

Cosmology

and Large Scale Structure



Today
Number counts
 $N(m)$; $N(z)$
Luminosity Functions

Observational Tests

Five Classic Tests

- Luminosity-redshift relation $D_L - z$ Standard Candle
- Angular size-redshift relation $D_A - z$ Standard Rod
- Number-redshift relation $N(z)$ Source counts with redshift
- Number-magnitude relation $N(m)$ Source counts with magnitude
- Tolman test $\Sigma(z)$ Surface brightness not distance independent in Robertson-Walker geometry

Hubble Ultra Deep Field

3' x 3'



Robertson-Walker metric

$$ds^2 = -c^2 dt^2 + a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right]$$

Volume

$$V = a^3 \int_0^r \frac{r^2 dr}{\sqrt{1 - kr^2}} \int_0^\pi \sin \theta d\theta \int_0^{2\pi} d\phi$$

integrates to

$$V = \frac{4\pi}{3} (ar)^3 f(r) \quad \text{for } k = \begin{cases} 1 \\ 0 \\ -1 \end{cases}$$

$$f(r) = \begin{cases} \frac{3}{2} \left[\frac{\sin^{-1} r}{r^3} - \frac{\sqrt{1 - r^2}}{r^2} \right] \\ 1 \\ \frac{3}{2} \left[\frac{\sqrt{1 + r^2}}{r^2} - \frac{\sinh^{-1} r}{r^3} \right] \end{cases}$$

in terms of the proper distance

$$D_p = a(t) \int_0^r \frac{dr}{\sqrt{1 - kr^2}}$$

we can make the Taylor expansion

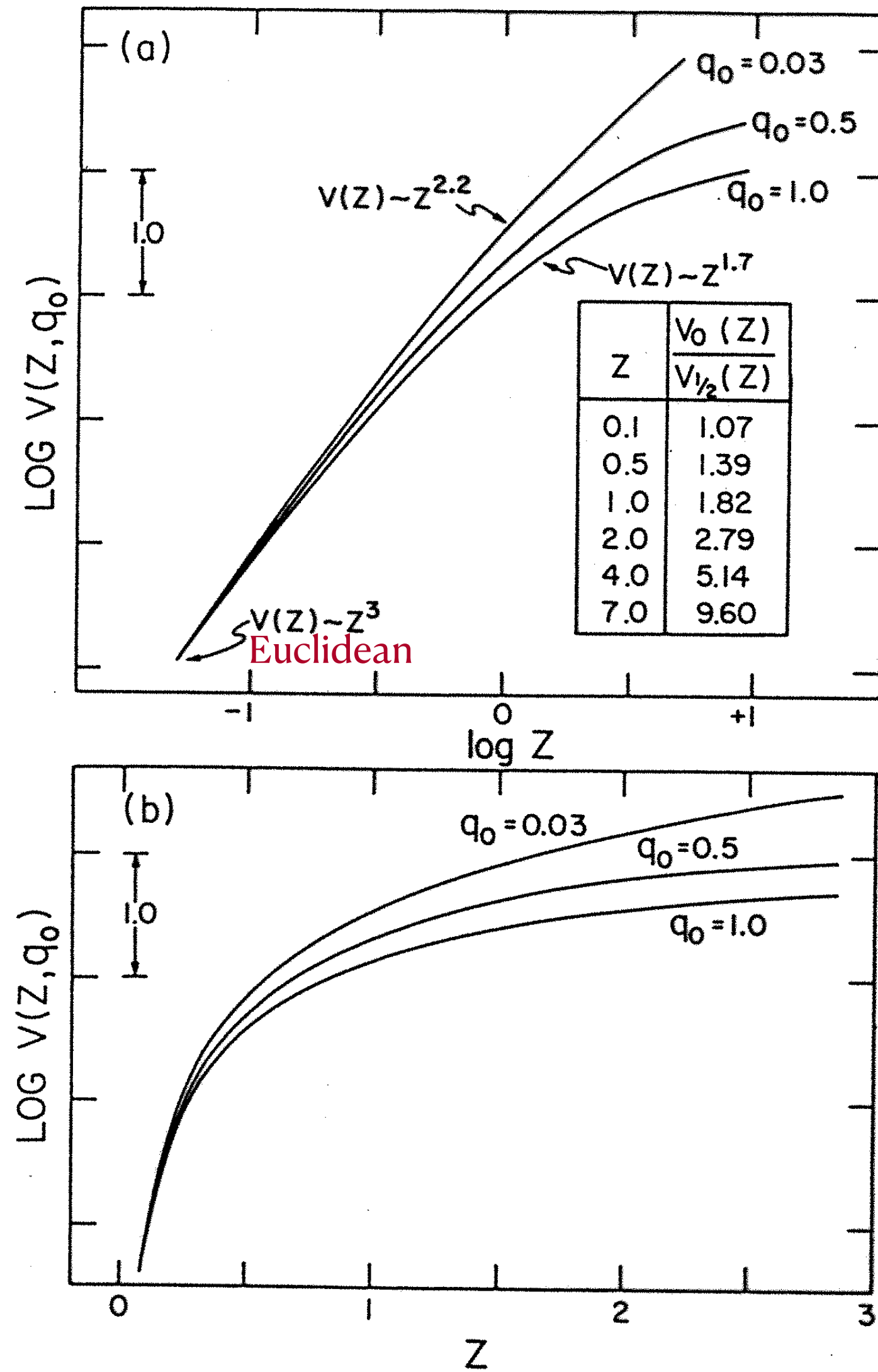
Sandage (1988, ARA&A, 26, 561)

