ASTR120
Homework 4 −Solutions

Ch. 7, Prob. 24.

a. $E_K = \frac{1}{2} mv^2$ where m is the mass and v is the velocity. We first need to find the mass of the asteroid from problem 23.

mass = density x volume = ρV

Assuming a spherical asteroid, $m = \rho \left(\frac{4}{3} \pi r^3\right) = (2500) \left\{\frac{4}{3} \pi (1 \times 10^3)^3\right\} = 1.0 \times 10^{13}$ kg

$E_K = \frac{1}{2} (1.0 \times 10^{13}) (25 \times 10^3)^2 = 3.2 \times 10^{21}$ J

b. 20 − kilotons of TNT would produce $20 \times 4.2 \times 10^{12} = 8.4 \times 10^{13}$ J. So the kinetic energy of the asteroid is $\frac{3.2 \times 10^{21}\ \text{J}}{8.4 \times 10^{13}\ \text{J}} = 3.9 \times 10^7$ times more energetic than the Hiroshima bomb.

Ch. 7, Prob. 26.

$E_K = \frac{3}{2} kT = \frac{1}{2} mv^2$

Solving for v, we get : $v = \sqrt{\frac{3 kT}{m}} = \sqrt{\frac{3 (1.38 \times 10^{-23}) (5800)}{1.673 \times 10^{-27}}} = 1.2 \times 10^4$ m / s = 12 km / s.

Note : the answer in the back of the book for this problem is wrong.

Ch. 9, Prob. 23.

Given : $P = 1.75 \times 10^{17}$ W

a. The albdeo (a) of a planet is the amount of light reflected by the planet. So :

$P_{relfected} = aP = 0.39 (1.75 \times 10^{17}\ W) = 6.8 \times 10^{16}$ W

b. The amount absorbed is the total minus the reflected. Therefore :

$P_{absorbed} = (1 - a) P = 0.61 P = 1.1 \times 10^{17}$ W

c. At equilibrium, $P_{radiated} = P_{absorbed}$, so the flux (power per square meter) is :

$$F_{radiated} = \frac{P_{radiated}}{\text{surface area of Earth}} = \frac{P_{radiated}}{4 \pi R_{Earth}{}^2} = \frac{1.1 \times 10^{17}}{4 \pi (6.378 \times 10^6)^2} = 215\ Wm^{-2}$$

d. Based on the flux, we can find the temperature :

$F_{radiated} = \sigma T^4$

$$T^4 = \frac{F_{radiated}}{\sigma} = \frac{215}{5.67 \times 10^{-8}} = 3.8 \times 10^9\ K^4$$

$$T = \sqrt[4]{3.8 \times 10^9} = 248.2\ K = 248.2 - 273 = -24.8\ °C$$

e. The Earth ’ s actual average temperature is much higher than the number calculated above because we have an atmosphere that holds heat in. The atmosphere is opaque to some in − coming IR radiation (see atmospheric window plots), it is opaque to out − going IR radiation as well. The atmosphere traps in the heat and keeps the Earth at a livable temperature.

Ch. 9, Prob. 25.

Knowing the rate of temperature increase, we can figure out the linear trend : $y = mx + b$

y : temperature at which water boils (100 °C)
m : rate of temperature increase (20 °C / km)
x : depth from surface (in km)
b : surface temperature (20 °C)

$100 = 20 x + 20$

$x = 4$ km

Ch. 9, Prob. 28.

a. $\rho = \frac{\text{mass}}{\text{volume}} = \frac{m_{Earth}}{\frac{4}{3} \pi R_{Earth}{}^3} = \frac{5.974 \times 10^{24}}{\frac{4}{3} \pi (6.378 \times 10^6)^3} = 5500 \frac{kg}{m^3}$

b. Knowing the average density of the mantle, we can find its mass. From table 9 − 3, we know that the inner radius of the mantle is 3500 km and the outer radius is the surface. For this problem, we are assuming that the Earth consists only of the mantle and core. The mass of the crust is very small and does not affect our calculations.

$$m_{mantle} = \rho_{mantle}\ (\text{volume of Earth} - \text{volume of core})$$
$$= \rho_{mantle}\left(\frac{4}{3}\pi R_{mantle,\,outer}{}^3 - \frac{4}{3}\pi R_{mantle,\,inner}{}^3\right)$$
$$= 3500\left(\frac{4\pi}{3}\right)\left\{(6.378\times10^6)^3 - (3.5\times10^6)^3\right\} = 3.2\times10^{24}\ \text{kg}$$
$$m_{core} = m_{Earth} - m_{crust} - m_{mantle} = 5.974\times10^{24} - 3.1\times10^{24} = 2.8\times10^{24}\ \text{kg}$$
$$\rho_{core} = \frac{m_{core}}{V_{core}} = \frac{2.8\times10^{24}}{\frac{4}{3}\pi R_{core}{}^3} = \frac{2.8\times10^{24}}{\frac{4}{3}\pi\,(3.5\times10^6)^3} = 1.6\times10^4\ \frac{\text{kg}}{\text{m}^3} = 16000\ \frac{\text{kg}}{\text{m}^3}$$

This average density of the core from this calculation is of the same order as the numbers given in table 9 – 3. The small difference may be account for by our assumptions. The Earth is not made up of just the mantle and core.

