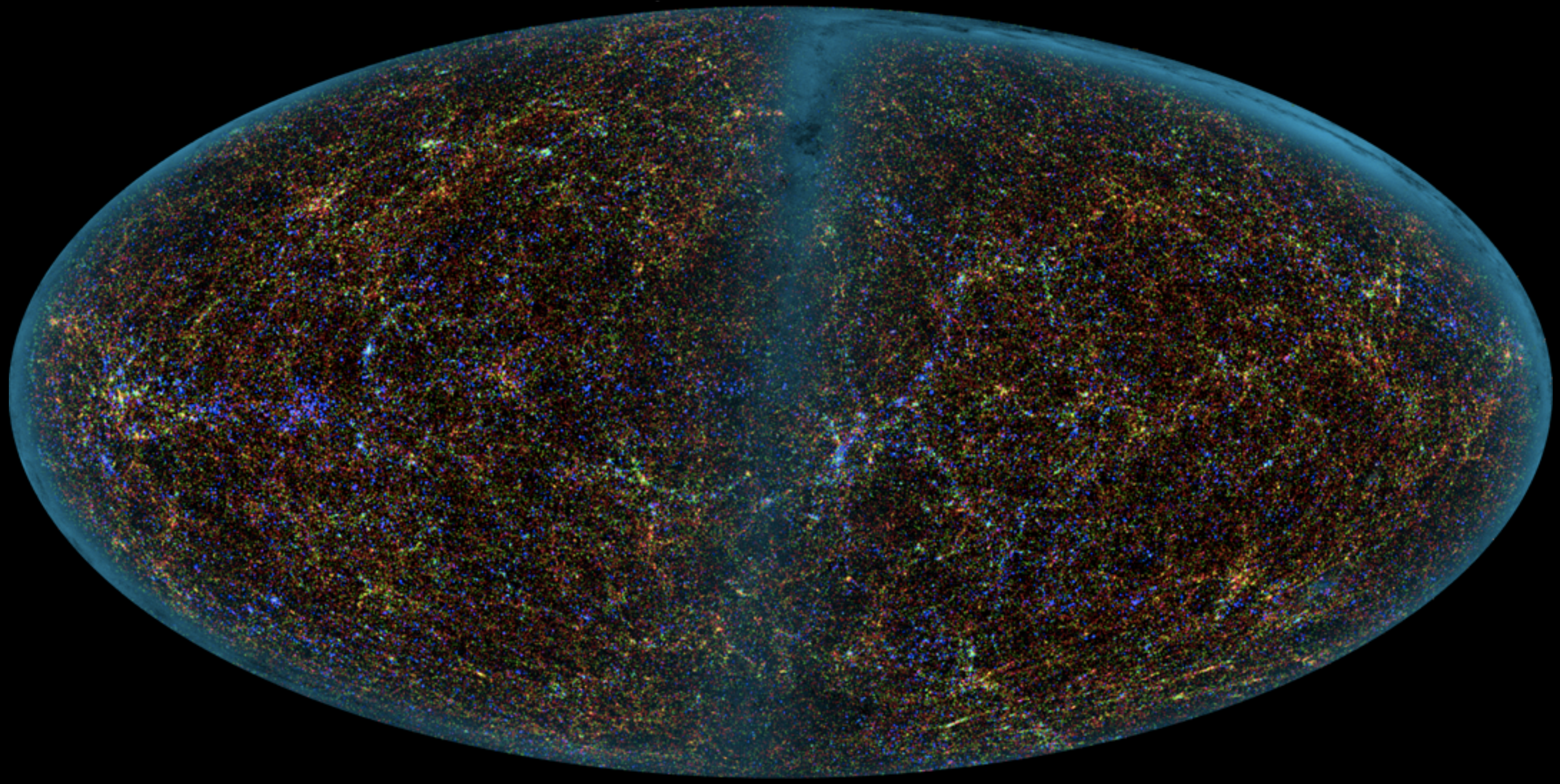
Cosmology
and Large Scale Structure



<u>Today</u>
Time and Distance
Observational Tests

Expansion history

$$\Omega_m + \Omega_r + \Omega_k + \Omega_\Lambda = 1$$

Friedmann equation

$$H = \frac{\dot{a}}{a}$$

$$H^2(z) = H_0{}^2[\Omega_{m_0}(1+z)^3 + \Omega_{r_0}(1+z)^4 + \Omega_{k_0}(1+z)^2 + \Omega_{\Lambda_0}]$$

$$a = (1+z)^{-1}$$

It is convenient to define the Expansion term

$$E^2(z) = \Omega_{m_0}(1+z)^3 + \Omega_{r_0}(1+z)^4 + \Omega_{k_0}(1+z)^2 + \Omega_{\Lambda_0}$$

or equivalently

$$E^2(a) = \Omega_{m_0}a^{-3} + \Omega_{r_0}a^{-4} + \Omega_{k_0}a^{-2} + \Omega_{\Lambda_0}$$

So that

$$H(z) = H_0 E(z)$$

or equivalently

$$\frac{\dot{a}}{a} = H_0 E(a)$$

Generalization of the search for two numbers: now want to measure H_0, $E(z)$ where $E(z)$ contains information about the various **Ω**.