$$V = \frac{4\pi}{3} D_p^3 \left[1 - \frac{k}{5} \left(\frac{D_p}{R} \right)^2 + \mathcal{O} \left(\frac{D_p}{R} \right)^4 \right]$$

$\frac{k}{R^2}$ is the curvature of space

Note that the volume increases as the curvature becomes more negative,
so a closed universe is “small”
and an open universe is “big”

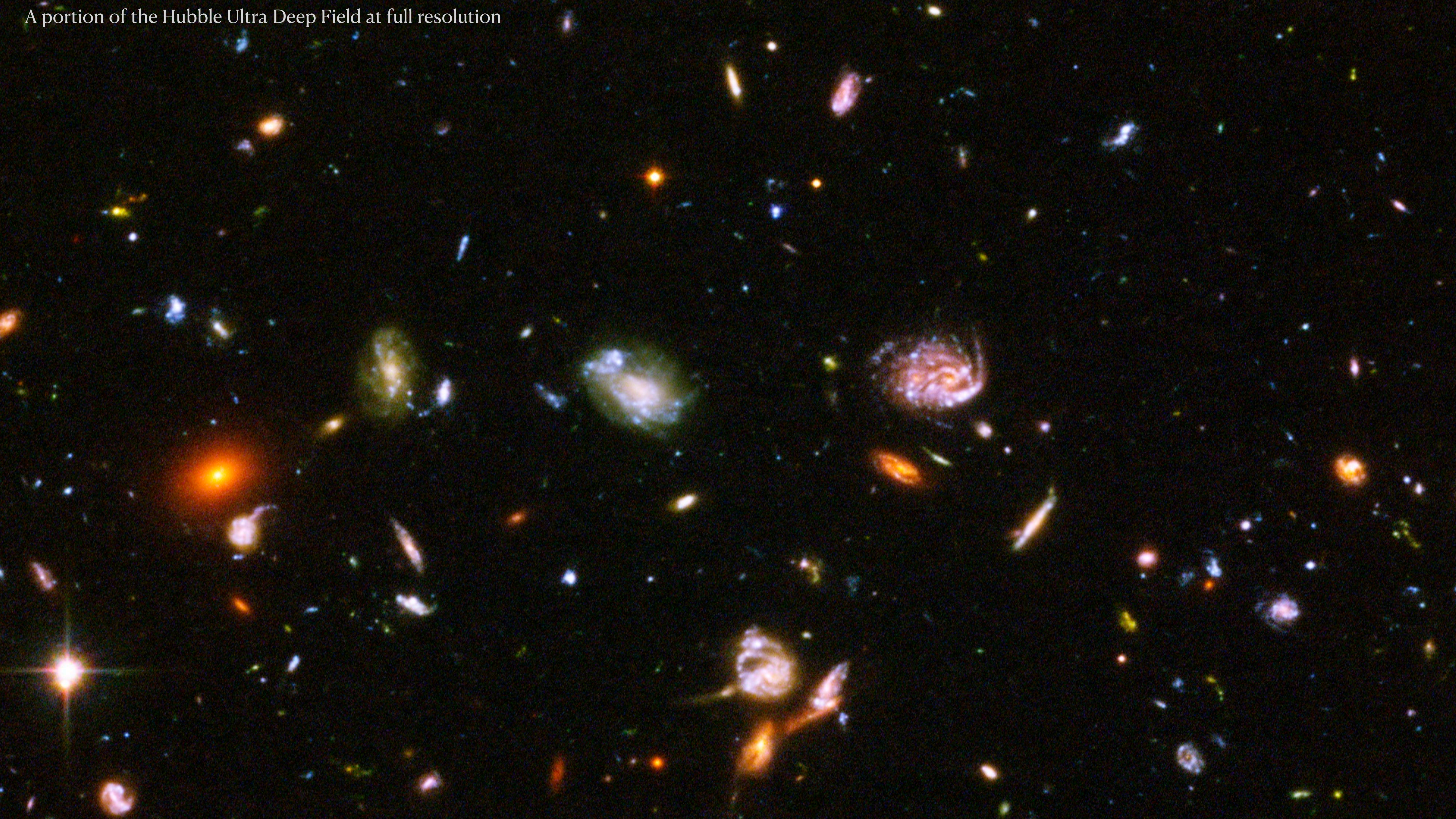
Predicted $N(<z)$



as $q_0 \downarrow, V \uparrow$

Figure 1 Theoretical $N(z, q_0)$ relations for three values of q_0 . Plotted is the integral count, i.e. the total number of galaxies in a complete (volume-limited) sample that have redshifts smaller than z . Parts (a) and (b) are the same function but plotted as $\log z$ (a) and z (b).

A portion of the Hubble Ultra Deep Field at full resolution



Since the volume depends on curvature, source counts provide a test

For sources of luminosities L and constant comoving number density $\Phi(L)$,

Number-redshift:

