ASTR 120

Fall 2006

Homework Assignment No. 4 – **Solutions**

1. (7.23)

   (a) $M = \rho V = \rho \frac{4}{3}\pi R^3 = (2500kg/m^3)\frac{4}{3}\pi(1000m)^3 = 1.05 \times 10^{13}kg$
   (NB: the question gives the diameter = 2 km; for the above calculation one needs the *radius* = D/2 = 1 km = 1000 m)

   (b)

   $$v_{esc} = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2(6.67 \times 10^{-11})(1.05 \times 10^{13})}{1000}} = 1.18m/s$$

   (c) Such an astronaut would "escape" from the asteroid's gravity - i.e. fly off the asteroid into space.

2. (7.24)

   (a) $KE = \frac{1}{2}mv^2 = .5(1.05 \times 10^{13})(25 \times 10^3)^2 = 3.28 \times 10^{21}Joules$

   (b) $3.28 \times 10^{21}/(20 \times 4.2 \times 10^{12}) = 3.9 \times 10^7$ times as much energy!!!

3. (9.23)

   (a) $F_{reflected} = A_b \times F_{\odot} = 0.39 \times 1.75 \times 10^{17}W = 6.825 \times 10^{16}W$

   (b) $F_{emitted} = F_{absorbed} = F_{\odot} \times (1 - A_b) = (1.0 - 0.39) \times 1.75 \times 10^{17}W = 1.0675 \times 10^{17}W$

   (c)

   $$\frac{F_{emitted}}{4\pi R_{\oplus}^2} = \frac{1.0675 \times 10^{17}W}{4\pi(6.378 \times 10^6 m)^2} = 208.8\ W/m^2$$

   (d) Use $F = \sigma T^4$ so $T = (F_{emitted}/\sigma)^{1/4} = (208.8/5.67 \times 10^{-8})^{1/4} = 246.3K = -26.7C$

   (e) The Earth's actual average temperature is higher due to the greenhouse effect caused by the atmosphere, which traps some "escaping" flux.

4. (9.26) The Core: $R_{core}^3/R_{\oplus}^3$ (the $\frac{4}{3}\pi$ factor cancels.) = $(\frac{3500km}{6378km})^3 = 0.165$

   The Mantle: $R_{mantle}^3/R_{\oplus}^3 - R_{core}^3/R_{\oplus}^3 = (\frac{6343km}{6378km})^3 - 0.165 = 0.818$

   The Crust: $1.0 - R_{mantle}^3/R_{\oplus}^3 - R_{core}^3/R_{\oplus}^3 = 1.0 - 0.165 - 0.818 = 0.017$

5. (9.27) Compare volumes $\times$ densities (again $\frac{4}{3}\pi$ factor cancels out):

   $$\frac{(1300 \times 10^3)^3(13000)}{(6378 \times 10^3)^3(3500)} = 0.03$$

   Alternatively, one could use $\rho \times 4/3\pi r^3$ and compare with the "listed" total mass of the Earth:

   $$\frac{\frac{4}{3}\pi(1300 \times 10^3)^3(13000)}{5.974 \times 10^24kg} = 0.02$$

6. (10.30) Perhaps some, but quite few in number. Impact breccias are rocks that were formed by impacts that break apart different rocks and mix them and fuse them together. Since the bombardment rate of the Moon seems to have been greatly reduced around 3.1 Gya (shortly after the major impacts that formed the maria) it is much less likely that impact breccias formed since then than before then.

7. (10.34) Use Newton/Kepler's Law:

$$P^2 = \frac{4\pi^2}{G(m_1+m_2)}a^3 \qquad a = \left(\frac{P^2G(m_1+m_2)}{4\pi^2}\right)^{1/3} =$$

$$\left(\frac{(47days \times 86400s/day)^2(6.67\times10^{-11})(5.9\times10^{24}+7.35\times10^{22}kg)}{4\pi^2}\right)^{1/3} =$$

$$5.5\times10^8 m = 5.5\times10^5 km$$

8. (11.23) See Fig. 11-1 in *Freedman*. To get the angular size ($\theta \sim Diameter_{Mercury}/distance$) we need to get the distance to Mercury at its greatest eastern elongation. Use cos(28°) = 1 AU / d (approximate the Sun-Mercury-Earth angle as 90°.)

   This gives us d $\sim$ 1.05 AU. (NB: It should also be clear now, retroactively, why we should use the greatest eastern elongation to get the largest angular size: the distance is inversely proportional to the cosine, and cos(18°) $>$ cos(28°), so using 18° gives a shorter distance to Mercury.)

$$\theta \sim \frac{D_{Merc}}{d} = \frac{4880km}{1.05AU(1.5\times10^8 km/AU)} = 3.09\times10^{-5}rad = 6.39''$$

   Many people used an arrangement of Mercury in inferior conjunction (like during a transit...) and made calculations based on that. This is another reasonable interpretation of the problem:

$$\theta \sim \frac{D_{Merc}}{d} = \frac{4880\ km}{(1.0-0.387)AU(1.5\times10^8 km/AU)} = 5.3\times10^{-5}rad = 10.9''$$

   Note that here I just used "a" - not aphelion ($r = a(1+e)$)... for that read on!

   Prof. McGaugh had this to say:

   > Yeah, that is a really poorly phrased question!
   >
   > What they really appear to have done is take Mercury to be at inferior conjunction AND at aphelion when Earth is also at perihelion. If I use the minimum earth-sun distance from table 9-1 (1.471×10^8 km) and the maximum mercury-sun distance from table 11-1 (6.98×10^7 km) I get 13.0″
   >
   > So I guess they want not only what alignment Mercury has, but also where along their orbits both Earth and Mercury would have to be in order to completely maximize its angular size. (Never mind that at the rate Mercury's perihelion position advances, we are in no danger of ever seeing this.)

   Kudos to all who correctly interpreted the authorial intent and did that! Any solution that was more or less like one of these was awarded the points (if calculated correctly...)

Extra Credit: (10.36)

(a) For the tidal force to overcome the Moon's gravity would require:

$$F_{tidal} = \frac{2\ G\ M_{\oplus}\ m\ d_{Moon}}{r^3_{\oplus-Moon}} > F_{grav} = \frac{G\ M_{Moon}\ m}{d^2_{Moon}}$$

Cancel the Gm on both sides and plug in the numbers:

$$\frac{(5.9 \times 10^{24})(3476 \times 10^3)}{(38440 \times 10^3)^3} > \frac{7.349 \times 10^{22}}{(0.5 \times 3476 \times 10^3)^2}$$

i.e. $2.2 \times 10^{17} > 4.23 \times 10^{19}$ which is *not* true, so the tidal force could not have pulled rocks off the Moon.

(b)

$$F_{tidal-net} \propto \frac{2GM_{\oplus} d_{Moon}}{r^6_{\oplus-Moon}}$$

$F_{tidal} \propto 1/r^6_{Earth-Moon}$ so if the Moon-Earth distance was 1/10 of today's value, the tidal force would have been 1,000,000 times greater!