Expansion history

$$\Omega_m + \Omega_r + \Omega_k + \Omega_\Lambda = 1$$

$$H = \frac{\dot{a}}{a}$$

$$a = (1+z)^{-1}$$

$$E(z) = \sqrt{\Omega_{m_0}(1+z)^3 + \Omega_{r_0}(1+z)^4 + \Omega_{k_0}(1+z)^2 + \Omega_{\Lambda_0}}$$

Friedmann equation

$$\frac{\dot{a}}{a} = H_0 E(a)$$

If we don't know the full details of `E(a)`, we can make a Taylor expansion

$$a(t) \approx 1 + H_0(t - t_0) - \frac{1}{2} q_0 H_0^2 (t - t_0)^2 + \ldots$$

where we see the deceleration parameter as the next term after the Hubble constant

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = -\frac{1}{H^2}\frac{\ddot{a}}{a}$$

so q_0 *becomes a proxy for E(z)*

Solutions from Felten & Isaacman (1986) Reviews of Modern Physics, **58**, 689

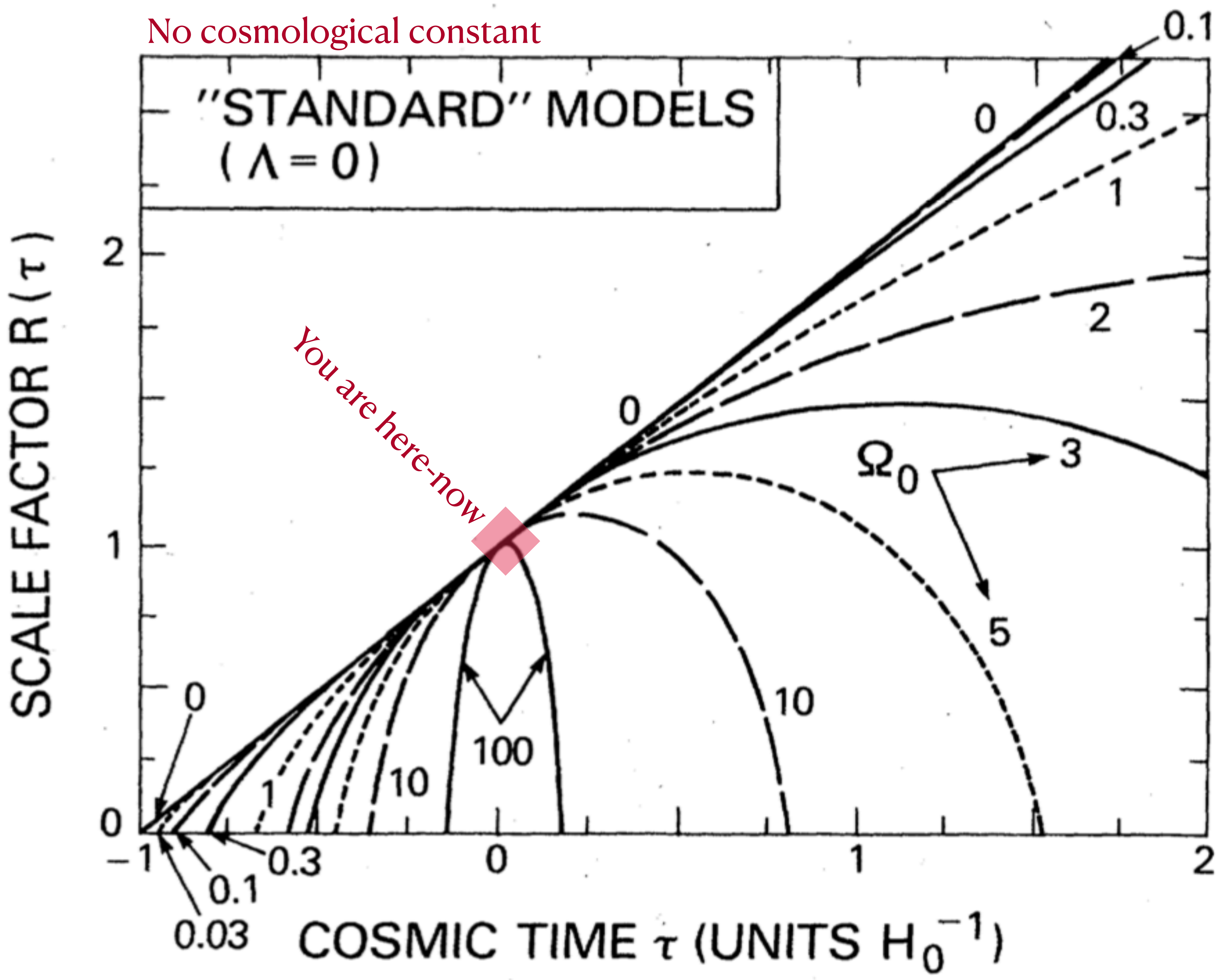
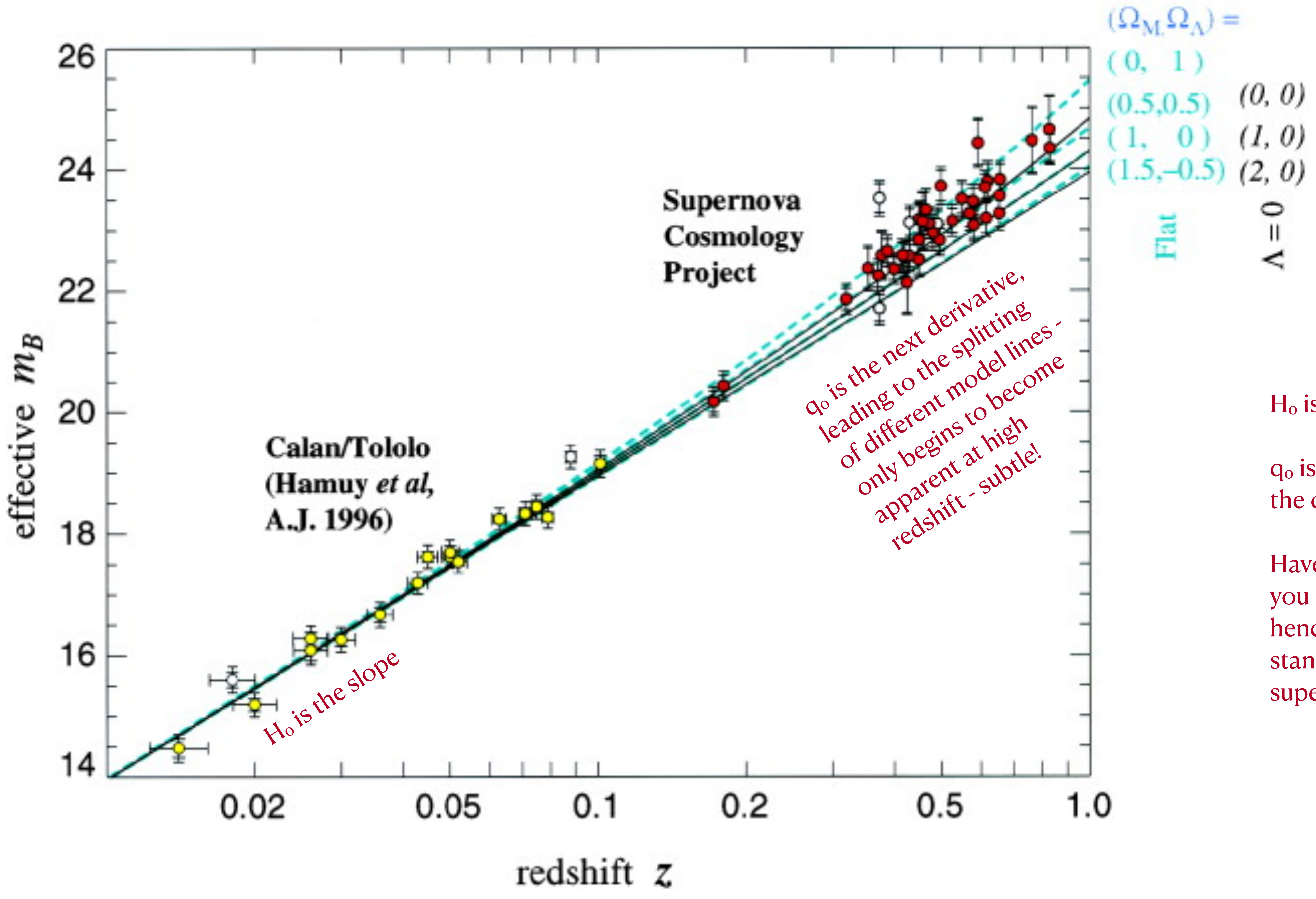


