ASTR120
Homework 3 −Solutions

Ch. 5, Prob. 25.
To find the wavelength of maximum emission for the black hole, we need to use Wien's Law:

$$\lambda_{max} = \frac{0.0029\ \text{K m}}{T} = \frac{0.0029\ \text{K m}}{10^6\ \text{K}} = 2.9 \times 10^{-9}\ \text{m} = 2.9\ \text{nm}$$

This wavelength is in the X − ray range of the electromagnetic spectrum.

Ch. 5, Prob. 26.
The solar constant given in Box 5 − 2 is 1370 W m^{-2}. The solar constant is the amount of the Sun's flux at the location of the Earth. So, to find the Sun's luminosity:

L_{sun} = sol. const x 4 πR^2 where R is the distance from the Sun to the Earth

$$L_{sun} = 1370 \times 4\pi \times (1.49 \times 10^{11})^2 = 3.8 \times 10^{26}\ \text{W}$$

Ch. 5, Prob. 27.
The book asks you to compare the energy emitted each second from each square meter of the surface of the star with that of the Sun. This is just a fancier way of saying compare the fluxes of the star and the Sun. Since:

$$F = \sigma T^4$$

$$\frac{F_{\alpha\ lupi}}{F_{sun}} = \frac{\sigma T_{\alpha\ lupi}{}^4}{\sigma T_{sun}{}^4} = \frac{T_{\alpha\ lupi}{}^4}{T_{sun}{}^4} = \left(\frac{T_{\alpha\ lupi}}{T_{sun}}\right)^4 = \left(\frac{21600}{5800}\right)^4 = 3.7^4 = 192.4$$

So, the star Alpha Lupi has about 192 times the flux of the Sun.

Ch. 6, Prob. 25.
Infrared radiation is just heat. So if you keep an infrared detector at room temperature, it will detect the infrared light it emits itself. Therefore, the detector must be kept cool so that its own radiation will not interfere with light from astronomical objects.

Ch. 6, Prob. 30.
Since the amount of starlight collected by a telescope is proportional to the area of the telescope, we just need to compare the area of the cage with that of the telescope.

$$\frac{A_{cage}}{A_{mirror}} = \frac{\pi R_{cage}{}^2}{\pi R_{mirror}{}^2} = \frac{R_{cage}{}^2}{R_{mirror}{}^2} = \left(\frac{R_{cage}}{R_{mirror}}\right)^2 = \left(\frac{0.5}{2.5}\right)^2 = \left(\frac{1}{5}\right)^2 = 0.04 = 4\ \%$$

So, only 4 % of the total incoming light is blocked by the cage.

Ch. 6, Prob. 31.
a] Again, light gathering power depends on the area of the telescope.

$$\frac{A_{Subaru}}{A_{HST}} = \frac{\pi R_{Subaru}{}^2}{\pi R_{HST}{}^2} = \left(\frac{R_{Subaru}}{R_{HST}}\right)^2 = \left(\frac{4.15}{1.2}\right)^2 = 12$$

So, the light gathering power of the Subaru telescope is about 12 times that of the HST.

b] The Subaru is more advantageous than the HST in its collecting area. It can collect more light than the HST and therefore see fainter objects, theoretically speaking. The HST is more advantageous than the Subaru in that it is above the atmosphere and therefore does not have to worry about absorption, scintillation, or twinkling effects from the atmosphere. In practice, this means it can take higher resolution images.

Ch. 6, Prob. 34.
For this problem, we need to use the small angle formula:

$$d = \frac{\alpha D}{206265}$$

Given that the resolution (α) is 0.1 areseconds, D is 6.28 x 10^8 km, we can find d.

$$d = \frac{0.1 \times 6.28 \times 10^8}{206265} = 304.5\ \text{km}$$

For the human eye, α is 1 arcmin or 60 arcsec, and the Moon is 3.84 x 10^5 km away.

$$d = \frac{60 \times 3.84 \times 10^5}{206265} = 111.8 \text{ km}$$

So the HST is much better at seeing finer details than the human eye. Although the numbers above show that the HST can only see objects 3 times as big as the human eye, one has to remember that these objects are located 2000 times farther away than the Moon.

Ch. 6, Prob. 35.

a] Again, we will use the small angle formula. Given that $\alpha = 0.1$ arcsec and $D = 7 \times 10^7$ ly.

$$d = \frac{0.1 \times 7 \times 10^7}{206265} = 33.9 \text{ ly}$$

b] Given that the dime is 1.8 cm, we need to find D.

$$1.8 \text{ cm} = \frac{0.1 \times D}{206265}$$

$$D = 3.71 \times 10^6 \text{ cm}$$
$$= 37.1 \text{ km}$$

Extra Credit:

From the problem, we're given:

$R_{Sirius} = 1.67\, R_{sun}$, $L_{Sirius} = 25\, L_{sun}$

a] We know that $F = \frac{L}{4\pi R^2}$, so:

$$F_{Sirius} = \frac{L_{Sirius}}{4\pi R_{Sirius}{}^2} = \frac{25\, L_{sun}}{4\pi\,(1.67\, R_{sun})^2} = \frac{25}{4\pi \times 1.67^2}\,\frac{L_{sun}}{R_{sun}{}^2}$$

$$= 0.71\,\frac{L_{sun}}{R_{sun}{}^2} = 0.71\left(\frac{3.8 \times 10^{26}}{(6.96 \times 10^8)^2}\right) = 5.72 \times 10^8 \text{ W m}^{-2}$$

b] We also know that $F = \sigma T^4$, so:

$$F_{Sirius} = \sigma T_{Sirius}{}^4$$

$$T_{Sirius} = \left(\frac{F_{Sirius}}{\sigma}\right)^{1/4} = \left(\frac{5.72 \times 10^8}{5.67 \times 10^{-8}}\right)^{1/4} = 1.0 \times 10^4 \text{ K}$$