ASTR 120
Fall 2006
Homework Assignment No. 6 – **Solutions**

1. The rate of mass loss is 1000 kg/s. Io's mass is 8.932×10^{22} kg. To lose 10% of that mass: $M_{lost} = 0.1 M_{Io} = 8.932 \times 10^{21}$ kg. Therefore:

$$t = \frac{M_{lost}}{mass\ loss\ rate} = \frac{8.932 \times 10^{21}}{1000} = 8.932 \times 10^{18} s = 2.83 \times 10^{11} yrs$$

   This time scale is much longer than the lifetime of the Sun, or the age of the solar system.

2. (15.52)

   (a) Since Enceladus and Dione have a 1:2 ratio of orbital periods, the time between successive oppositions would be the orbital period of Dione, i.e. 65.7 hours

   (b) Use the small angle formula. According to the text, the linear diameter of Dione is 1.0×10^6 m. Enceladus is 2.38×10^8 m from the center of Saturn and Dione is 3.774×10^8 m from Saturn. At opposition, the distance between Enceladus and Dione is $3.774 \times 10^8 - 2.38 \times 10^8 = 1.394 \times 10^8$ m. The angular diameter of Dione can be found using the small angle formula :

$$\alpha = \frac{206265d}{D} = \frac{206265(1.0 \times 10^6)}{1.394 \times 10^8} = 1479'' = 25' = 0.41°$$

   The angular diameter of Dione as seen from Enceladus is slightly smaller than the angular diameter of the Moon as seen from Earth ($\sim 0.5°$).

3. (16.38) Use Kepler's Law (since the orbit is around the Sun, we don't even need to use Newton's version.) The semi-major axis of the orbit will be (30 + 1)/2 = 15.5 AU.

$$P = \sqrt{a^3} = \sqrt{(15.5)^3} = 61.02\ yrs.$$

   But, since this is a one way trip, the trip time will be half that orbital period, i.e. ~ 30.5 yrs.

   The New Horizons trip will take about 9 years, so the time saved by swinging by Jupiter is considerable, about 20 years!

4. (17.36) The caption to Fig. 17-23 states that Hale-Bopp's tail was more than $10°$ across the sky. Once again, use the small angle formula:

$$\alpha = \frac{206265\ d}{D}$$

$$d = \frac{\alpha\ D}{206265} = \frac{(10°)(1.39 AU)}{206265} = \frac{(10 \times 3600)''(1.39 \times 1.5 \times 10^8\ km)}{206265} = 3.6 \times 10^7\ km = 0.24\ AU$$

5. (17.40)

   (a) For a cube, volume, $V = r^3$, where r is the length of a side. So

$$M = \rho V = \rho r^3 = 1000\ kg/m^3 (1 \times 10^4\ m)^3 = 1 \times 10^{15}\ kg$$

(b) $M_{tail} = 0.01 M_{nulceus} = 1 \times 10^{13}$ kg If we assume the tail has a square cross section (like the nucleus) of $10^6 \ km = 1 \times 10^9 \ m$ and its length is $10^8 \ km = 10^{11} \ m$ then its volume is $V_{tail} = (1 \times 10^9 \ m)^2 \times 10^{11} \ m = 10^{29} \ m^3$ thus

$$\rho_{tail} = \frac{M_{tail}}{V_{tail}} = \frac{1 \times 10^{13} \ kg}{10^{29} \ m^3} = 1 \times 10^{-16} \ kg/m^3$$

Note that if we assumed the tail had a circular cross section, then we'd get

$$\rho_{tail} = \frac{1 \times 10^{13} \ kg}{10^{11}(\pi(0.5 \times 10^9)^2) \ m^3} \sim 1.27 \times 10^{-16} \ kg/m^3$$

(c) The tail is not dense enough to have a lot of material to affect humans on Earth (e.g. our air is about 10^{16} times denser than the tail!).

6. (8.23) ^{40}K has a half life of 1.3 billion years. This means, 1/2 of ^{40}K will decay into ^{40}Ar in that amount of time. So, it takes 1.3 billion years for 1/2 of the original sample of 40K to decay, and another 1.3 billion years for 1/2 of the remaining half (1/4 of the original sample) to decay. So it takes 2.6 billion years for 3/4 of the original 40K to decay and the rock is 2.6 billion years old.

7. (8.30) Use Newton's version of Kepler's third law:

$$P^2 = \frac{4\pi^2}{GM_{70Vir.}} a^3$$

(we can ignore M_{planet} in the denominator since $M_{star} >> M_{planet}$). Now solve for the mass of the star: (Of course, convert P and a to proper units first: P = 116.7 days $\sim 10^7$ seconds and a = 0.48 AU = 7.2 × 10^{10} m.)

$$M_{70Vir.} = \frac{4\pi^2 a^3}{GP^2} = \frac{4\pi^2(7.2 \times 10^{10})^3}{6.67 \times 10^{-11}(10^7)^2} = 2.2 \times 10^{30} \ kg$$

$M_\odot = 2 \times 10^{30}$ kg, so $M_{70Vir.} \sim 1.1 \ M_\odot$

8. (8.31)

(a)

$$v = \frac{2\pi r}{t} = \frac{2\pi(7.42 \times 10^8 \ m)}{11.86 \ yrs(3600 \times 24 \times 365 \ s/yr)} = 12.5 \ m/s$$

(b)

$$\alpha = \frac{206265 \ d}{D} = \frac{206265(2 \times 7.42 \times 10^8)}{25 \ ly(9.46 \times 10^{15} \ m/ly)} = 0.0013''$$

This would just barely be discernable to the alien astronomer.

(c)

$$\alpha = \frac{206265 \ d}{D} = \frac{206265(2 \times 7.42 \times 10^8)}{360 \ ly(9.46 \times 10^{15} \ m/ly)} = 9 \times 10^{-5} ''$$

This would not be discernable to the alien astronomer.

Extra Credit: (17.34)

The Trojan asteroids have orbits that are 60° ahead and 60° behind Jupiter in its orbit. This means that the asteroids, Jupiter, and Sun will always make a 60° angle. Therefore, Jupiter will always appear to be in a gibbous phase. Whether it is "waxing" or "waning" depends on whether the asteroid is 60° ahead (waxing) or 60° behind (waning) Jupiter. (NB: These phases do not change, so they do not actually "wax" or "wane," but they appear as the same shape as the "waxing" or "waning" lunar phases. Also, note that they could only be seen with a telescope, of course, since they are ~ 5 AU away!)

Jupiter will appear to move against the background stars. It will take 1 Jupiter year (~ 11.8 yrs) for Jupiter to come back to the same place.