FIG. 3. "Standard" Friedmann models. The family of scale factors $R(\tau)$ for the "standard models" ($\Lambda=0$). The free parameter, shown on the curves, is Ω_0. As shown by the τ intercepts, all models have ages ≤ 1 ($\leq H_0^{-1}$ yr).

H_o is the slope

q_o is the next derivative - the change in the slope

Have to see far away before you can start to perceive q_o, hence the desire for bright standard candles like supernovae.



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Expansion history

$$\Omega_m + \Omega_r + \Omega_k + \Omega_\Lambda = 1$$

There is an analytic solution for matter domination - can parameterize the expansion as a cycloid

$$H = \frac{\dot{a}}{a}$$

$$a = \frac{\Omega_m}{2(1 - \Omega_m)}(\cosh \eta - 1)$$

$$a = (1 + z)^{-1}$$

$$E(z) = \sqrt{\Omega_{m_0}(1+z)^3 + \Omega_{r_0}(1+z)^4 + \Omega_{k_0}(1+z)^2 + \Omega_{\Lambda_0}}$$

$$H_0 t = \frac{\Omega_m}{2(1 - \Omega_m)^{3/2}}(\sinh \eta - \eta)$$

where η is the development parameter - related to the conformal time

The current value of the development parameter is

$$\cosh \eta_0 = \frac{2}{\Omega_{m_0}} - 1$$

This is of no use now because of Lambda, BUT it does become useful for the growth of structure in the early universe, when every protogalaxy can be considered its own little island universe with $\Omega_m \gtrsim 1$

