ASTR 120

Fall 2006

Homework Assignment No. 2 – **Solutions**

1. (a) The Moon's sidereal angular velocity (rate of rotation):

$$\omega = \frac{360^\circ}{27.3\ days} = \frac{360^\circ}{27.3\ days \times 1440\ min/day} = 9.16 \times 10^{-3}\ min^{-1}$$

$$t = \frac{.5^\circ}{\omega} = \frac{.5^\circ}{9.16 \times 10^{-3}\ min^{-1}} = 54.6\ minutes = .91\ hrs$$

(b)

$$\theta = \omega\, t = 9.16 \times 10^{-3}\ min^{-1} \times (12\ hrs \times 60\ min/hr) = 6.59^\circ$$

Yes. As per the hinted fig. (1-10) such a change in angle is easily visible/measurable with the naked eye.

2. A sidereal month is $\sim$ 27.3 days and a synodic $\sim$ 29.5.
So there are (365.25 days/year)/27.3 days $\sim$ 13.38 sidereal months in a year
and there are (365.25 days/year)/29.5 days = 12.38 synodic months in a year.
Thus there is $\sim$ 1 more sidereal than synodic month per year.

3. F= m a = 3 kg $\times$ 2 m/s^2 = 6 Newtons
and $\frac{2F}{m} = 2a = 4m/s^2$

4. (a) Total major axis = 2 AU + 6 AU, so semi-major axis, $a = \frac{2+6}{2} = 4AU$
(b) Kepler's 2nd Law: $P^2 = a^3 = (4AU)^3 = 64$
So $P = \sqrt{64} = 8\ yrs$

Extra "A":

(a) Ceres' semi-major axis: $a = \frac{2.99+2.55}{2} = 2.77AU$
(b) $P^2 = a^3 = (2.77AU)^3 = 21.25$ So $P = \sqrt{21.25} = 4.61\ yrs$

5. (a) Again, use Kepler's 2nd Law: P = 64 yrs
So... $a = P^{\frac{2}{3}} = 64^{\frac{2}{3}} = 16AU$
(b) Aphelion distance (d_{Ap}) + Perihelion distance (d_{Per}) = 2 a
In this case: $d_{Per} = 2a - d_{Ap} = (2 \times 16) - 31.5 = .5AU$

Extra "B': $a = P^{\frac{2}{3}} = 2500^{\frac{2}{3}} = 184.2AU$
$d_{Ap} = 2a - d_{Per} = (2 \times 184.2) - .9 = 367.5AU$

6. Gravitational acceleration: $g = \frac{GM}{R^2}$
On this other planet $M_{PlanetX} = 4M_\oplus$ and $R_{PlanetX} = 4R_\oplus$ so

$$g_{PlanetX} = \frac{GM_{PlanetX}}{R^2_{PlanetX}} = \frac{G(4M_\oplus)}{(4R_\oplus)^2} = \frac{g_\oplus}{4}$$

Extra "C":

Phobos:

$M_{Phobos} = 1.1 \times 10^{16}\ kg$, $D_{Phobos} = 23\ km$ (I chose the middle value...)

Convert these to "Earth Masses" and "Earth Radii":

$M_{Phobos} = \frac{1.1 \times 10^{16}\ kg}{5.9 \times 10^{24}\ kg/M_{\oplus}} = 1.86 \times 10^{-9}\ M_{\oplus}$

$R_{Phobos} = .5 \times D_{Phobos} = \frac{(.5 \times 23\ km)}{6378.137 km/R_{\oplus}} = 1.8 \times 10^{-3}\ R_{\oplus}$

$$g_{Phobos} = \frac{GM_{Phobos}}{R^2_{Phobos}} = \frac{G(1.86 \times 10^{-9}\ M_{\oplus})}{(1.8 \times 10^{-3}\ R_{\oplus})^2} = 5.74 \times 10^{-4}\ g_{\oplus}$$

Callisto:

$M_{Callisto} = 1.077 \times 10^{23}\ kg = \frac{1.077 \times 10^{23}\ kg}{5.9 \times 10^{24}\ kg/M_{\oplus}} = 1.83 \times 10^{-2}\ M_{\oplus}$

$D_{Callisto} = 4800\ km$; $R_{Callisto} = .5 \times D_{Callisto} = \frac{(.5 \times 4800\ km)}{(6378.137 km/R_{\oplus})} = 3.76 \times 10^{-1}\ R_{\oplus}$

$$g_{Callisto} = \frac{GM_{Callisto}}{R^2_{Callisto}} = \frac{G(1.83 \times 10^{-2}\ M_{\oplus})}{(3.76 \times 10^{-1}\ R_{\oplus})^2} = 1.29 \times 10^{-1}\ g_{\oplus}$$

Titan:

$M_{Titan} = 1.34 \times 10^{23}\ kg = \frac{1.34 \times 10^{23}\ kg}{5.9 \times 10^{24}\ kg/M_{\oplus}} = 2.27 \times 10^{-2}\ M_{\oplus}$

$D_{Titan} = 5150\ km$; $R_{Titan} = .5 \times D_{Titan} = \frac{(.5 \times 5150\ km)}{(6378.137 km/R_{\oplus})} = 4.04 \times 10^{-1}\ R_{\oplus}$

$$g_{Titan} = \frac{GM_{Titan}}{R^2_{Titan}} = \frac{G(2.27 \times 10^{-2}\ M_{\oplus})}{(8.075 \times 10^{-1}\ R_{\oplus})^2} = 0.139\ g_{\oplus}$$

7.

$$F_{Saturn} = \frac{GM_{\odot}M_{Saturn}}{d^2_{Saturn}} = \frac{GM_{\odot}(100M_{\oplus})}{(10d_{\oplus})^2} = F_{\oplus}$$

$$a = \frac{F}{m} \quad so \quad a_{Saturn} = \frac{F_{Saturn}}{m_{Saturn}} \quad and \quad a_{\oplus} = \frac{F_{\oplus}}{m_{\oplus}}$$

Since $M_{Saturn} = 100M_{\oplus}$ $\quad a_{Saturn} = \frac{a_{\oplus}}{100}$

Extra Credit:

1. The period is one day.
2. Use Newton's form of Kepler's 2nd Law: $P^2 = \frac{4\pi^2}{GM_{\oplus}}a^3$ (we can ignore $M_{Satellite}$ in the denominator since $M_{\oplus} >> M_{Satellite}$) so

$$a = (P^2 \frac{GM_{\oplus}}{4\pi^2})^{1/3} =$$

$$\left((1\ day \times 8.64 \times 10^4\ sec/day)^2 \frac{(6.67 \times 10^{-8} dyn\ cm^2\ g^{-2})(5.9 \times 10^{27}\ g)}{4\pi^2} \right)^{1/3} =$$

$$4.2 \times 10^9\ cm = 4.2 \times 10^4\ km$$

3. If its orbit was inclined with respect to Earth's equator then the point directly under the satellite would not remain the same; it would oscillate in latitude even if the orbit was synchronous with the Earth's rotation.

   Think about the Sun's motion in the sky; it changes in latitude (i.e. the sub-solar point does) due to the fact that Earth's axis is tilted 23.5 degrees.

   Also, don't be confused into thinking that only a geosynchronous orbit can have a one day period; that's not correct.