VIRIAL THEOREM

Relates potential and kinetic energy of system in equilibrium

\[ 2 \text{k.e} + \text{p.e} = 0 \]

\[ \text{Q} \quad \text{How might we summarize potential kinetic energy for a giant elliptical?} \]
\[ \text{(a quantity we can measure fairly easily)} \]

\[ \rightarrow \text{use velocity dispersion } \sigma^2, \text{ specially if there is no rotational support} \]

\[ \text{Q} \quad \text{What could we measure that would quantify potential energy?} \]
\[ cf \frac{-GM}{r} \]
Structure of ellipticals - Trends

Elliptical luminosity decreases with radius by the $R^{-4}$ law:

$$\log I \propto \left( \frac{r}{r_e} \right)^{-4}$$

Here, effective radius gives a measure of the size of the galaxy (it's the radius containing half the total light).

We can also measure the galaxy's surface brightness: $I_e$ is the mean surface brightness of the galaxy inside $r_e$ (in magnitudes).

How do size and mean surface brightness correlate with galaxy luminosity?
Figure 23.29 Normal ellipticals and dwarf systems define separate sequences of effective radius as a function of $M_B$. The bulges of spiral galaxies lie along the sequence of normal ellipticals. (Data from Bender et al., Ap. J., 399, 462, 1992.)

Brighter ellipticals are BIGGER

(note different sequence for dE & dSph galaxies)
Figure 23.30 Normal ellipticals and dwarf systems define separate sequences of average surface brightness versus $M_B$. The bulges of spiral galaxies lie along the sequence of normal ellipticals. (Data from Bender et al., Ap. J., 399, 462, 1992.)

Brighter ellipticals have LOWER (fainter) average surface brightness
More trends for ellipticals

Define $L_e = \text{luminosity interior to } R_e$

$\sigma_0 = \text{central velocity dispersion}$

Faber-Jackson relation connects luminosity & kinematics (of Tully-Fisher relation for spirals)

$L_e \propto \sigma_0^4$

We can define different relations between $R_e$, surface brightness within $R_e$ ($<I_e>$), $L_e$ and $\sigma$. 

The fundamental plane for (giant) elliptical galaxies relates these quantities.

$\log R_e = 0.36 <\mu_e> + 1.4 \log \sigma$

measured in B band surf. brightness

$<\mu_e> = -2.5 \log <I_e> + \text{const}$
Figure 4.43 Correlations between four shape-independent parameters of elliptical galaxies. The parameters are the effective radius $R_e$, the mean surface brightness within $R_e$, $(I)_e$, the central velocity dispersion $\sigma_0$ and $L_e$, the luminosity in Djorgovski's G band interior to $R_e$. The luminosity and the surface brightness are expressed in magnitudes and in magnitudes per square arcsecond, respectively. [From data published in Djorgovski & Davis (1987)]

The lower right panel of Figure 4.43 shows the correlation of which this is the mean regression.

More luminous ellipticals have larger central velocity dispersions. The upper right panel in Figure 4.43 illustrates this correlation, which is called the Faber–Jackson relation after its discoverers (Faber & Jackson 1976). Quantitatively one has

$$L_e \sim \sigma_0^2.$$  \hspace{1cm} (4.38)

Since $\sigma_0$ is correlated with $L_e$ and $L_e$ is correlated with $R_e$ it follows that $\sigma_0$ must be correlated with $R_e$. The top left panel in Figure 4.43 displays this correlation.

Since $\sigma_0$ is strongly correlated with line-strengths and colors, the existence of a host of additional correlations involving $Mg_2$, $B - V$ etc is implied by the top two panels of Figure 4.43. One of the earliest of these to be discovered was the color–magnitude relation: Faber (1973) showed that more luminous elliptical galaxies have stronger absorption lines, and Visvanathan & Sandage (1977) showed that more luminous galaxies are redder.

In all the correlations of Figure 4.43 there is cosmic scatter; the scatter of the points about the mean relations is larger than can be accounted for by measurement errors alone. The positions errors between the points are correlated. For example, the magnitude relation in the top right panel in Figure 4.43 is useful to imagine each...
Figure 4.44 An edge-on view of the fundamental plane as defined by the data of Figure 4.43. Note how much narrower is the distribution of points than in either of the left-hand panels of Figure 4.43.