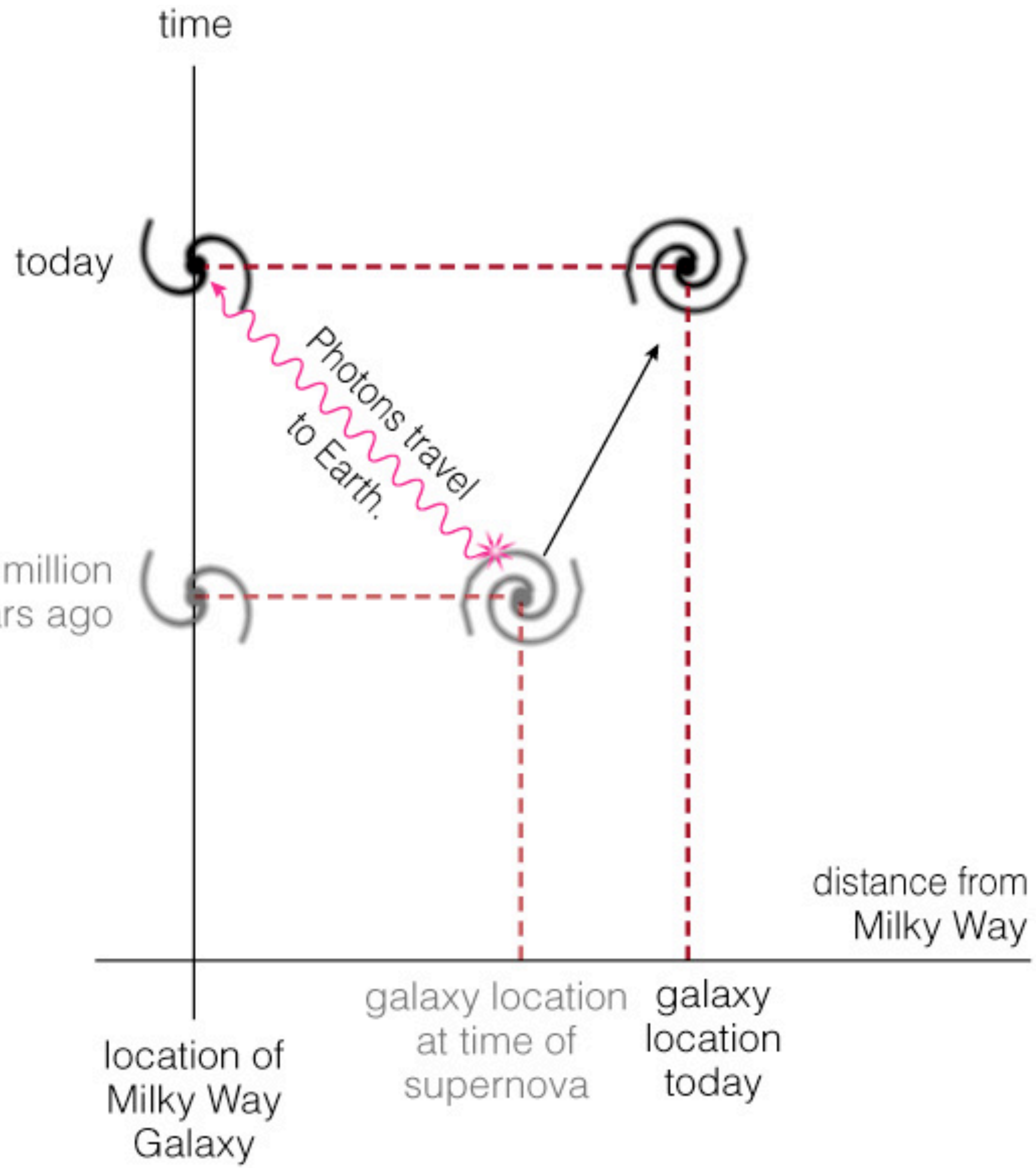
The proper distance to be the current comoving separation

$$D_p(t_0) = c \int_{t_e}^{t_0} \frac{dt}{a(t)}$$

$$a(t) \approx 1 + H_0(t - t_0) - \frac{1}{2} q_0 H_0^2 (t - t_0)^2 + \dots$$

t_0

t_e



In terms of redshift,

$$D_p(z_e) = \frac{c}{H_0}\int_0^{z_e} \frac{dz}{E(z)}$$

For **zero** cosmological constant, there is an exact solution known as Mattig's equation:

