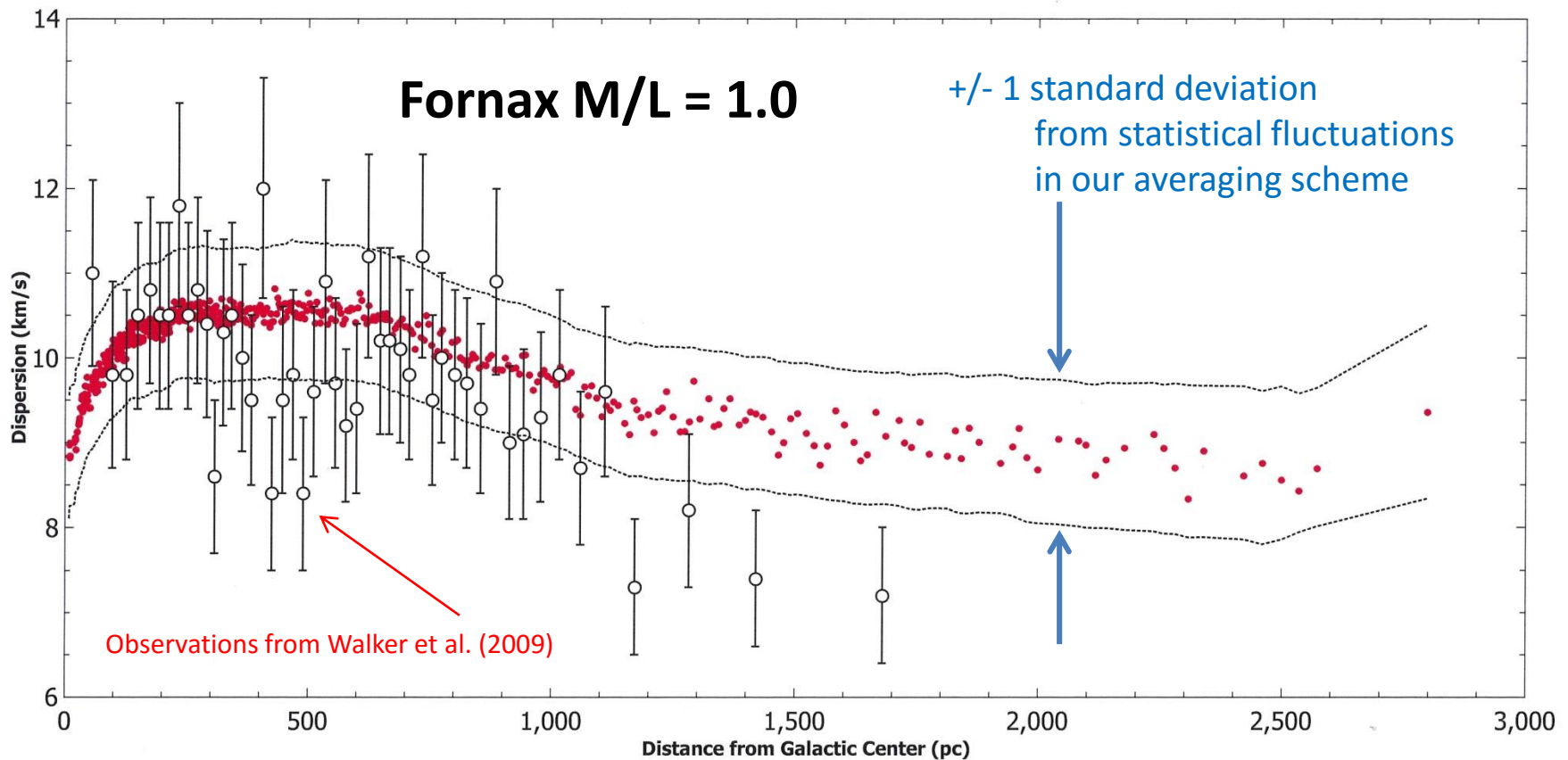
Simulations of MW dSph's: Dispersion Profiles

- Using the value of M/L that gives the best match to the observed bulk dispersion, we then calculate a dispersion profile for that M/L .
- Open circles with error bars** are the observed profiles from Walker et al. (2009)
- The red curves are the isolated MOND results** for the M/L shown.
- The blue curves are the Newtonian results** following the same procedure of adjusting the M/L until the bulk dispersion comes close to that observed.



A Closer Look at Fornax

- The previous slide shows a profile for Fornax with $M/L = 1.5$ that is well above the observations, yet gives us a bulk dispersion close to that observed.
- Below is a profile for Fornax with a smaller mass of $M/L = 1$, but here the bulk dispersion is less than the observed value.
- This agrees better with Walker's observed profile, except for the last four points.



