

ASTR 120
Fall 2006
Homework Assignment No. 3 – Solutions

1. (5.22) Use Wein's law (but first convert from Fahrenheit to Kelvin, of course...)

$$T_K = \frac{5}{9}(T_F - 32) + 273 \quad T_K = \frac{5}{9}(98.6 - 32) + 273 = 310K$$

Wein's law:

$$\lambda_{max} = \frac{0.0029 \text{ K m}}{T} \quad \lambda_{you} = \frac{0.0029}{310} = 9.35 \times 10^{-6} \text{ m} = 9.35 \mu\text{m}$$

This in in the IR part of the spectrum.

2. (5.28) (a) Wein's law again:

$$\lambda = \frac{0.0029 \text{ K m}}{320 + 273} = 4.9 \mu\text{m}$$

This in in the IR part of the spectrum.

(b) $\Delta F = \left(\frac{T_{Pele}}{T_{surf.}}\right)^4 = \left(\frac{593}{273-150}\right)^4 = 540.25$ times as much flux

Note: Some interpreted the question to mean "what is the difference" in flux, which seems like a reasonable interpretation. For that, the correct answer would be:

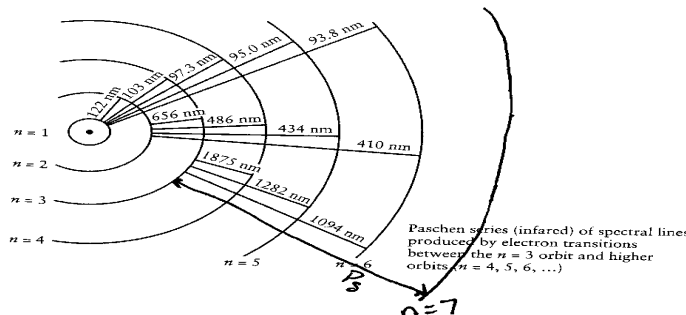
$$\Delta F_2 = \sigma(T_{Pele}^4 - T_{surf.}^4) = 6.9 \times 10^3 \text{ W/m}^2$$

3. (5.32) (a) To get the wavelength and/or energy of a transition in a Hydrogen atom use:

$$\frac{1}{\lambda_{P_\delta}} = R \left(\frac{1}{n_i^2} - \frac{1}{n_f^2} \right)$$

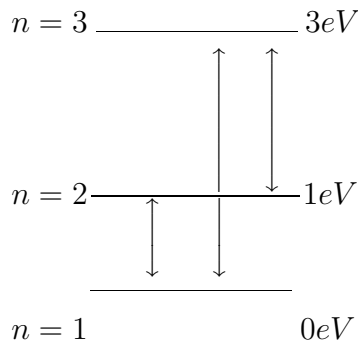
$$\lambda_{P_\delta} = \left[1.097 \times 10^7 \left(\frac{1}{7^2} - \frac{1}{3^2} \right) \right]^{-1} = 1.005 \times 10^{-6} \text{ m} = 1.005 \mu\text{m}$$

(b) This diagram was borrowed from *Freedman*, and modified by yours truly. The (clearly) hand drawn additions (labeled "P_δ" and "n=7") show the Paschen-δ transitions requested.



(c) IR

4. (5.36) Call the 3 levels $n=1$, $n=2$, $n=3$. $E_1 = 0 \text{ eV}$, $E_2 = 1 \text{ eV}$, $E_3 = 3 \text{ eV}$.



The differences between levels are $\Delta E_{12} = 1 \text{ eV}$. $\Delta E_{13} = 3 \text{ eV}$. $\Delta E_{23} = 2 \text{ eV}$.

The Planck constant in eV: $h_{eV} = 4.135 \times 10^{-15}$. And $c = 3 \times 10^8 \text{ m/s}$

$$E = \frac{hc}{\lambda} \text{ so } \lambda = \frac{hc}{\Delta E} = \frac{1.24 \times 10^{-6}}{\Delta E}$$

$$\lambda_{12} = 1.24 \times 10^{-6} \text{ m} = 1.24 \mu\text{m} \text{ This is IR.}$$

$$\lambda_{13} = 4.135 \times 10^{-7} \text{ m} = 0.4 \mu\text{m} \text{ This is in the Visible.}$$

$$\lambda_{23} = 6.2 \times 10^{-7} \text{ m} = 0.62 \mu\text{m} \text{ This is also Visible.}$$

5. (6.25) If it were not cooled, the telescope's own blackbody radiation would emit more photons than would be detected for most targets of observation. This would overwhelm the signal detected from a target, thus making the telescope fairly useless.
6. (6.31) The light gathering power of a telescope is proportional to its primary's *area*. So here $8.3^2/2.4^2 = 11.96$.

This is Subaru's advantage over the Hubble - its light gathering power; it will get more photons from the same target for equal amounts of exposure, thus making a "brighter" image.

HST's advantage over Subaru is its greater resolving power. Because it is above Earth's atmosphere, "seeing" (blurring of images due to refraction by air) is not an issue. So its images will be much clearer and able to resolve smaller (angular)/farther objects observed.