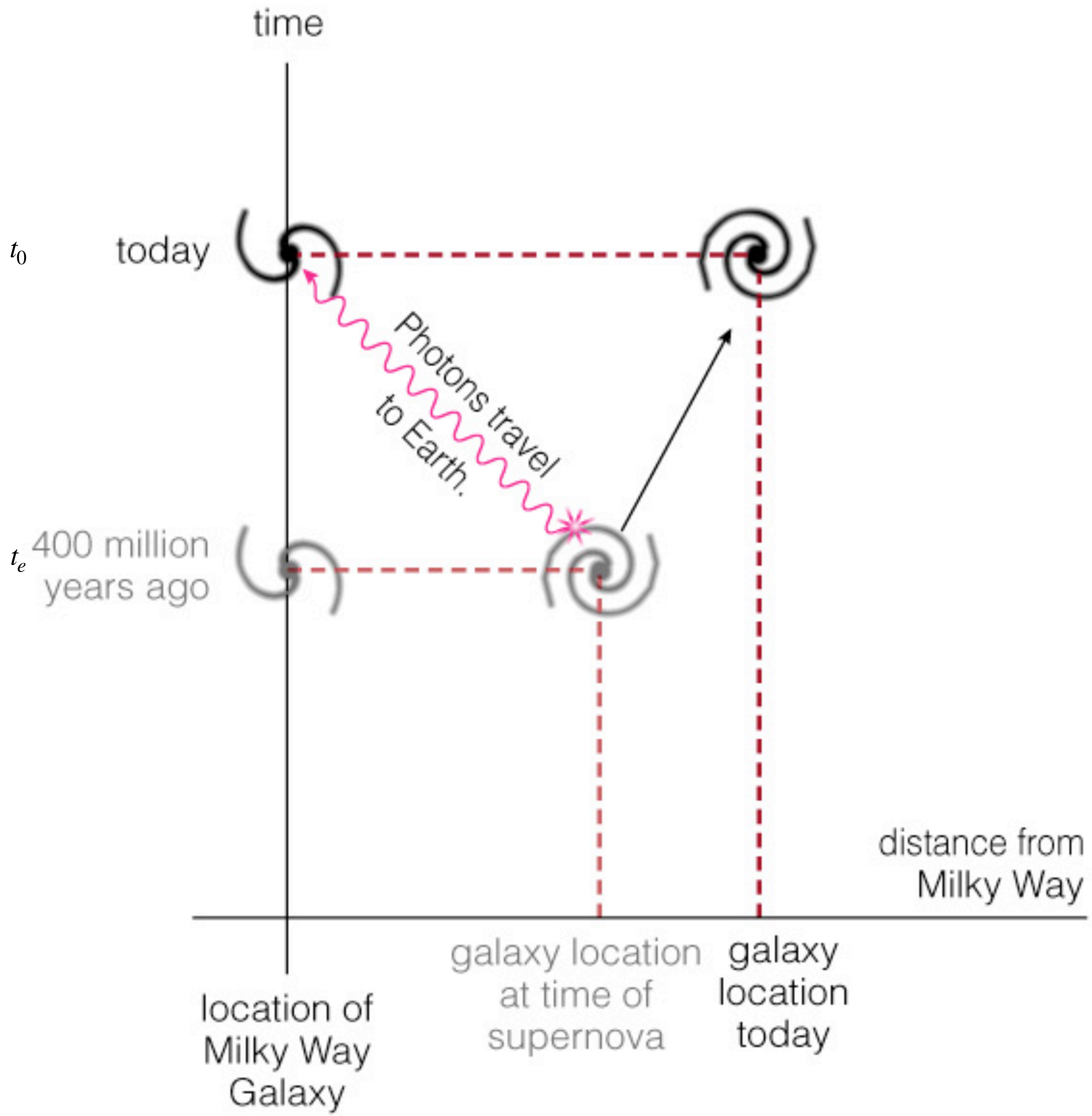
$$D_p(z) = \frac{2c}{H_0}\frac{[z\Omega_m + (\Omega_m - 2)(\sqrt{1+z\Omega_m} - 1])}{\Omega_m^2(1+z)}$$

In general, there is no analytic solution, but can approximate with the Taylor expansion:

$$D_p(z) = \frac{c}{H_0}\left[z - \frac{1}{2}(1+q_0)z^2\right]$$

Where the leading term is Hubble's Law

$$D_p(z) = \frac{cz}{H_0}$$



For time rather than distance

Friedmann equation

$$\frac{\dot{a}}{a} = H_0 E(a)$$

$$H_0 \int_{t_e}^{t_o} dt = \int_{a_e}^{1} \frac{da}{aE(a)} = \int_0^{z_e} \frac{dz}{(1+z)E(z)}$$