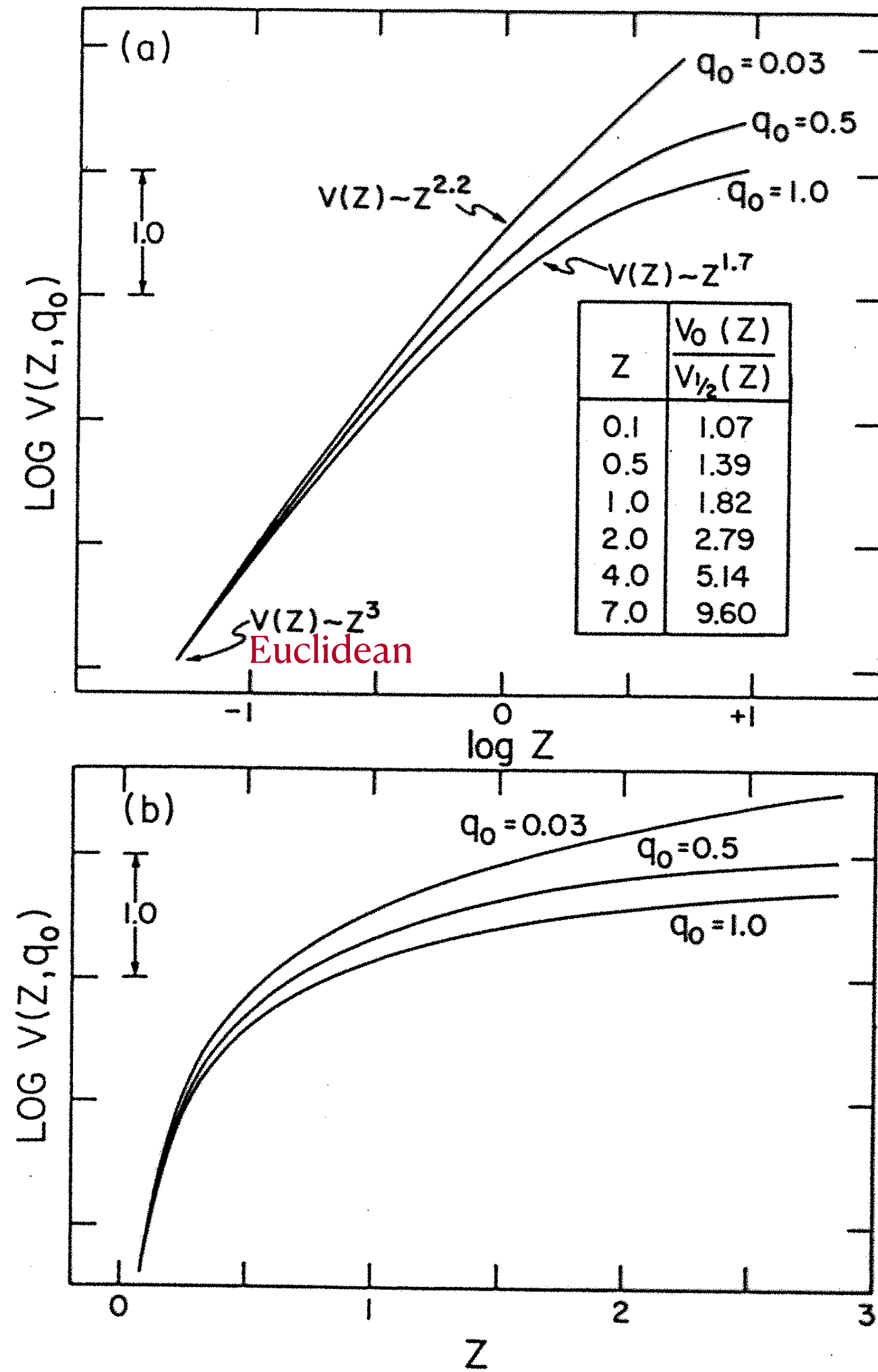
$$N(< z) = \frac{4\pi}{3H_0^2} z^3 \int_0^\infty \Phi(L) \left[1 + \frac{3}{2} z(1 + q_0) \right] dL$$

Number-magnitude:

$$N(< f) = \frac{4\pi}{3} (4\pi f)^{-3/2} \int_0^\infty \Phi(L) \left[1 - 3H_0 \left(\frac{L}{4\pi f} \right)^{1/2} \right] L^{3/2} dL$$

Historically, radio source counts in the 1960s played an important role in excluding the Steady State cosmology.

