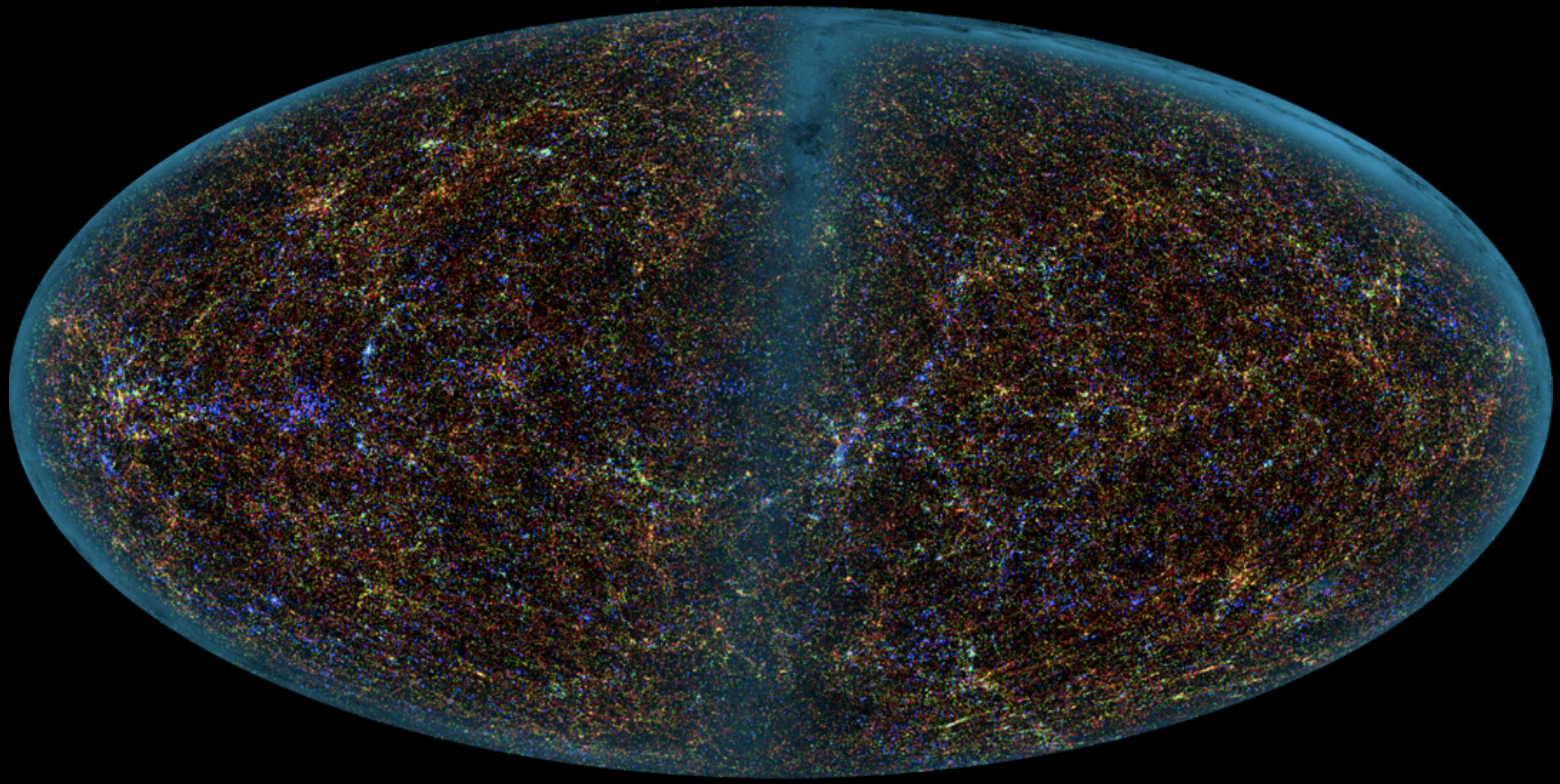
Cosmology

and Large Scale Structure



<u>Today</u>

Newtonian Cosmology

Expansion dynamics

16 September 2020

http://astroweb.case.edu/ssm/astr328/

Expansion dynamics

$$H = \frac{\dot{a}}{a}$$

$$a = (1 + z)^{-1}$$

The Acceleration equation with the cosmological constant:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P) + \frac{1}{3}\Lambda$$

can usually be replaced with a single variable, as $P = w\rho$ for a single medium.