$$H_0(t_0 - t_e) = \int_0^{z_e} \frac{dz}{(1+z)E(z)}$$

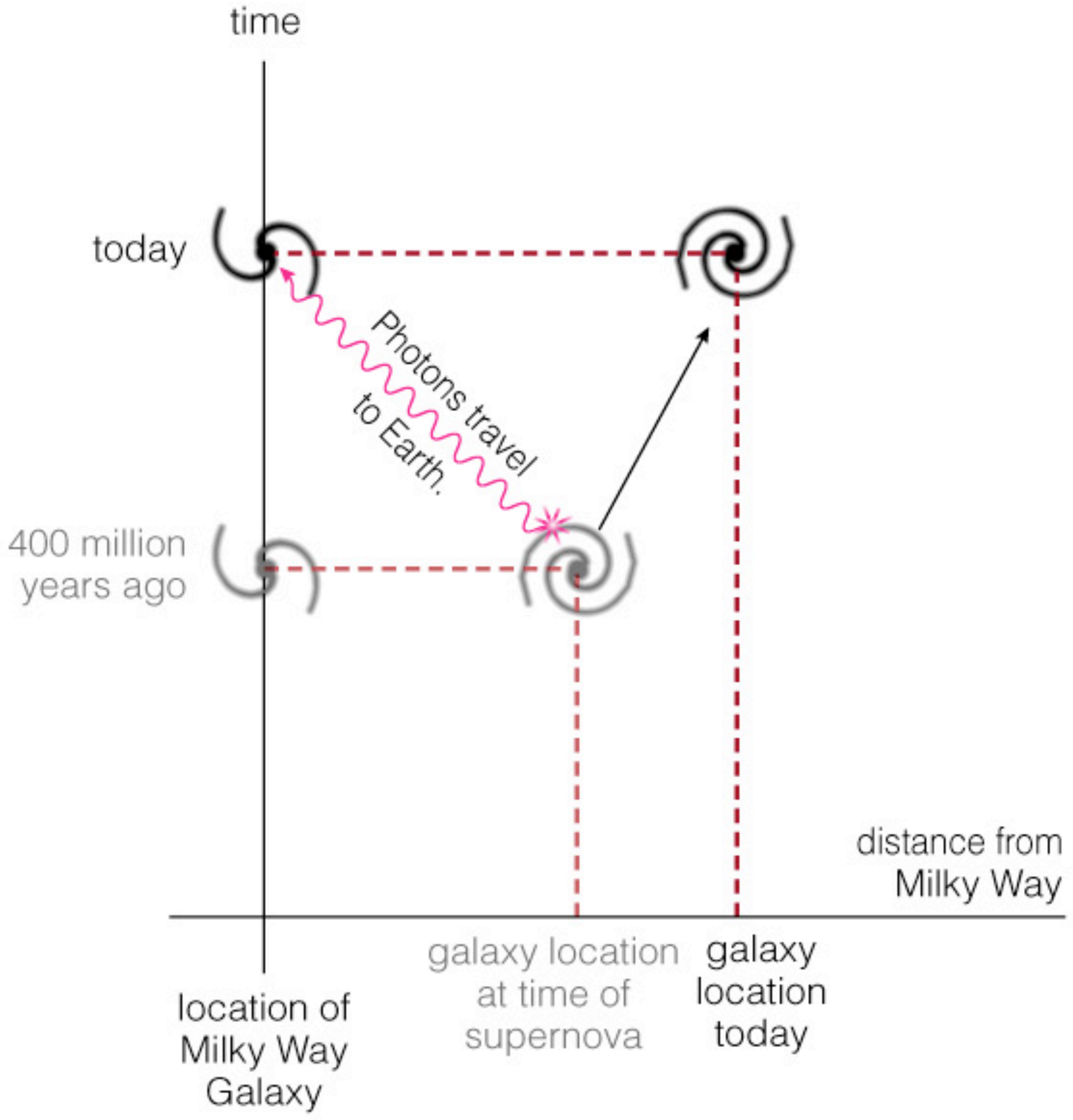
$(t_0 - t_e)$ is the **Lookback time** - the time since the photon was emitted.

$$\Omega_m + \Omega_r + \Omega_k + \Omega_\Lambda = 1$$

$$H = \frac{\dot{a}}{a}$$

$$a = (1+z)^{-1}$$

$$E(z) = \sqrt{\Omega_{m_0}(1+z)^3 + \Omega_{r_0}(1+z)^4 + \Omega_{k_0}(1+z)^2 + \Omega_{\Lambda_0}}$$



The age of the universe is obtained by setting

$$t_e = 0 \; ; \; z \to \infty$$

$$H_0 t_0 = \int_0^\infty \frac{dz}{(1+z)E(z)}$$

which can be approximated by

$$t_0 \approx \left(\frac{2}{3H_0}\right)(0.7\Omega_{m_0} + 0.3 - 0.3\Omega_{\Lambda_0})^{-0.3}$$

There is no deep theory in this last formula.
It is just a fitting formula that approximates the answer to a few %.

Similarly, the redshift-age of a matter dominated universe can be approximated as

$$\frac{1}{t(z)} \approx H(z)[1 + \frac{1}{2}\Omega_m^{0.6}(z)]$$ Peacock (3.46)

$$\Omega_m + \Omega_r + \Omega_k + \Omega_\Lambda = 1$$

$$H = \frac{\dot{a}}{a}$$

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$$E(z) = \sqrt{\Omega_{m_0}(1+z)^3 + \Omega_{r_0}(1+z)^4 + \Omega_{k_0}(1+z)^2 + \Omega_{\Lambda_0}}$$