Predicted $N(<z)$

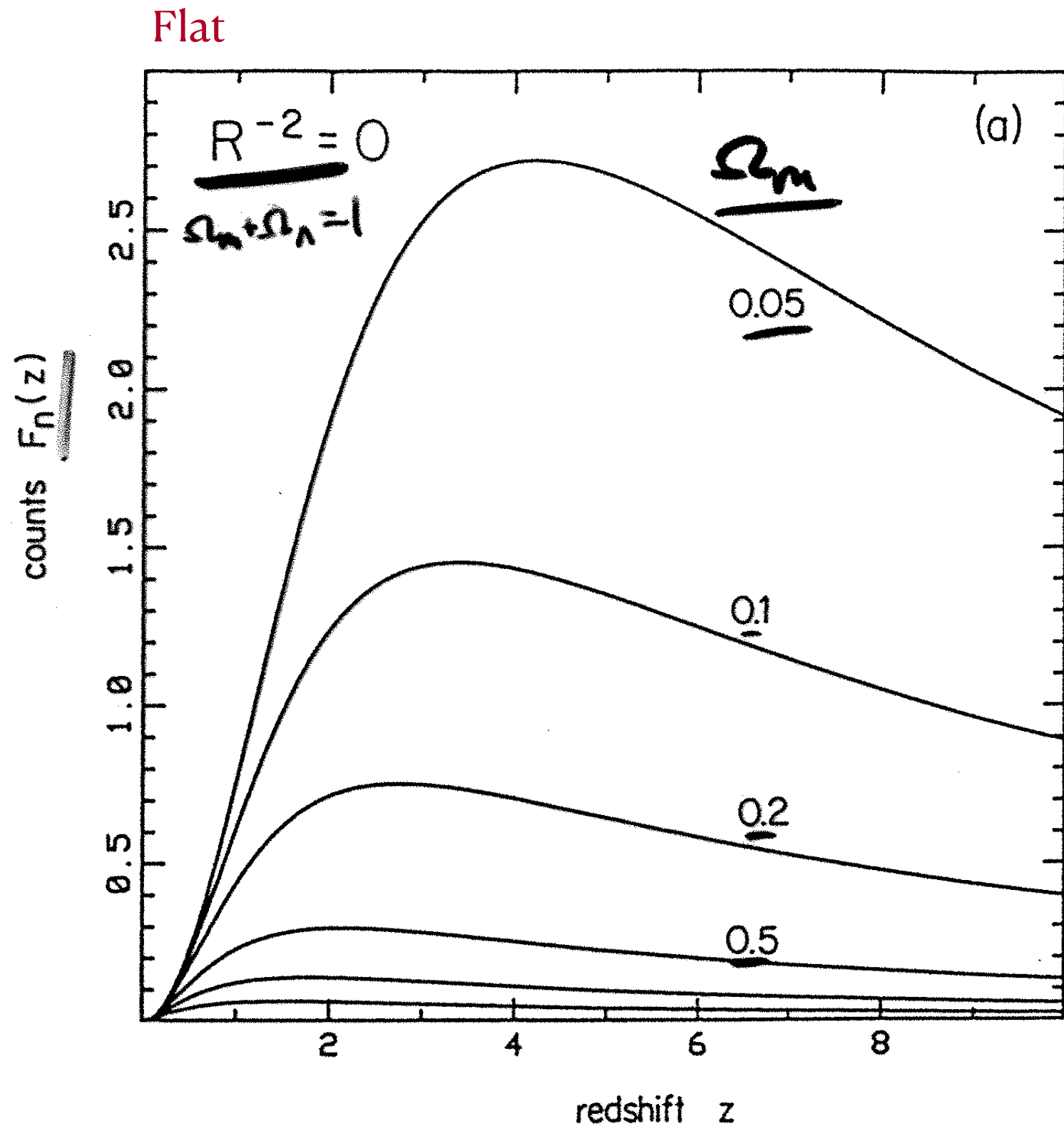


as $q_0 \downarrow, V \uparrow$

Figure 1 Theoretical $N(z, q_0)$ relations for three values of q_0 . Plotted is the integral count, i.e. the total number of galaxies in a complete (volume-limited) sample that have redshifts smaller than z . Parts (a) and (b) are the same function but plotted as $\log z$ (a) and z (b).

Peebles

Figure 13.8. Counts as a function of redshift. The vertical axis is the dimensionless function $F_n(z)$ in dN/dz in equation (13.61). The parameters are arranged as in figure 13.1.



