ASTR120
Homework 6 −Solutions

Ch. 15, Prob. 42.
The rate of mass loss is 1000 kg / s. Io has a mass of 8.932×10^{22} kg and it will lose 10 % of that mass in its lifetime. So:

$$M_{loss} = 0.1\, M_{Io} = 8.932 \times 10^{21}\ \text{kg}$$

Therefore:

$$t = \frac{\text{mass loss}}{\text{mass loss rate}} = \frac{8.932 \times 10^{21}}{1000} = 8.932 \times 10^{18}\ \text{s} = 2.83 \times 10^{11}\ \text{yrs}$$

This time scale is much longer than the lifetime of the Sun, or the age of the solar system.

Ch. 15, Prob. 52.
a. Since Enceladus and Dione have a 1 : 2 ratio of orbital periods, the time between successive oppositions would be the orbital period of Dione –– 65.7 hours

b. For this part, we want to use the small angle formula. According to the text, the linear diameter of Dione is 1.0×10^6 m. Enceladus is 2.38×10^8 m from the center of Saturn and Dione is 3.774×10^8 m from Saturn. At opposition, the distance between Enceladus and Dione is $3.774 \times 10^8 - 2.38 \times 10^8 = 1.394 \times 10^8$ m. The angular diameter of Dione can be found using the small angle formula:

$$\alpha = \frac{206265\, d}{D} = \frac{206265\,(1.0 \times 10^6)}{1.394 \times 10^8} = 1479\ \text{arcsec} = 0.41\,°$$

The angular diameter of Dione as seen from Enceladus is slightly smaller than the angular diameter of the Moon as seen from Earth.

Ch. 17, Prob. 33.
a. $\rho_{Dactyl} = 2500\ \text{kg}/\text{m}^3;\ r = 700\ \text{m}$

$$M = \rho V = \rho \left(\frac{4}{3}\pi r^3\right) = 2500 \left(\frac{4}{3}\pi 700^3\right) = 3.6 \times 10^{12}\ \text{kg}$$

b. $$v_{esc} = \sqrt{\frac{2\,GM}{r}} = \sqrt{\frac{2\,(6.67 \times 10^{-11})\,(3.6 \times 10^{12})}{700}} = 0.83\ \text{m}/\text{s}$$

A professional pitcher can throw the ball fast enough to escape Dactyl.

Ch. 17, Prob. 36.
We don't have to worry about projection effects for this problem. We can use the small angle formula here again. $\alpha = 10\,°$ and $D = 1.39$ AU.

$$d = \frac{\alpha D}{206265} = \frac{(10)\,(3600)\,(1.39)}{206265} = 0.24\ \text{AU} = 3.6 \times 10^7\ \text{km}$$

Ch. 17, Prob. 40.
a. The nucleus is a cube, 10 km across on each side.

$$M = \rho V = \rho r^3 = 1000\,(1 \times 10^4)^3 = 1 \times 10^{15}\ \text{kg}$$

b. The mass of the tail is 1 % that of the nucleus. So:

$$M_{tail} = 0.01\, M = 1 \times 10^{13}\ \text{kg}$$

To get the average density of the tail, we need find the volume. The tail is 1×10^{11} m long. Since the nucleus is a cube, we will assume the tail has square cross sections 1×10^9 m on each side. Therefore:

$$\rho_{tail} = \frac{M_{tail}}{V} = \frac{1 \times 10^{13}}{(1 \times 10^{11})\,(1 \times 10^9)^2} = 1 \times 10^{-16}\ \text{kg}/\text{m}^3$$

This is a lot less dense than the air on Earth

c. The tail is not dense enough to have a lot of material to affect humans on Earth.

Ch. 8, Prob. 23.
^{40}K has a half life of 1.3 billion years. This means, $\frac{1}{2}$ of ^{40}K will decay into ^{40}Ar in that

amount of time. So, it takes 1.3 billion years for $\frac{1}{2}$ of the original sample of ^{40}K to decay, and another 1.3 billion years for $\frac{1}{2}$ of the remaining half $\left(\frac{1}{4}\text{ of the original sample}\right)$ to decay. So it takes 2.6 billion years for $\frac{3}{4}$ of the original ^{40}K to decay.

Ch. 8, Prob. 30.

Given : D = 59 ly, a = 0.48 AU, e = 0.40, P = 116.7 days. To find the mass of the star, we need to use Newton's version of Kepler's third law :

$$P^2 = \frac{4\pi^2}{G\,(M_{star} + M_{planet})}\,a^3$$

We need to conver P to seconds, a to meters, and since the star is much more massive than the planet, $M_{star} + M_{planet} = M_{star}$.

$$M_{star} = \frac{4\pi^2\,a^3}{GP^2} = \frac{4\pi^2\,(0.48 \times 1.49 \times 10^{11})^3}{6.67 \times 10^{-11}\,(116.7 \times 24 \times 3600)^2} = 2.12 \times 10^{30}\text{ kg}$$

This is 1.1 times the mass of the Sun.

Ch. 8, Prob. 31.

a. $v = \frac{\text{distance}}{\text{time}} = \frac{2\pi r}{\text{time}} = \frac{2\pi\,(7.42 \times 10^8)}{11.86 \times 3600 \times 24 \times 365} = 12.5\text{ m/s}$

b. $\alpha = \frac{206265\,d}{D} = \frac{206265\,(2 \times 7.42 \times 10^8)}{25 \times 9.46 \times 10^{15}} = 0.0013\text{ arcsec}$

c. $\alpha = \frac{206265\,d}{D} = \frac{206265\,(2 \times 7.42 \times 10^8)}{360 \times 9.46 \times 10^{15}} = 9.0 \times 10^{-5}\text{ arcsec}$

Extra Cedit

The Trojan asteroids have orbits that are 60° ahead and 60° behind the orbit Jupiter in its orbit. This means that the asteroids, Jupiter, and Sun will always make a 60° angle. Therefore, Jupiter will always appear to be in the gibbous phase. Whether it is waxing or waning depends on whether the asteroid is 60° ahead (waning) or 60° behind (waxing) Jupiter. Jupiter will appear to move against the background stars. It will take 1 Jupiter year for Jupiter to come back to the same place.