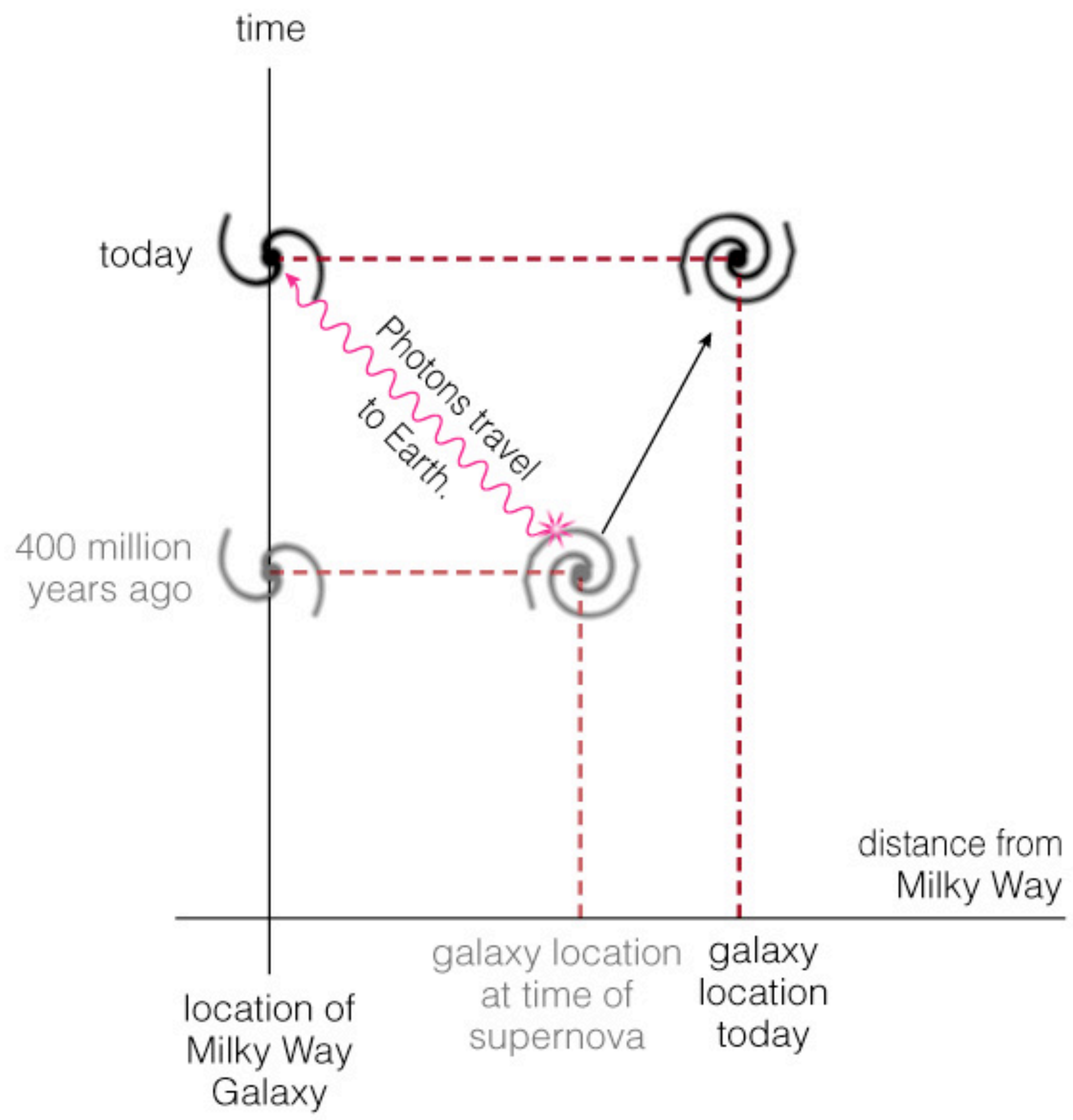
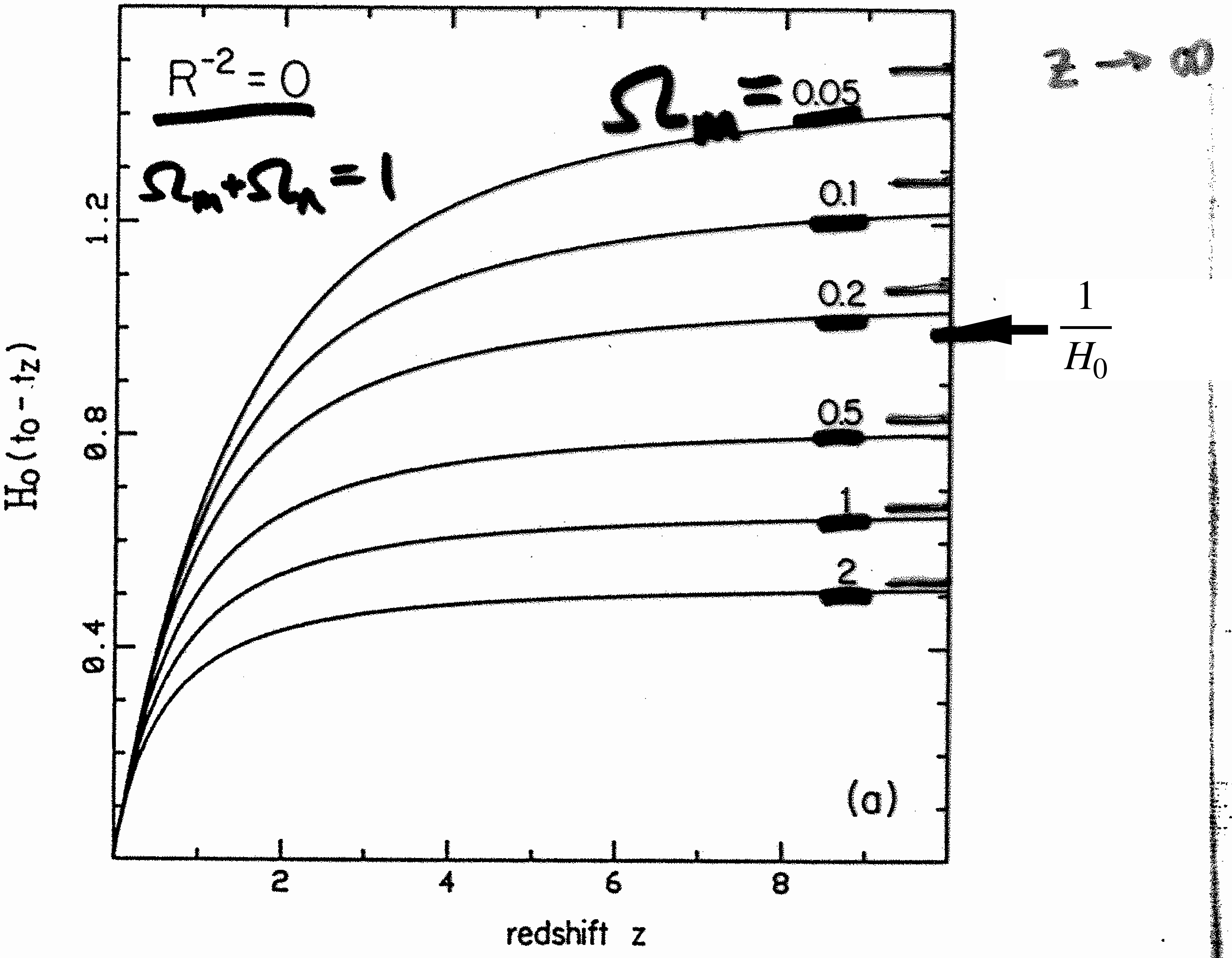
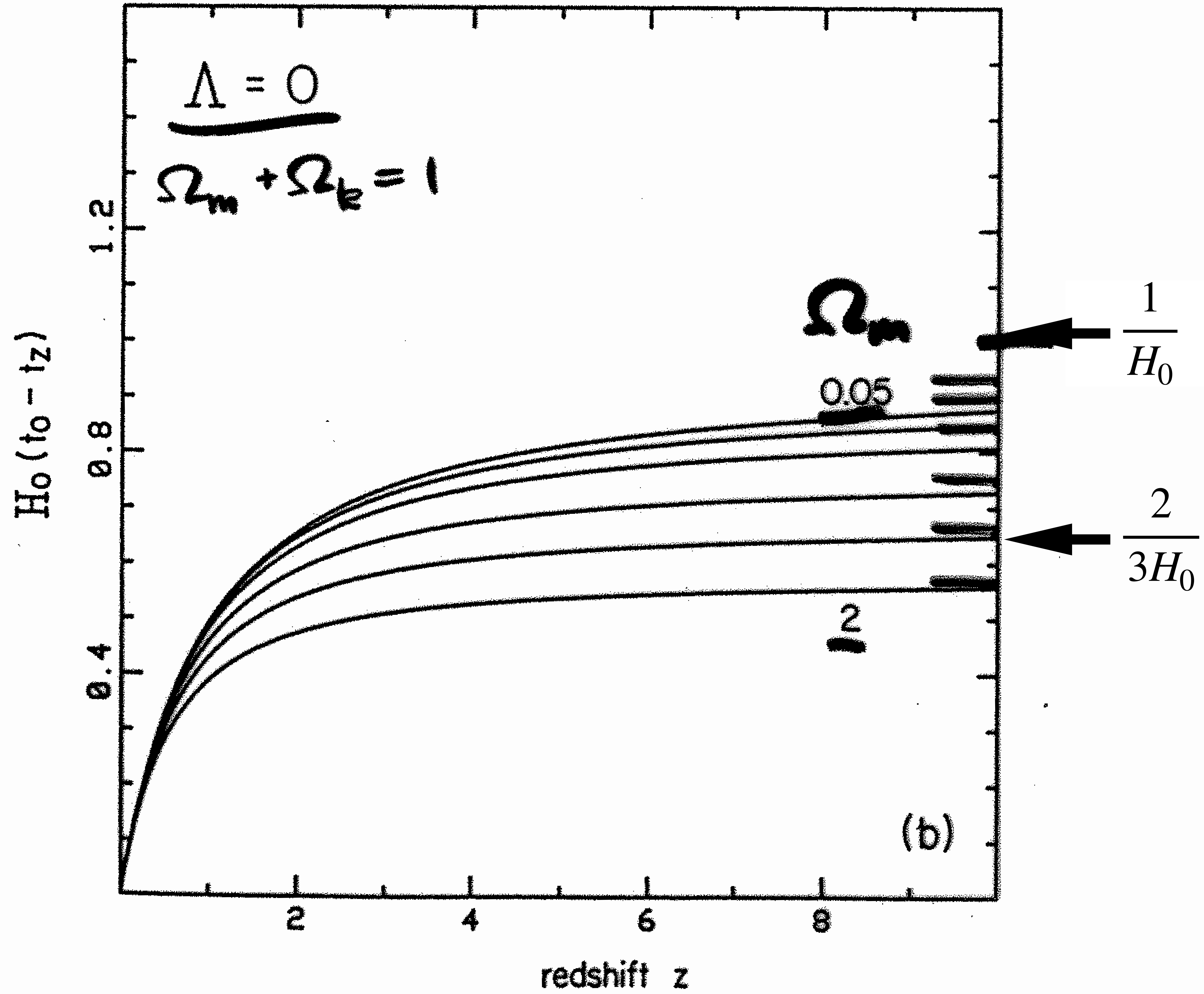


Figure 13.1. Lookback time as a function of redshift. The long dashes on the right-hand axis show the age t_0 of the universe computed from $z \to \infty$. In panel (a) space curvature is negligible, and in panel (b) the cosmological constant, Λ, is negligibly small. The curves are labeled by the density parameter, Ω.



Zero cosmological constant

These cosmologies have only decelerated, so must have ages less than one Hubble time.



Observational Tests

of cosmology

• Luminosity-redshift relation	$D_L - z$	Standard Candle
• Angular size-distance relation	$D_A - z$	Standard Rod
• Number-redshift relation	$N(z)$	Source counts with redshift
• Number-magnitude relation	$N(m)$	Source counts with magnitude
• Tolman test	$\Sigma(z)$	Surface brightness not distance independent in Robertson-Walker geometry
• redshift time derivative	$\dot{z}$	Could conceivably measure expansion of universe directly in a human lifetime