The Pressure P is zero when matter dominates.
It is simply related to the energy density when radiation dominates.

You can see why the cosmological constant leads to acceleration!

$$\ddot{a} \sim \Lambda$$

Expansion dynamics

Friedmann equation

$$H = \frac{\dot{a}}{a}$$

$$H^2 = H^2(\Omega_m + \Omega_r + \Omega_k + \Omega_\Lambda)$$

$$a = (1+z)^{-1}$$

$$\Omega_m + \Omega_r + \Omega_k + \Omega_\Lambda = 1$$

Looks trivial, but H and Ω evolve. So really

$$\left(\frac{H}{H_0}\right)^2 = \Omega_{m_0}(1+z)^3 + \Omega_{r_0}(1+z)^4 + \Omega_{k_0}(1+z)^2 + \Omega_{\Lambda_0}$$

matter

radiation

curvature

cosmological constant

In general, must solve numerically.
But often we can ignore irrelevant terms -

Only one matters unless close to the redshift of matter-radiation equality z_{eq}.

Zero in a flat universe

Zero in a sane universe

So often only two terms matter.
In the early universe, only one,
as the mass-energy dominates.

Expansion dynamics

Friedmann equation

$$H = \frac{\dot{a}}{a}$$

$$H^2 = H^2(\Omega_m + \Omega_r + \Omega_k + \Omega_\Lambda)$$

$$a = (1+z)^{-1}$$

$$\Omega_m + \Omega_r + \Omega_k + \Omega_\Lambda = 1$$

Simplifies to

$$\left(\frac{H}{H_0}\right)^2 = \Omega_{m_0}(1+z)^3 + \Omega_{k_0}(1+z)^2$$

for a universe without a cosmological constant in the matter dominated era.

Or just $$\left(\frac{H}{H_0}\right)^2 = \Omega_{r_0}(1+z)^4$$ for the early, radiation dominated universe.

It is useful to consider the limit for domination by each case (matter, radiation, curvature, cosmological constant)

