Points:
- Velocity dispersion \( \uparrow \) w/ mean age
- \( \sigma_v > \sigma_v > \sigma_w \)
- \( \sigma_v \sim 2 \sigma_w \)
- Young stars not in equilibrium
  "Vertex deviation"

Asymmetric Drift
- Stars tend to lag the LSR
- Lag \( \uparrow \) w/ \( \uparrow \) \( \sigma \)/age
- Drift velocity for old disk \( \sim 15 \text{ km/s} \)
  
  Why?
  - Think orbits
  - Why asymmetric?

Thick Disk
\( \sigma_v \approx 60 \text{ km/s} \)
\( \sigma_v \approx 40 \text{ km/s} \)
\( \sigma_w \approx 40 \text{ km/s} \)
asymmetric drift \( \sim 30 \text{ km/s} \)
Solar motion

\[ u_0 = -10-11 \text{ km/s} \]
\[ v_0 = 5-7 \text{ km/s} \]
\[ w_0 = 7-8 \text{ km/s} \]

LSR circular velocity

\[ \Theta_0 \sim 220 \text{ km/s} \] (see Korr & Lupton-Bell)


\[ \Theta_0 = 180 \text{ km/s} \]

Kinematics of Disk Stars

- Circular orbits w/ small deviations
- Dispersion around motion of LSR
- \( \sigma_u \)
- \( \sigma_v \)
- \( \sigma_w \)

"The velocity ellipsoid"

B&G 10.3
MB 7
Orbits & epicycles

Think of orbits as being a sum of two motions:

- Circular motion around galactic center
  "the guiding center"

- Motion around that guiding center
  "the epicycle"

Examples:

- Ptolemy (no good!)
- Pointmass (works!)
  Gives closed orbit
- General mass distribution
  Orbits not closed in general
The orbits of the stars

Figure 3.9 Path of the star of Figure 3.7, viewed from above the Galactic plane; the orbit started with \( (R = 1.3, \phi = 0) \) and \( (\dot{R} = 0, R\dot{\phi} = 0.4574) \).

Near the Sun, the period of the epicycles is about 170 Myr, far too long for us to watch stars complete their circuits. But we can measure the velocities of stars close to us, at \( R \approx R_0 \). Some of these will have guiding centers further out than the Sun, and so are on the inner parts of their epicycles, while others have their guiding centers at smaller radii. Because of its epicyclic motion, a nearby star with its guiding center at \( R_g > R_0 \) moves faster in the tangential direction than a circular orbit at our radius. Equation 3.71 gives its relative speed \( v_y \) as

\[
v_y = R_0[\dot{\phi} - \Omega(R_0)] \approx R_0 \left[ \Omega(R_g) - 2x \frac{\Omega(R_g)}{R_g} - \Omega(R_0) \right]; \tag{3.73}
\]

recalling that \( R_0 = R_g + x \), and dropping terms in \( x^2 \), we have

\[
v_y \approx -x \left[ 2\Omega(R_0) - R_0 \left( \frac{d\Omega}{dR} \right)_{R_0} \right] = 2Bx. \tag{3.74}
\]
3.3 Orbits of disk stars: epicycles

Figure 3.8 A star following an elliptical epicycle around its guiding center at \((x = 0, y = 0)\), which is carried around the Galactic center with angular speed \(\Omega(R_g)\).


During its epicyclic motion, the star’s azimuthal speed \(\dot{\phi}\) must vary so that the angular momentum \(L_z\) remains constant:

\[
\dot{\phi} = \frac{L_z}{R^2} = \frac{\Omega(R_g)R_g^2}{(R_g + x)^2} \approx \Omega(R_g) \left(1 - \frac{2x}{R_g} + \cdots\right). \tag{3.71}
\]

Substituting from Equation 3.68 for \(x\) and integrating, we have

\[
\phi(t) = \phi_0 + \Omega(R_g)t - \frac{1}{R_g} \frac{2\Omega}{\kappa} X \sin(\kappa t + \psi), \tag{3.72}
\]

where \(\phi_0\) is an arbitrary constant. Here, the first two terms give the guiding center motion. The third represents harmonic motion with the same frequency as the \(x\) oscillation in radius, but 90° out of phase, and larger by a factor of \(2\Omega/\kappa\) (see Figure 3.8). The epicyclic motion is *retrograde*, in the opposite sense to the guiding center motion; it speeds the star up closer to the center, slowing it down when it is further out.

In two simple cases, the epicyclic frequency \(\kappa\) is a multiple of the angular speed \(\Omega\) of the guiding center. In the gravitational field of a point mass, \(\Omega(r) \propto r^{-3/2}\).
So the epicycle is an oscillation around circular motion. Notes:

- the epicycle is **retrograde**

- the frequency of this oscillation is known as the **epicyclic frequency** \( K \)

- the epicyclic frequency & the angular speed of the guiding center are both governed by the potential & are related:

\[
K^2 = \left[ R \frac{d\Omega^2}{dR} + 4 \Omega^2 \right]_{R_g}
\]

- epicycle has axis ratios

\[
\frac{X}{Y} = \frac{K}{2\Omega}
\]

*see* SEG 3.3