Including the External Field Effect (EFE)

- All of the results that I have shown here are for the **Isolated MOND Case**, i.e. the gravity of any host or external galaxy is ignored.
- dSph's are at different distances from their host galaxies and have different sizes, so the effect of the gravity of the host, i.e. the External Field Effect, may be important.

- **What is the External Field Effect?**

In his original 1983 work, Milgrom cited open clusters as systems that have internal acceleration $< a_0$, but show no mass discrepancy, and hence are not in the MOND regime. Based on this observed fact, he postulated:

“We are then compelled to conclude that the internal dynamics of the open clusters embedded in the field of the Galaxy is different from that of a similar but isolated cluster. When the external field g (and hence the resultant acceleration) become comparable to or larger than a_0 , the internal dynamics approaches and eventually becomes Newtonian even when the internal accelerations themselves are much smaller than a_0 .” (Milgrom, ApJ, 270, 1983)

- The essential point here for dSph's is that it's the total acceleration, internal plus external, that should matter.

Our Model of a dSph: Including MOND & EFE

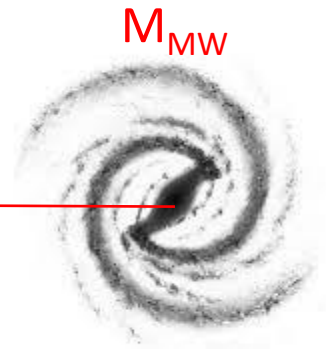
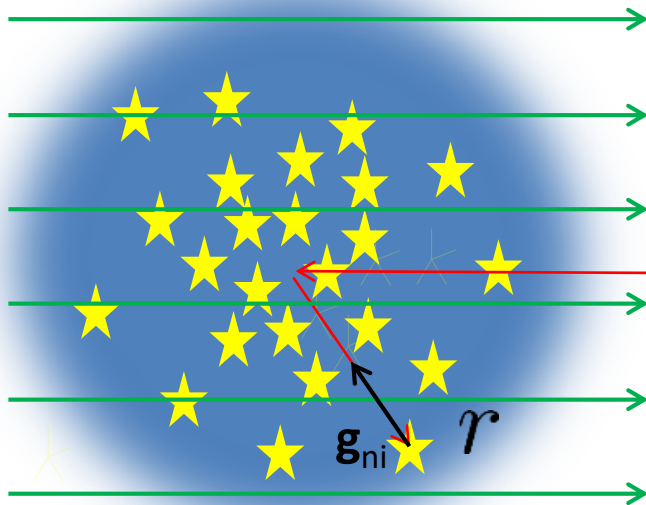
(keep it as simple as possible)

1. As before, treat a dSph as a spherically symmetric baryon cloud described by a Hernquist distribution with baryonic mass M and half-mass radius $r_{1/2}$. So the magnitude of the **Internal Newtonian Acceleration** of a star at radius r is:

$$g_{ni} = \frac{GM_{\text{int}}}{r^2}$$

2. Assume that the gravitational field of the Milky Way is uniform with magnitude equal to its value at the dSph center. Using M_{MW} as the baryon mass of the Milky Way, the uniform **External Newtonian Acceleration** is:

$$g_{ne} = \frac{GM_{\text{MW}}}{r_{\text{MW}}^2}$$



Our Model of a dSph: Including MOND & EFE

3. Our starting point to include the EFE is the MOND relation is Sanders & McGaugh (2002)

$$\mu\left(\frac{|\mathbf{g}_e + \mathbf{g}_i|}{a_0}\right) \mathbf{g}_i = \mathbf{g}_{n_i}$$

where \mathbf{g}_e and \mathbf{g}_i are the **actual external and internal acceleration** vectors and \mathbf{g}_{n_i} is the Newtonian internal acceleration, and μ is, again, the matching function.

4. The magnitude of the external acceleration is corrected for MOND as in the isolated case. Using the same matching function that was used in the isolated case, gives for the **Actual External Acceleration**:

$$g_e = \frac{g_{n_e}}{2} \left[1 + \sqrt{1 + 4 \left(\frac{a_0}{g_{n_e}} \right)} \right]$$

5. Keeping things as simple as possible, replace the magnitude of the vector sum $|\mathbf{g}_e + \mathbf{g}_i|$ with the sum of the magnitudes: $|\mathbf{g}_e| + |\mathbf{g}_i|$.



