

ASTR 120
Fall 2006
Homework Assignment No. 5 – Solutions

- (9.37) The Hydrogen would have escaped into space. The Carbon and Nitrogen remained on Earth as their thermal velocities would not have exceeded the Earth's escape velocity.
- (12.37) The weight of the column of water will be 9.8 N per kg and each n meters has 1000 kg. So we want $9 \times 10^6 \text{ N/m}^2 = 9.8 \text{ N/kg} \times n \times 1000 \text{ kg/m}^2$ and $n = \frac{9 \times 10^6}{9.8 \times 1000} = 918.37 \text{ m}$.
- (13.49) The sky is not just black (with stars, moons, sun...) Some gas is scattering sunlight and causing the sky to have diffuse lighting and color. The dunes in the picture are also evidence of an atmosphere. Dunes form only by wind carrying sand/dust and depositing it on the side of some "obstacle".

- (14.42) Jupiter's total mass is $1.899 \times 10^{27} \text{ kg}$. 1/4 of that is Helium so $M_{He} = 0.25 \times 1.899 \times 10^{27} = 4.75 \times 10^{26} \text{ kg}$. 3/4 is Hydrogen so $M_H = 0.75 \times 1.899 \times 10^{27} = 1.42 \times 10^{27} \text{ kg}$. The mass of a hydrogen atom: $m_H = 1.67 \times 10^{-27} \text{ kg}$, so to get the number of atoms, just divide:
 $n_H = \frac{1.42 \times 10^{27}}{1.67 \times 10^{-27}} = 8.5 \times 10^{53} \text{ H atoms}$

And for Helium: $m_{He} = 4 \times 1.67 \times 10^{-27} \text{ kg}$

$$n_{He} = \frac{4.75 \times 10^{26}}{4 \times 1.67 \times 10^{-27}} = 7.1 \times 10^{52} \text{ He atoms}$$

- (14.48)

$$F = \frac{GM_J m_{Gal.}}{R^2}$$

Since we know Galileo's weight on Earth, it is convenient here to express Jupiter's mass and radius in terms of Earth masses and radii:

$$F_J = F_{\oplus} \frac{M_J}{R_J^2} = (3320N) \frac{317.8M_{\oplus}}{11.209R_{\oplus}^2} = 8397 \text{ N}$$

- (14.49) In section 14-6 we are given the mass of Jupiter's core as $8 M_{\oplus}$ and its diameter as 11000 km (thus $R = 5500 \text{ km}$.)

$$\rho = \frac{M}{\frac{4\pi R^3}{3}} = \frac{8(5.974 \times 10^{24}) \text{ kg}}{\frac{4\pi(5.5 \times 10^6 \text{ m})^3}{3}} = 68577 \text{ kg/m}^3$$

The average density of the Earth (Table 9-1) is 5515 kg/m^3 so Jupiter's core's density is ~ 12 times denser than Earth's average density. The average density of the Earth's inner core (Table 9-3) is 13000 kg/m^3 , ~ 5 times Earth's inner core.

- (15.47)

$$v_{esc} = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2(6.67 \times 10^{-11})(1.34 \times 10^{23} \text{ kg})}{2575 \times 10^3 \text{ m}}} = 2635 \text{ m/s} \sim 2.6 \text{ km/s}$$

$$v_{thermal} = \sqrt{\frac{3kT}{m}}$$

To retain an atmospheric gas, we want $v_{esc} \geq 6 v_{thermal}$ so

$$\frac{3kT}{m_{lim}} = \frac{v_{esc}^2}{36} \quad \text{so} \quad m_{lim} = \frac{108kT}{v_{esc}^2} = \frac{108(1.38 \times 10^{-23})(95)}{2635^2} = 2.04 \times 10^{-26} \text{ kg} = 12 \text{ AMU}$$

8. (16.32)

$$r_{ap} = a(1 + e) \text{ and } r_{peri} = a(1 - e)$$

The Solar Flux (at Earth, 1 AU) is $F_{\odot} = 6.41 \times 10^7 \text{ W/m}^2$. For Pluto $a = 39.5 \text{ AU}$ and $e = 0.25$ so

$$r_{ap} = 39.5(1 + 0.25) = 49.375 \text{ AU} \quad \text{and} \quad r_{peri} = 39.5(1 - 0.25) = 29.625 \text{ AU}$$

$$F_{peri} = \frac{F_{\odot}}{r_{peri}^2} = \frac{6.41 \times 10^7}{29.625^2} = 7.3 \times 10^4 \text{ W/m}^2$$

and

$$F_{ap} = \frac{F_{\odot}}{r_{ap}^2} = \frac{6.41 \times 10^7}{49.375^2} = 2.63 \times 10^4 \text{ W/m}^2$$

$$\frac{F_{peri}}{F_{ap}} = \frac{7.3 \times 10^4}{2.63 \times 10^4} = 2.77$$

Extra Credit: (14.52)

(a) The radius of the outer edge of the A ring is $r_A = 137,000 \text{ km}$ and the inner edge of the B ring is $r_B = 92,000 \text{ km}$. Kepler's/Newton's Law:

$$P^2 = \frac{4\pi^2}{G(m_1 + m_2)} a^3$$

$$P_A = \sqrt{\frac{4\pi^2}{6.67 \times 10^{-11}(5.685 \times 10^{26} \text{ kg})} (1.37 \times 10^8 \text{ m})^3} = 5.17 \times 10^4 \text{ s} = 14.4 \text{ hrs.}$$

$$P_B = \sqrt{\frac{4\pi^2}{6.67 \times 10^{-11}(5.685 \times 10^{26} \text{ kg})} (9.2 \times 10^7 \text{ m})^3} = 2.85 \times 10^4 \text{ s} = 7.9 \text{ hrs.}$$

(b) Saturn's (equatorial) rotation period = $10^h 13^m 59^s$. This is in between the two periods for the rings, so the B ring rotates faster and the A ring slower than the observer on Saturn's clouds. Thus from the observer's viewpoint, the B ring will be moving one way (east) and the A ring the other way (west).