Ch. 10, Prob. 34.

For this problem, we need Newton's version of Kepler's third law. We're given that th e Moon's period will be 47 days: P = 47 days = 4060800 s

$$P^2 = \left\{\frac{4\pi^2}{G\,(m_{Earth} + m_{Moon})}\right\} a^3$$
$$a^3 = \frac{G P^2\,(m_{Earth} + m_{Moon})}{4\pi^2} = \frac{(6.67\times10^{-11})\,(4060800)^2\,(5.974\times10^{24} + 7.349\times10^{22})}{4\pi^2}$$
$$= 1.68\times10^{26}$$
$$a = 5.5\times10^8\ \text{m}$$

Ch. 10, Prob. 37.

By volume, the Moon's iron core (relative to the Moon's size) is a lot smaller than the Earth's iron core. This suggests that the Moon formed from material that contained a smaller amount of iron. If the Moon and Earth formed at the same time, one would expected they would both contain the same amount of iron. However, the fact that the Moon has a smaller amount of iron supports the idea that the Moon formed by collisional ejection from the Earth. If the Earth had been differentiated at the time of the collision, then most of its iron would be in the core. The mantle contains relatively little iron. So when the collision happened, material from the mantle was ejected into space, the Moon formed from the iron – depleted (relative to the Earth) material. From this scenario, it makes sense that the Moon has a smaller iron core than the Earth.

Ch. 11, Prob. 27.

During the day, Mercury's surface heats up to 623 K.

$$\text{From Wien's law: } \lambda_{max} = \frac{0.0029}{T} = \frac{0.0029}{623} = 4.7\times10^{-6}\ \text{m} = 4.7\ \mu\text{m}$$

This is in the infrared part of the spectrum.

Extra Credit:

a. If the Earth's tidal force was strong enough to lift a rock off the Moon's surface, then it must be greater than the Moon's gravitational pull on the rock. The distance from the Earth to the Moon is assumed to be 1 / 10 the current value = 3.84×10^7 m.

$$\frac{F_{tidal}}{F_{Moon}} = \frac{\dfrac{2\,GM_{Earth}\,m_{rock}\,d_{Moon}}{r_{Earth-Moon}{}^3}}{\dfrac{GM_{Moon}\,m_{rock}}{R_{Moon}{}^2}} = \frac{2\,GM_{Earth}\,m_{rock}\,d_{Moon}}{r_{Earth-Moon}{}^3}\ \frac{R_{Moon}{}^2}{GM_{Moon}\,m_{rock}}$$
$$= \frac{2\,M_{Earth}\,d_{Moon}}{r_{Earth-Moon}{}^3}\ \frac{R_{Moon}{}^3}{M_{Moon}} = \frac{2\,M_{Earth}\,(2\,R_{Moon})}{r_{Earth-Moon}{}^3}\ \frac{R_{Moon}{}^2}{M_{Moon}} = \frac{4\,M_{Earth}\,R_{Moon}{}^3}{r_{Earth-Moon}{}^3\,M_{Moon}}$$
$$= 4\ \frac{M_{Earth}}{M_{Moon}}\left(\frac{R_{Moon}}{r_{Earth-Moon}}\right)^3 = 4\left(\frac{5.974\times10^{24}}{7.349\times10^{22}}\right)\left(\frac{1.738\times10^6}{3.84\times10^7}\right)^3 = 0.03$$

The tidal force from the Earth is about 3 % of the Moon's gravity, so it is not strong enough to pull the rock off the surface.

b. Comparing the the tidal force then and the tidal force now:

$$\frac{F_{then}}{F_{now}} = \frac{\dfrac{2\,GM_{Earth}\,m_{rock}\,d_{Moon}}{r_{Earth-Moon,\,then}{}^3}}{\dfrac{2\,GM_{Earth}\,m_{rock}\,d_{Moon}}{r_{Earth-Moon,\,now}{}^3}} = \frac{r_{Earth-Moon,\,now}{}^3}{r_{Earth-Moon,\,then}{}^3} = \left(\frac{r_{Earth-Moon,\,now}}{r_{Earth-Moon,\,then}}\right)^3 = 10^3$$

So the tidal force then was about 1000 times the tidal force now.