What's the red flag? We've added two vectors by adding their magnitudes – oh, the irony! **We recognize this . . . let's call this the zeroth order attempt see where it goes.**

6. Using our simple matching function, this gives for the magnitude of the **Actual Internal Acceleration**:

$$\mu(x) = \frac{x}{1+x}$$
$$g_i = \frac{g_{n_i} - g_e}{2} + \frac{g_{n_i} + g_e}{2} \sqrt{1 + \frac{4g_{n_i}a_0}{(g_{n_i} + g_e)^2}}$$

MOND EFE Sample Accelerations

- Our relation for the MOND EFE acceleration should give the necessary limiting cases, and perhaps something different in between.
- To illustrate this, we consider a toy model dSph near the Milky Way with:

$$\begin{array}{ll} \text{Baryon Mass } M = 10^7 M_{\odot} & \text{(Baryon)MW Mass, } M_{MW} = 6.5 \times 10^{10} M_{\odot} \\ \text{Half-mass radius } r_{1/2} = 500 \text{ pc} & \text{Distance to dSph, } r_{MW} = 200 \text{ kpc} \end{array}$$

- We consider three cases:

1. **Case 1:** $g_e < g_i < a_0$ (just reduce M_{MW} by 10^{-4})
Should give g_i the same as the isolated MOND case.

2. **Case 2:** $g_i < a_0 < g_e$ (just increase M_{MW} by 10^4)
Should give g_i the same as the Newtonian case.

3. **Case 3:** $g_i \sim g_e < a_0$ (keep data as above)

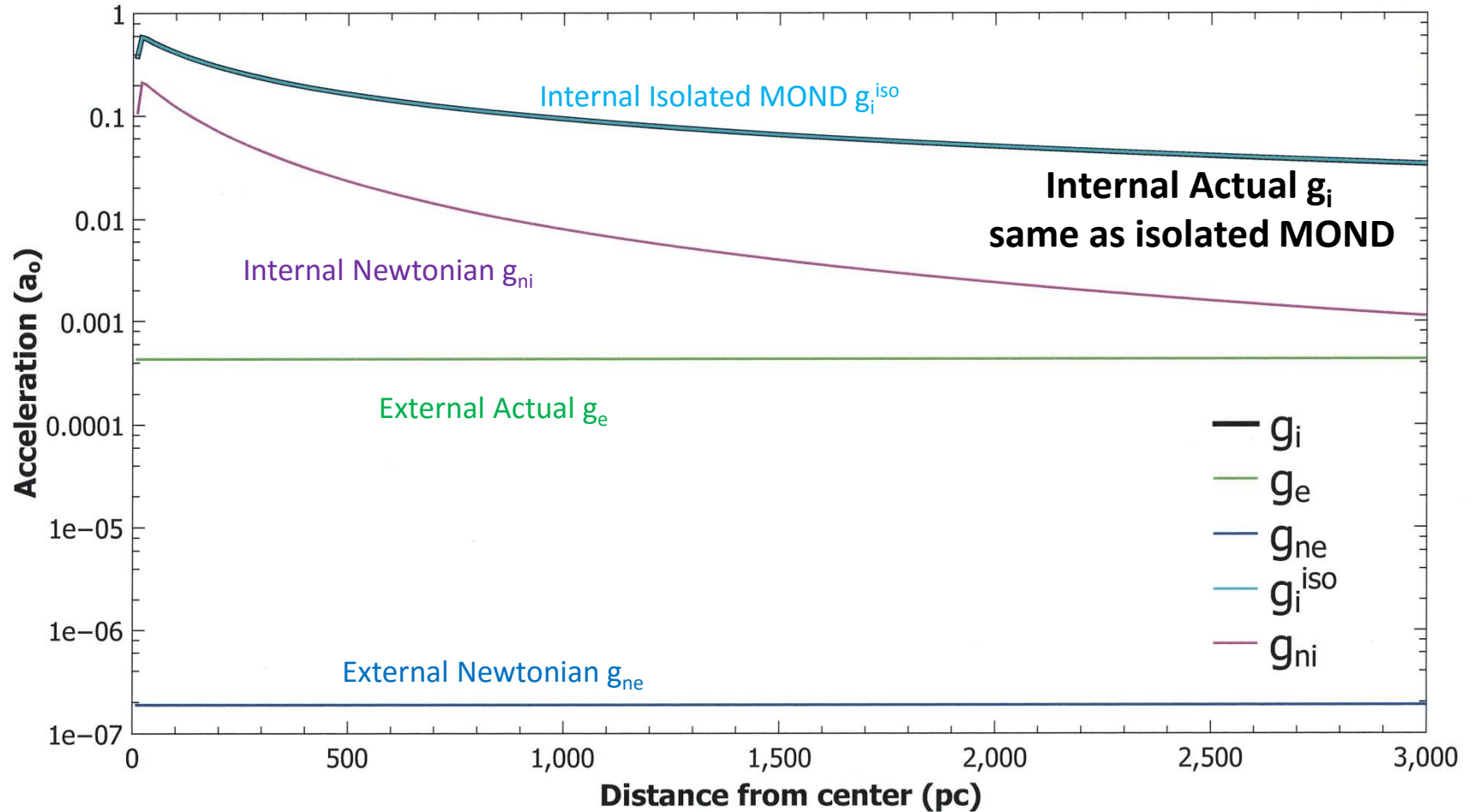
- Another case that is widely used is the “**Effective Newtonian Case**” where where $g_i \ll g_e \ll a_0$. In this limit:

$$g_i = g_{\text{eff}} \approx \frac{G a_0}{g_e} \frac{M_{\text{int}}}{r^2} = G_{\text{eff}} \frac{M_{\text{int}}}{r^2}$$

which gives a quasi-Newtonian acceleration with a reduced gravitation constant.

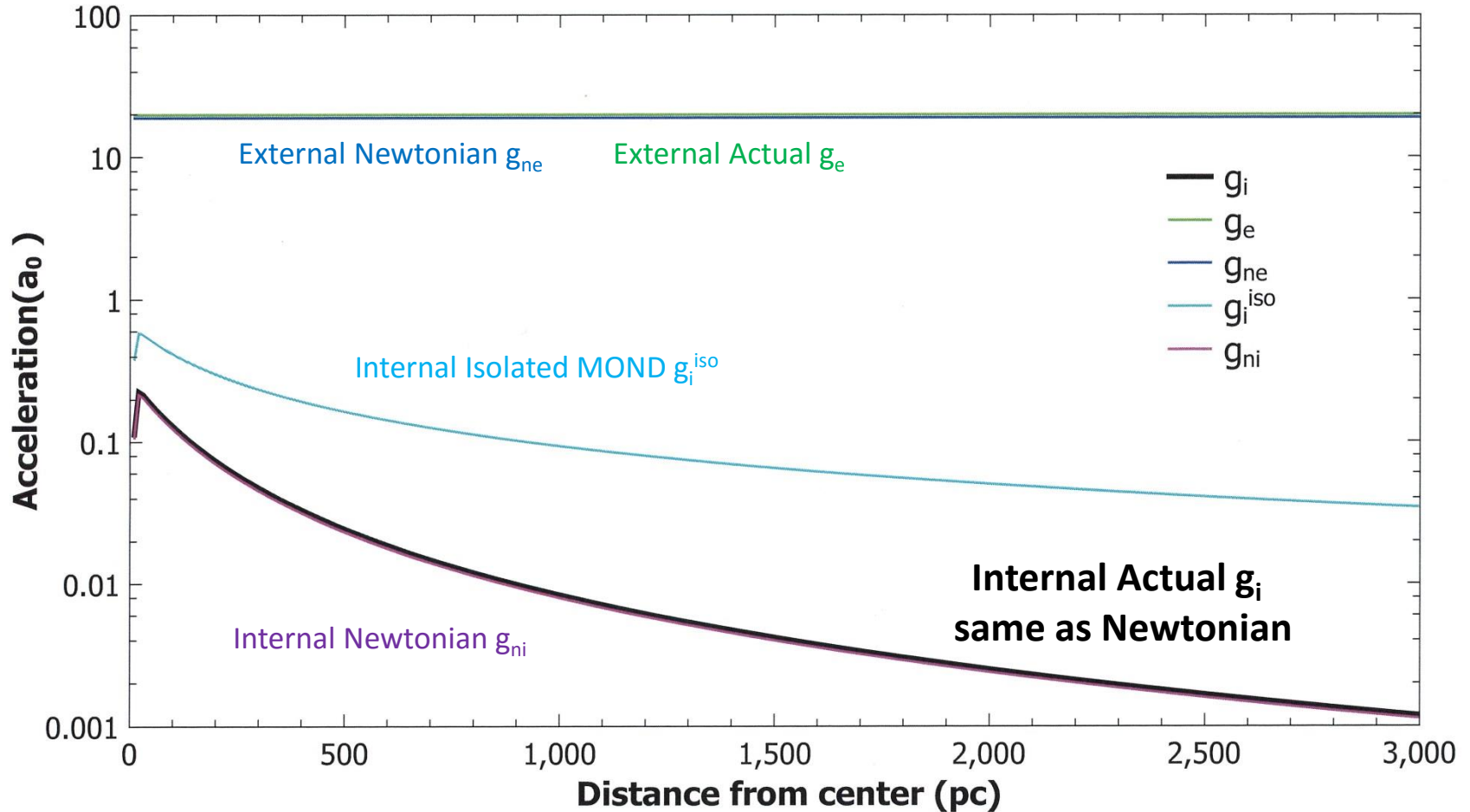
Case 1: Acceleration Profile

$$g_e < g_i < a_0$$



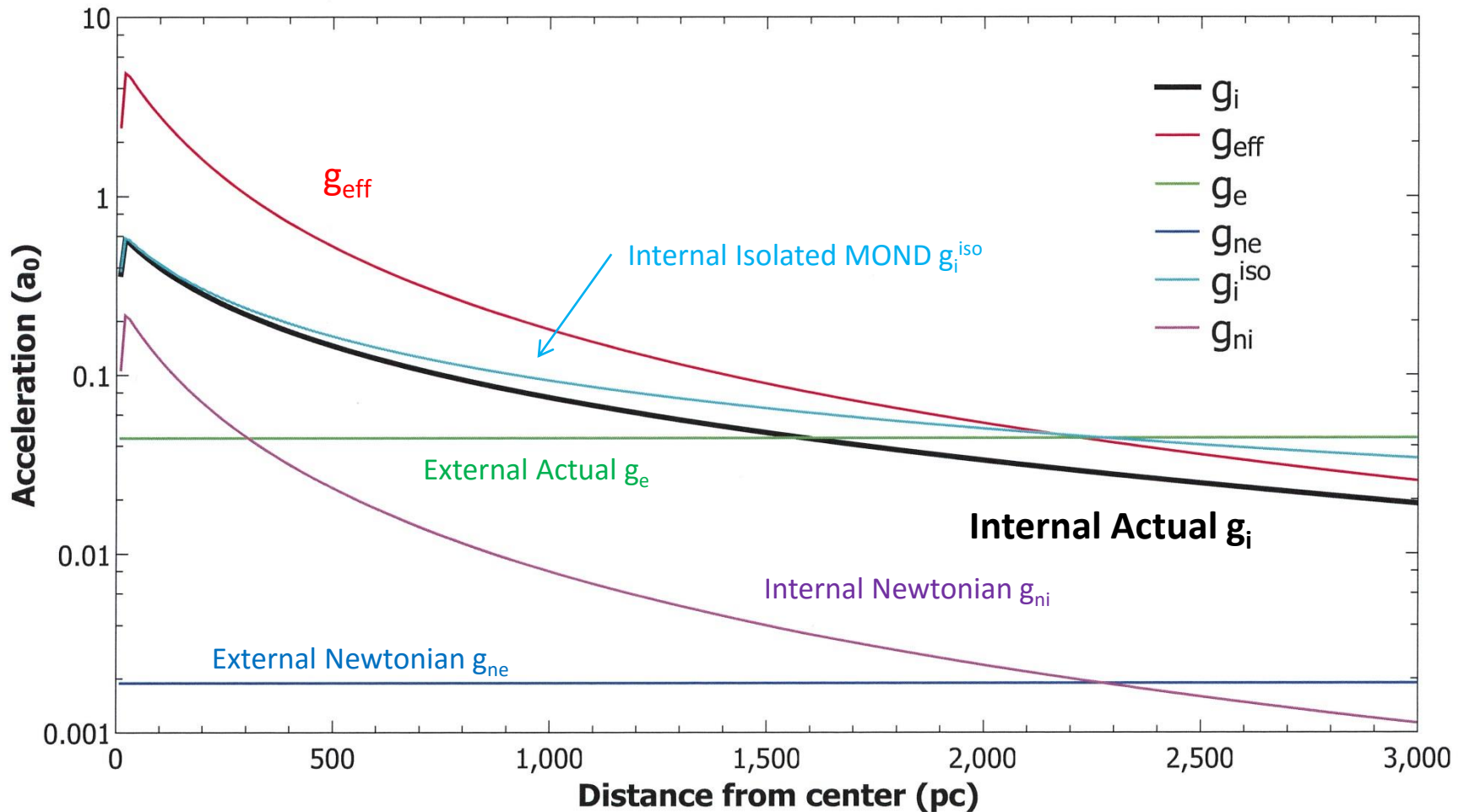
Case 2: Acceleration Profile

$$g_i < a_0 \ll g_e$$



Case 3 & Effective Newtonian: Acceleration Profile

$$g_i \approx g_e < a_0 \quad \text{and} \quad g_{\text{eff}} \approx \frac{Ga_0}{g_e} \frac{M_{\text{int}}}{r^2} = G_{\text{eff}} \frac{M_{\text{int}}}{r^2}$$



dSph Dispersion Profiles Using our EFE Model

(recent preliminary results)