The Cartesian coordinates \( x_i \) of a point in this space are the three numbers of the set \( (\log R_e, \langle I \rangle_e, \log \sigma_0) \), where \( \langle I \rangle_e \) is in units of \( \mu_B \). If there were no correlations between the variables, the points of individual galaxies would be fairly uniformly distributed within a cuboidal region of our three-dimensional space. Correlations between the variables will confine the data points to a sub-volume of this cuboid. For example, if two of the variables are genuinely independent and the third dependent on these two, the points will be confined to a plane, while if there is only one independent variable, the points will lie on a line. Our first step in analyzing such data is to ask whether the points are nearly confined to a plane; if we can find a suitable plane, we can further enquire whether the points lie on a line within that plane. In statistics books this type of investigation is called **principal component analysis**. Box 4.2 explains what has to be done.

For the data in Figure 4.43 one finds that the points are as nearly confined to a plane as observational errors allow; in the absence of observational errors, they might lie precisely on a plane! If the position vector of a galaxy in our three-space is \( \mathbf{g} = (\log R_e, \langle I \rangle_e, \log \sigma_0) \), where \( R_e \) is measured in kpc, \( \langle I \rangle_e \) in units of \( \mu_B \) and \( \sigma_0 \) in \( \text{km s}^{-1} \), then the equation of this **fundamental plane** is \( \mathbf{n} \cdot \mathbf{g} = 1 \) where \( \mathbf{n} = (-0.65, 0.22, 0.86) \).

Naturally one wants to 'see' the fundamental plane. One way to do this is to choose an axis, for example the \( R_e \)-axis, as the horizontal axis, and rotate the space about this axis until the plane appears edge-on. In this orientation, the plane's normal, \( \mathbf{n} \), lies in the plane of the projection, which is spanned by the unit vector, \( \mathbf{e}_R \), that runs parallel to \( R_e \)-axis and some other, orthogonal, unit vector \( \mathbf{e} \). Thus \( \mathbf{n} = \alpha \mathbf{e}_R + \beta \mathbf{e} \), where \( \alpha \) and \( \beta \) are suitable numbers. Comparing this with the value of \( \mathbf{n} \) given above we see that \( \beta \mathbf{e} = 0.22 \mathbf{e}_I + 0.86 \mathbf{e}_e \). Thus the fundamental plane will be seen edge-on if we plot \( \log R_e \) against \( 0.22 (\langle I \rangle_e/\mu_B) + 0.86 \log \sigma_0 \). Actually, it is conventional to plot \( 0.26 (\langle I \rangle_e/\mu_B) + \log \sigma_0 \), which is simply \( 1/0.86 \) times the linear combination we have derived. Figure 4.44 shows this plot. The data points nearly lie on fundamental plane. The \( \log R_e \)

is also an equation for the

\[ I_n = \frac{1}{\sigma_0} \]

The \( D_n - \sigma_0 \) correlation measured photometric properties virtue of the fundamental which the mean surface brightness than \( \log R_e \) luminosity. To quantify the same surface-brightness profile

\[ I(R) = I_e f(R/R_e) \]

From Figure 4.25 we deduce will come from radii at which magnification to evaluate the int

When equation (4.39) is used

The weak dependence upon correlation between \( D_n \) an

If we adopt a distance of relation becomes

\[ D = \frac{k}{k} \]

with a 15% scatter from g

Dwarf elliptical galaxies brightness profiles of giant

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14 \( L_e \) need not be included in the set since it is related to \( R_e \) and \( \langle I \rangle_e \) by \( L_e = \pi \langle I \rangle_e R_e^2 \), where \( L_e \) is measured in \( \text{W} \), \( I_e \) in \( \text{W m}^{-2} \text{sterad}^{-1} \) and \( R_e \) in \( \text{m} \).

15 Recall that \( \langle I \rangle_e/\mu_B \propto - \).