ASTR120
Homework 2 −Solutions

Ch. 2, Prob. 37.
The Sun will pass 19 °N on its way up to the Tropic of Cancer (23.5 °N) and on its way down toward the equator and Tropic of Capricorn each year. So it will be over head at 19 °N exactly twice a year.

Ch. 3, Prob. 29.
The month is defined as 1 period of the Moon's orbit around the Earth. The Moon's synodic period is 29.5 days. So in 1 year (365 days), we get $\frac{365}{29.5} = 12.37$ synodic months. The sidereal period of the Moon is 27.3 days, so in 1 year, we have $\frac{365}{27.3} = 13.37$ sidereal months. So we get one more sidereal month than synodic month in a year.

Ch. 4, Prob. 30.
For inferior planets, the relationship between sidereal and synodic periods is :

$$\frac{1}{P} = \frac{1}{E} + \frac{1}{S}$$

So for Mercury, S = 115.88 days, E = 1 year = 365 days :

$$\frac{1}{P} = \frac{1}{365} + \frac{1}{115.88} = 0.01137$$

$$P = 88 \text{ days}$$

Ch. 4, Prob. 35.

a) $2a = d_{peri} + d_{aph}$

$a = 0.5 \times (2 + 6)$

$= 4$ AU

b) $P^2 = a^3$

$= 4^3$

$= 64$

$P = 8$ yrs.

extra : For Ceres

$a = 0.5 \times (2.55 + 2.99)$

$= 2.77$ AU

$P^2 = 2.77^3 = 21.25$

$P = 4.6$ AU

Ch. 4, Prob. 36.

a) $a^3 = P^2 = 64^2 = 4096$

$a = 16$ AU

b) $2a = d_{peri} + d_{aph}$

$32 = d_{peri} + 31.5$

$d_{peri} = 0.5$ AU

extra : For Hale − Bopp

$d_{peri} = 0.9$ AU

$a^3 = (2500)^2$

$a = 184.2$ AU

$d_{aph} = 2a - 0.9 = 368.4 - 0.9 = 367.5$ AU

Ch. 4, Prob. 39.
Gravitational pull due to Earth :

$$F_{Earth} = \frac{G M_E m}{{r_E}^2}$$

Gravitational pull due to planet :

$$F_{planet} = \frac{GM_p\, m}{r_p^{\,2}}$$

Find the ratio of the pull by Earth and the planet : know $M_p = 4\, M_E$, $r_p = 4\, r_E$

$$\frac{F_{planet}}{F_{Earth}} = \frac{\frac{GM_p\, m}{r_p^{\,2}}}{\frac{GM_E\, m}{r_E^{\,2}}} = \frac{GM_p\, m}{r_p^{\,2}} \frac{r_E^{\,2}}{GM_E\, m} = \frac{M_p}{r_p^{\,2}} \frac{r_E^{\,2}}{M_E} = \frac{M_p}{M_E} \left(\frac{r_E}{r_p}\right)^2 = 4 \left(\frac{1}{4}\right)^2 = \frac{1}{4}$$

Therefore : $F_{planet} = \frac{1}{4} F_{Earth}$. The Earth's gravity is grater by a factor of 4.

extra : surface gravity of satellites. Surface gravity equation : $g = \frac{GM}{r^2}$

$$g_{Phobos} = \frac{G\,(1.6 \times 10^{16})}{(1.2 \times 10^4)^2} = 0.0074\ m/s/s = \frac{0.0074}{9.8} = 7.6 \times 10^{-4}\ g$$

* since Phobos is not spherical, the average distance to the center is used.

$$g_{Callisto} = \frac{G\,(1.077 \times 10^{23})}{(2.4 \times 10^6)^2} = 1.25\ m/s/s = \frac{1.25}{9.8} = 0.13\ g$$

$$g_{Titan} = \frac{G\,(1.34 \times 10^{23})}{(2.575 \times 10^6)^2} = 1.35\ m/s/s = \frac{1.35}{9.8} = 0.14\ g$$

Ch. 4, Prob. 42.

a) Since the satellite must be directly over the same point on Earth all the time, it must have the same period as the Earth's sidereal rotation period. The period is 23 hours 56 minutes.

b) For this part, we need to use Newton's version of Kepler's third law :

$$P^2 = \left(\frac{4\pi^2}{G\,(M_E + M_{sat})}\right) a^3$$

$P = 23$ hr 56 minutes $= 23.933$ hr $= 8.616 \times 10^4$ s

Since the mass of Earth is much greater than the mass of the satellite, $M_E + M_{sat} = M_E$

$$a^3 = \left(\frac{GM_E}{4\pi^2}\right) P^2 = 7.5 \times 10^{22}$$

$a = 4.2 \times 10^7$ m from the center of the Earth

or 3.6×10^7 m from the surface of the Earth (

c) The orbit must be in the plane of the equator. It must orbit so that the same point on the earth is beneath it. If the orbit is tilted, then the satellite will appear to move in the sky (much like the Sun's motion over the year). Thus, the orbit must be in the plane of the equator.

Extra Credit :

Again, we will use Newton's version of Kepler's third law :

$$P^2 = \left(\frac{4\pi^2}{G\,(M_E + M_{Sun})}\right) a^3$$

Since $M_{Sun} >> M_E$, $M_E + M_{Sun} = M_{Sun}$. $M_{star} >> M_E$, $M_E + M_{star} = M_{star} = 4\, M_{Sun}$

$a = 1$ AU in both cases. So we get the following :

$$P_{Earth}^{\,2} = \left(\frac{4\pi^2}{GM_{Sun}}\right) a^3$$

$$P_{planet}^{\,2} = \left(\frac{4\pi^2}{GM_{star}}\right) a^3$$

Therefore :

$$\frac{P_{Earth}^{\,2}}{P_{planet}^{\,2}} = \frac{\frac{4\pi^2 a^3}{GM_{Sun}}}{\frac{4\pi^2 a^3}{GM_{star}}} = \frac{4\pi^2 a^3}{GM_{Sun}} \frac{GM_{star}}{4\pi^2 a^3} = \frac{M_{star}}{M_{Sun}} = 4$$

$$P_{Earth}^{\,2} = 4\, P_{planet}^{\,2}$$

$$P_{Earth} = 2\, P_{planet}$$

Therefore, the planet's sidereal period is half that of the Earth. Its period is 0.5 years.