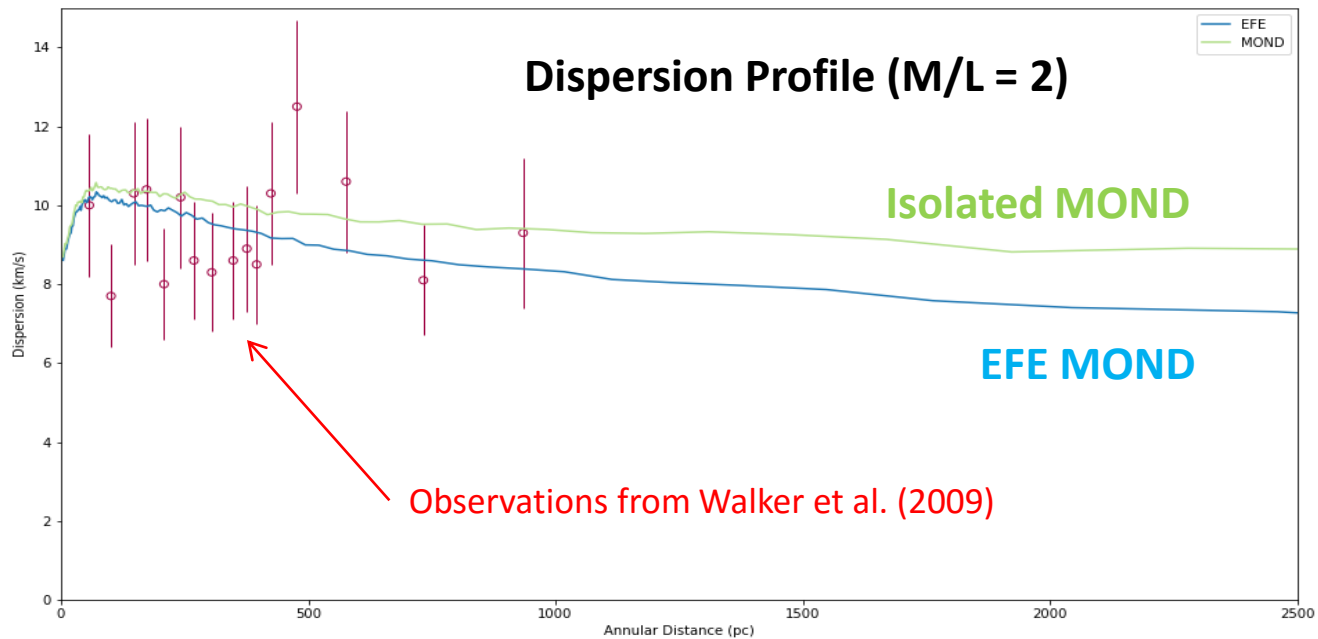
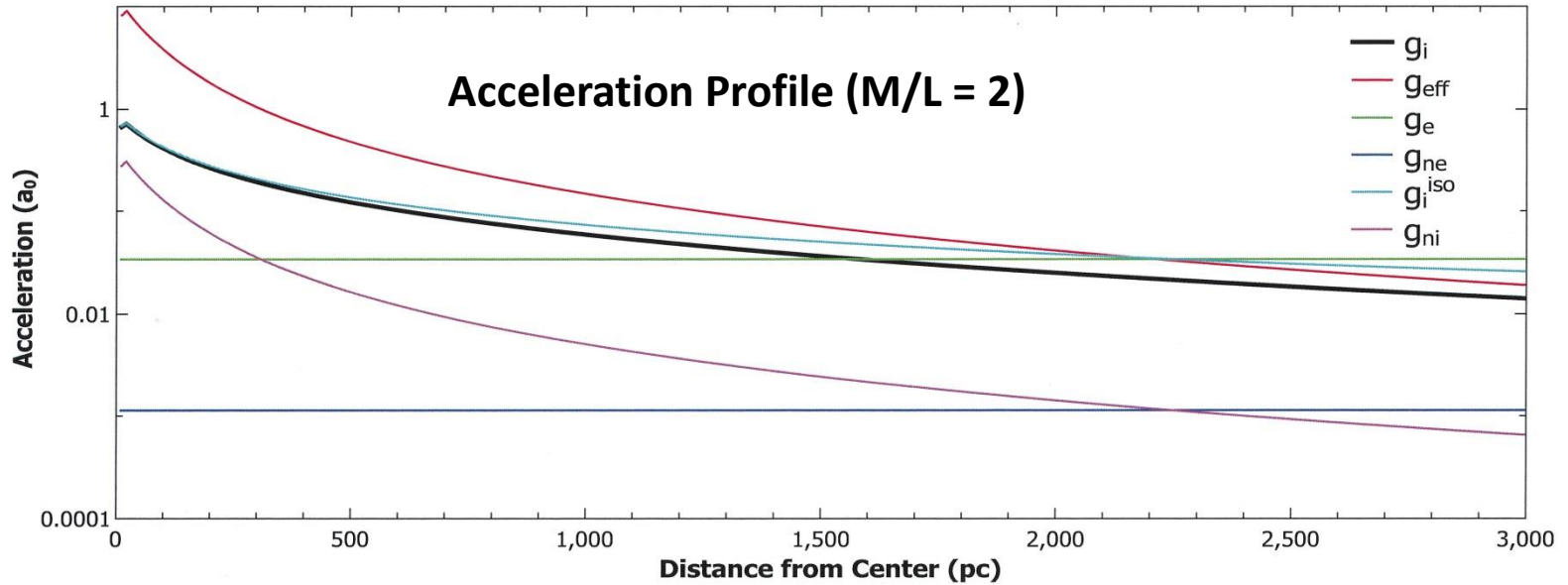
- To calculate dispersion profiles, we use the EFE acceleration as above in vector form, differentiate it, and code it into our HITS integrator – *I'm assuming no one wants to see these equations – I can pull them up if you want.*
- To date, we've looked at two galaxies: **Leo I** and **Fornax**. We used the updated luminosities and half-mass radii in the appendix of Lelli et al. (2017):