7. (6.32) (a) The magnification power is

$$P = \frac{f_{\text{objective}}}{f_{\text{eyepiece}}} = \frac{2\text{m}}{9\text{mm}} = \frac{2\text{m}}{9 \times 10^{-3}\text{m}} = 222$$

(b)

$$P = \frac{f_{\text{objective}}}{f_{\text{eyepiece}}} = \frac{2\text{m}}{20\text{mm}} = \frac{2\text{m}}{20 \times 10^{-3}\text{m}} = 100$$

(c)

$$P = \frac{f_{\text{objective}}}{f_{\text{eyepiece}}} = \frac{2\text{m}}{55\text{mm}} = \frac{2\text{m}}{55 \times 10^{-3}\text{m}} = 36.36$$

(d)

$$\theta = 2.5 \times 10^5 \frac{600\text{nm}}{20\text{cm}} = 0.75'' \text{ (arcsec)}$$

(e) The “seeing disk” is $\sim 0.5''$, so one *might* be able to resolve something that was $0.75''$, if the weather was perfect. The seeing disk is the (angular) size that a point source (e.g. a star) would be blurred to by that amount of “seeing” - so if two features/sources were $0.75''$ apart they would have seeing disks that overlap (by $0.25''$) but one could probably still determine that they were disks and where the center of each is located (which is the actual point source’s location.)

8. (6.36) The long and short of it is: not really, but sort of...

As above (32e) $\theta = 2.5 \times 10^5 \frac{\lambda}{D}$ What are λ and D for this situation? D is the mirror of HST, which is 2.4 m. λ could theoretically be anywhere in the range of wavelengths that HST can observe: $115 \text{ nm (UV)} < \lambda < 1 \mu \text{ m (IR)}$. The boundaries of HST’s range would have angular resolutions of:

$$\theta_{1\mu m} = 2.5 \times 10^5 \frac{10^{-6}m}{2.4m} = 0.1''$$

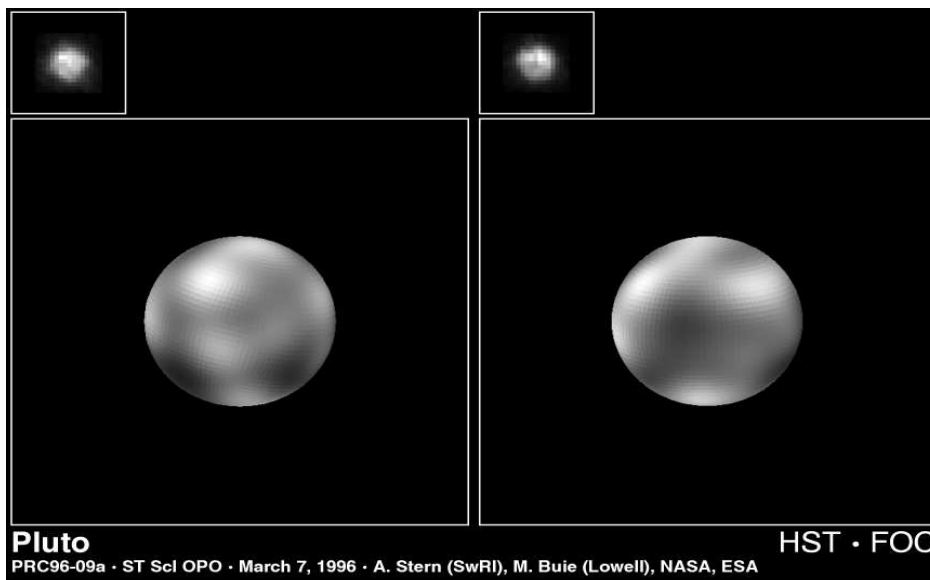
$$\theta_{115nm} = 2.5 \times 10^5 \frac{115 \times 10^{-9}m}{2.4m} = 0.012''$$

What is θ of the target here? The angular size of the “feature” on Pluto. What size is a feature of interest? A *maximal* value might be $\sim R_{Pluto}$ - i.e. assume that something that makes about half the disk of the planet viewed look different from the other half could potentially be a “feature” of interest. (By analogy, on Earth, this might include the Eurasian continents but not the Americas...) Using the small angle approximation, this would make

$$\theta \sim \frac{2300km}{39.5AU(1.5 \times 10^8km/AU)} \sim 3.88 \times 10^{-7}radians \sim 0.08''$$

That is between the two angular resolutions above, so with *some part* of the spectrum it would be less than the angular resolution of HST.

See these pictures of Pluto by HST (<http://www.solarviews.com/raw/pluto/pluto4.gif>)



Note that the two smaller inset pictures at the top are *actual* images from Hubble. The larger images (bottom) are from a global map constructed through computer image processing performed on the Hubble data. The tile pattern is an artifact of the image enhancement technique.

Extra Credit: (6.38)

(a) $\nu = 557 \text{ GHz}$ so $\lambda = c/\nu = 5.39 \times 10^{-4} \text{ m} = 0.539 \text{ mm}$. This is just on the borderline between IR and Microwave, sometimes called the “sub-millimeter” region.

(b) Atmospheric absorption - at that wavelength the atmosphere is opaque.

(c)

$$\theta = 4' = 240'' = 2.5 \times 10^5 \frac{5.39 \times 10^{-4}}{D}$$

Thus, solving for D, we get $D = 0.56 \text{ m}$