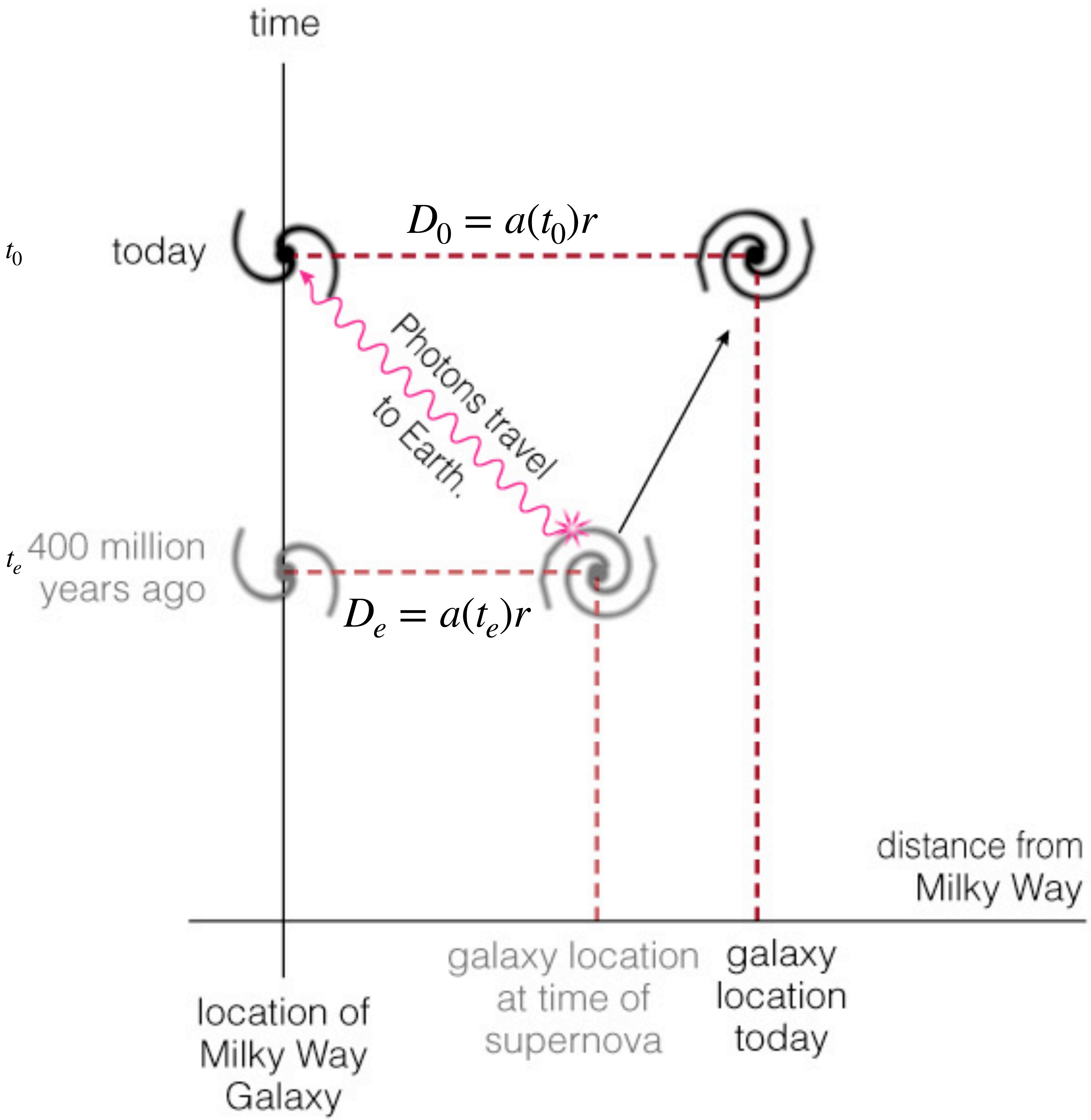
comoving coordinates constant

Robertson-Walker metric

$$ds^2 = -c^2dt^2 + a^2(t)[dr^2 + S_k^2(r)d\Omega^2]$$

for photons, $ds = 0$ so this becomes

$$cdt = a(t)dr$$



Once we know (or assume) what kind of universe we live in,
we specify the expansion history *a(t)*.

we know the expansion factor from the redshift

$$\frac{a(t_0)}{a(t_e)} = 1 + z$$

comoving coordinates constant

Once we know (or assume) what kind of universe we live in, we specify the expansion history *a(t)*.