$$\begin{aligned} L_V &= 5.5 \times 10^6 L_\odot \\ \text{Leo I: } r_{1/2} &= 298 \text{ pc} \\ r_{MW} &= 258 \text{ kpc} \end{aligned}$$

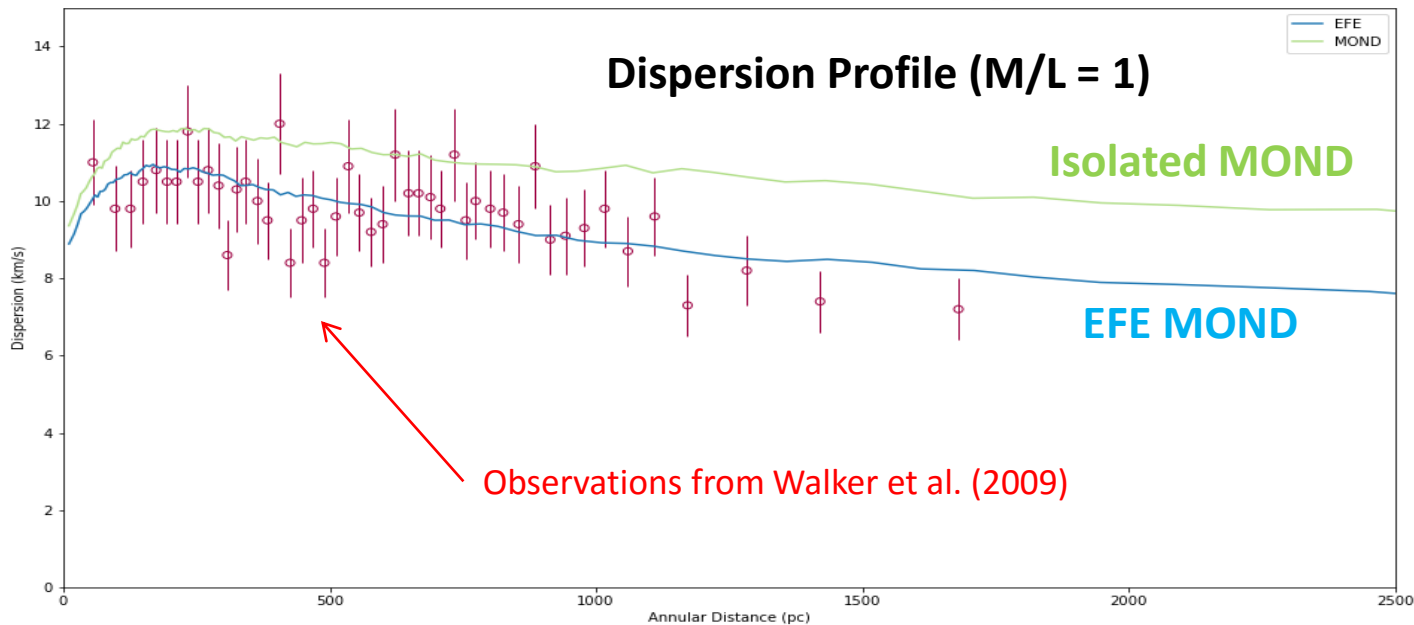
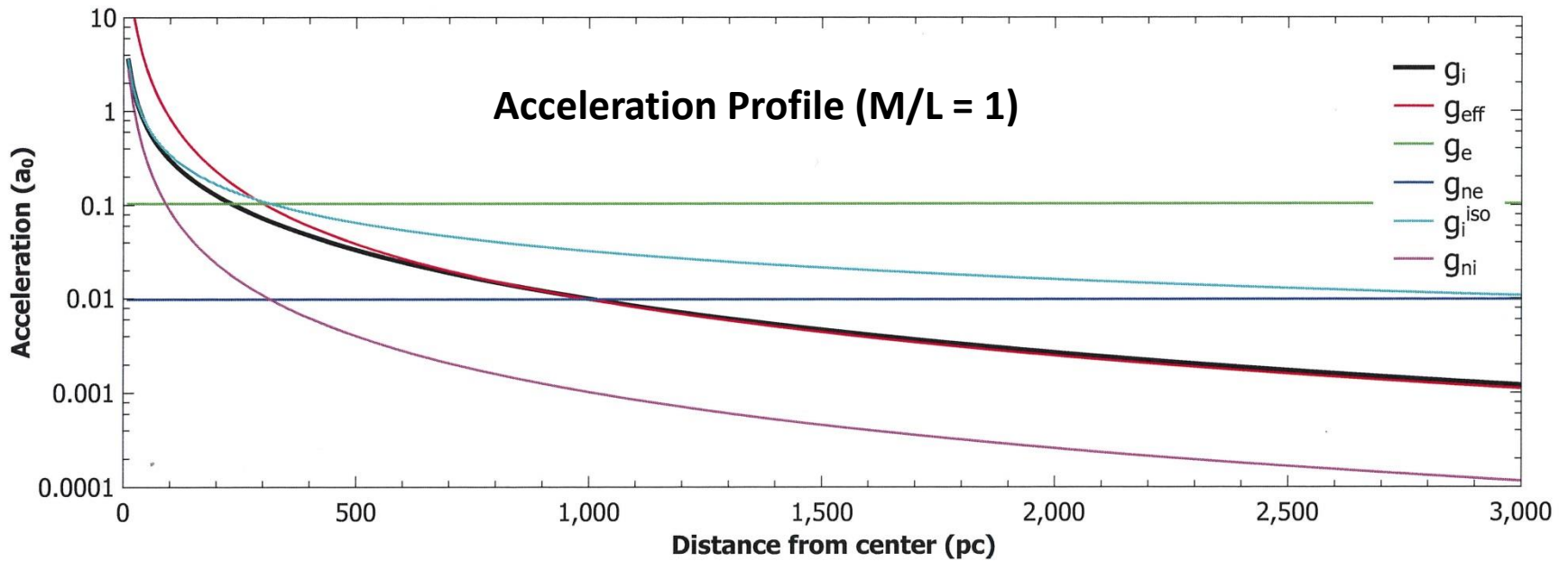
$$\begin{aligned} L_V &= 2.4 \times 10^7 L_\odot \\ \text{Fornax: } r_{1/2} &= 792 \text{ pc} \\ r_{MW} &= 149 \text{ kpc} \end{aligned}$$

- As in the isolated case, we populate the space with 10,000 stars according to the Hernquist distribution, assign random initial velocities, and integrate the motion for 2.5 Gy.
- One difference here is that we now adjust the M/L for each galaxy until we get the best chi-squared match with the observed dispersion profile.

Acceleration and Dispersion for Leo I ($M/L = 2$)



Acceleration and Dispersion for Fornax (M/L = 1)



Where to Go From Here?

(Please feel free to make any suggestions)

1. Perform simulations of the other MW dSph's that have observed profiles?
Other dSph's (*Crater II?*) - or other objects of interest?
2. Try different initial velocities?
3. Try other density profiles to model dSph mass distribution,
e.g. the Plummer profile?
4. Go back and treat acceleration vector sum more rigorously in the MOND
EFE equation? (*breaks spherical symmetry*)
5. Include tidal effects in our calculations? (*also breaks spherical symmetry*)
6. Include binary stars in the simulation?
7. Use our dSph model with Dark Matter Models instead of MOND?