we know the expansion factor from the redshift $\frac{a(t_0)}{a(t_e)} = 1 + z$

a photon propagating through the expanding universe traverses a distance element

$$d\ell = cdt = a(t)dr$$

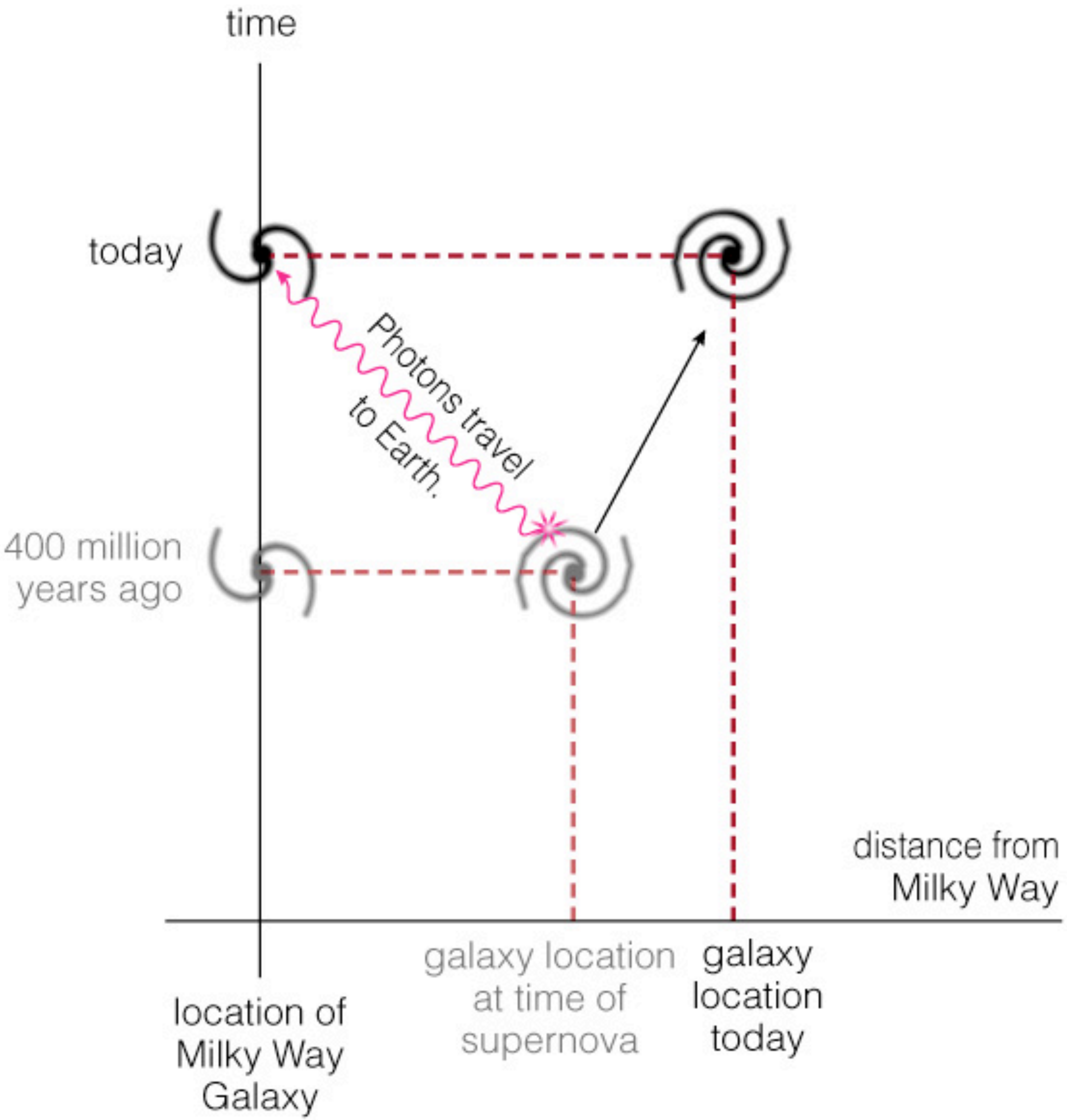
The comoving separation between two points is fixed, so

$$r = \int_0^r dr = c\int_{t_e}^{t_0} \frac{dt}{a(t)}$$

Relates observed redshift to the time of photon emission (400 Myr ago in the example at right).

t_0

t_e



comoving coordinates constant

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a photon propagating through the expanding universe traverses a distance element

$$d\ell = cdt = a(t)dr$$

The comoving separation between two points is fixed, so

$$r = c\int_{t_1}^{t_2} \frac{dt}{a(t)} = c\int_{t_2}^{t_3} \frac{dt}{a(t)}$$

and

$$\frac{a(t_2)}{a(t_1)} = 1 + z_{1\to2} \qquad \frac{a(t_3)}{a(t_2)} = 1 + z_{2\to3}$$

relates the redshift to the expansion factor