Differential counts $\frac{dN}{dz}$

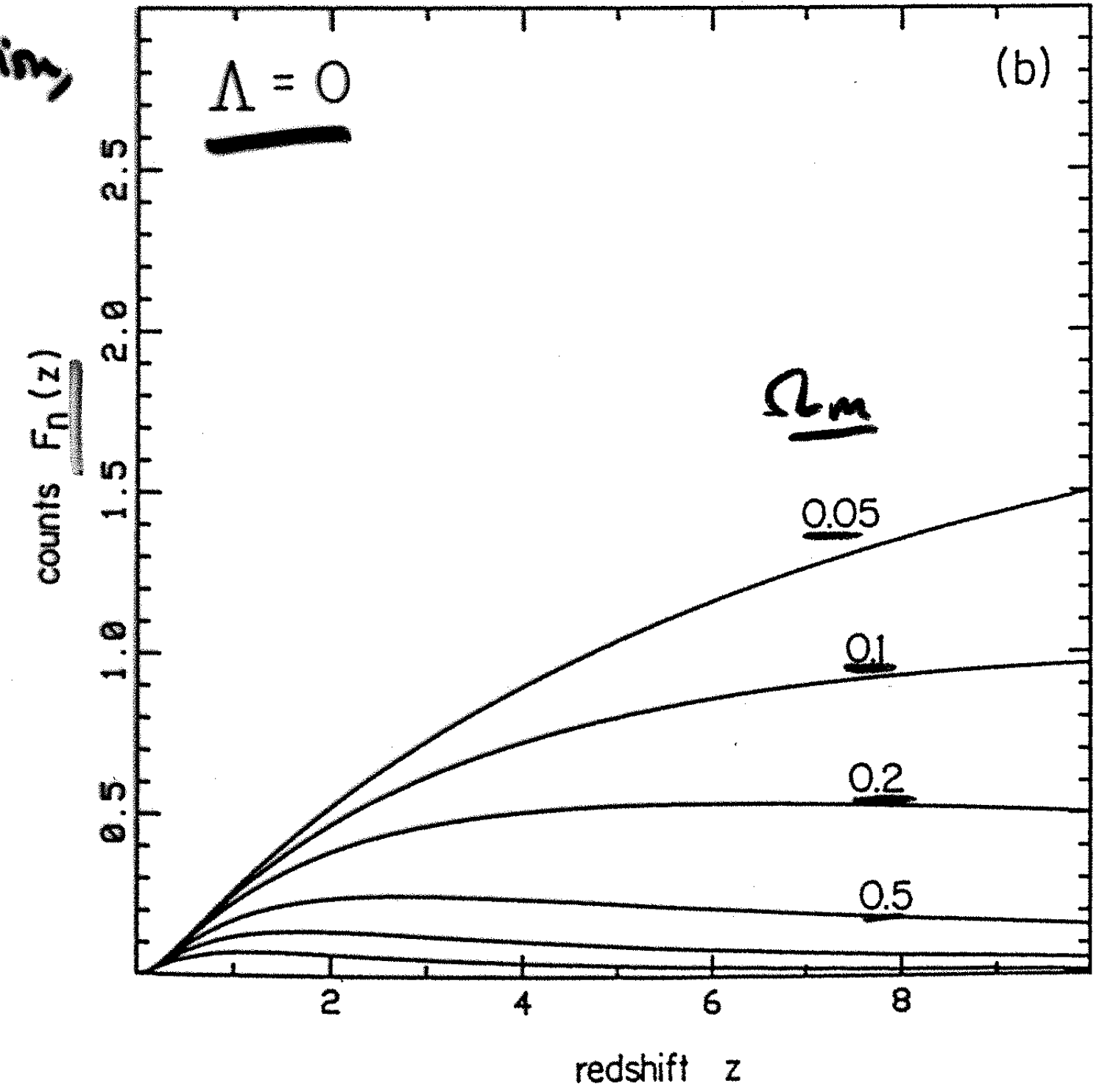
for number conservation,

$$n = n_0 \left(\frac{R_0}{R}\right)^3$$

$$\frac{dN}{dz} = n_0 \frac{F_n(z)}{H_0^3}$$

$$F_n(z) = \frac{[H_0 R_0 r(z)]^2}{E(z)}$$

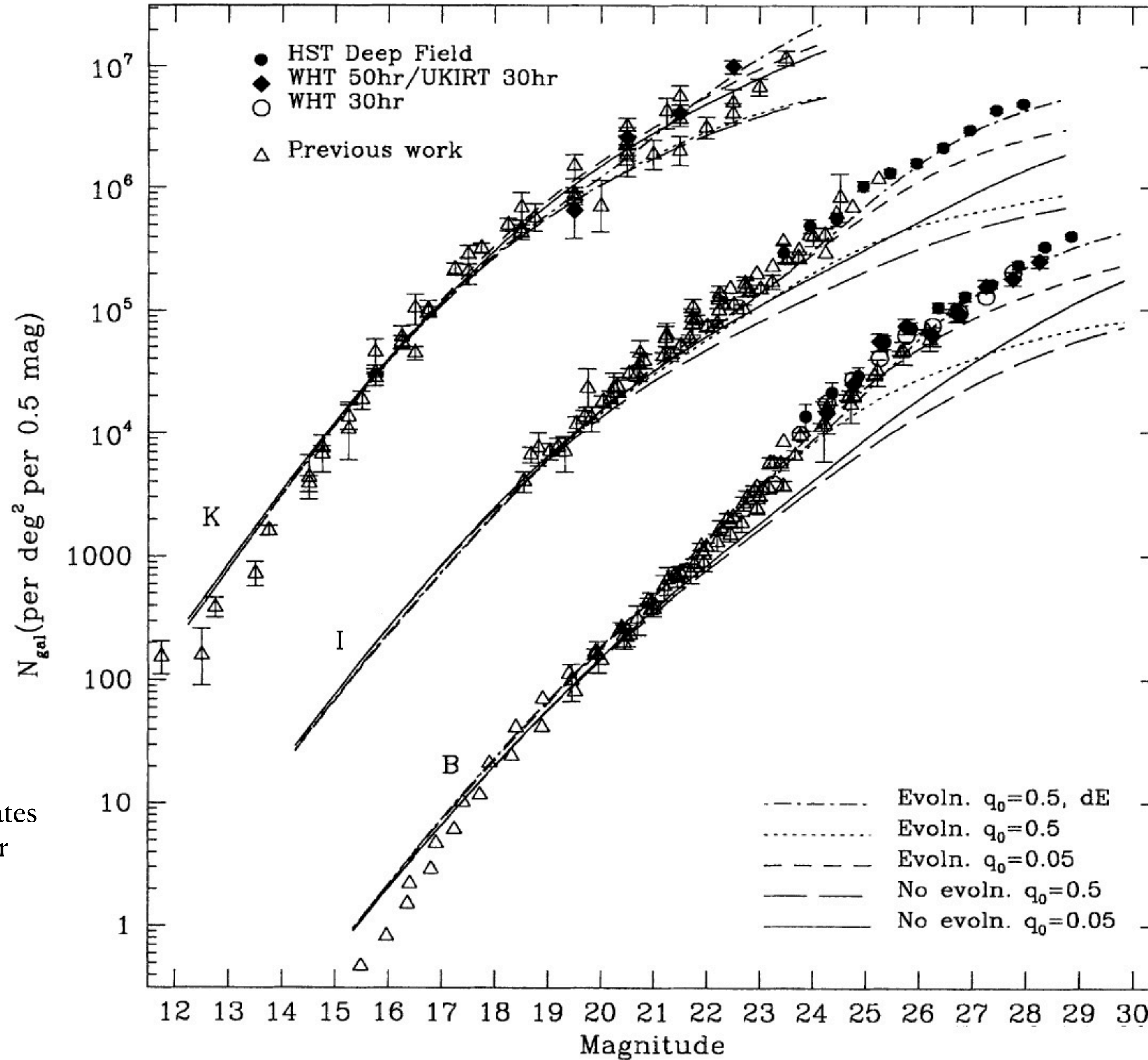
No cosmological constant



Differential counts $\frac{dN}{dz}$

Number-magnitude:

Metcalfe et al. (1996)



A “no evolution” model extrapolates the locally measured Schechter function to high redshift.

Number-magnitude:

Only test that does not explicitly require redshift information.
 Basically integrate over all the relevant distributions.

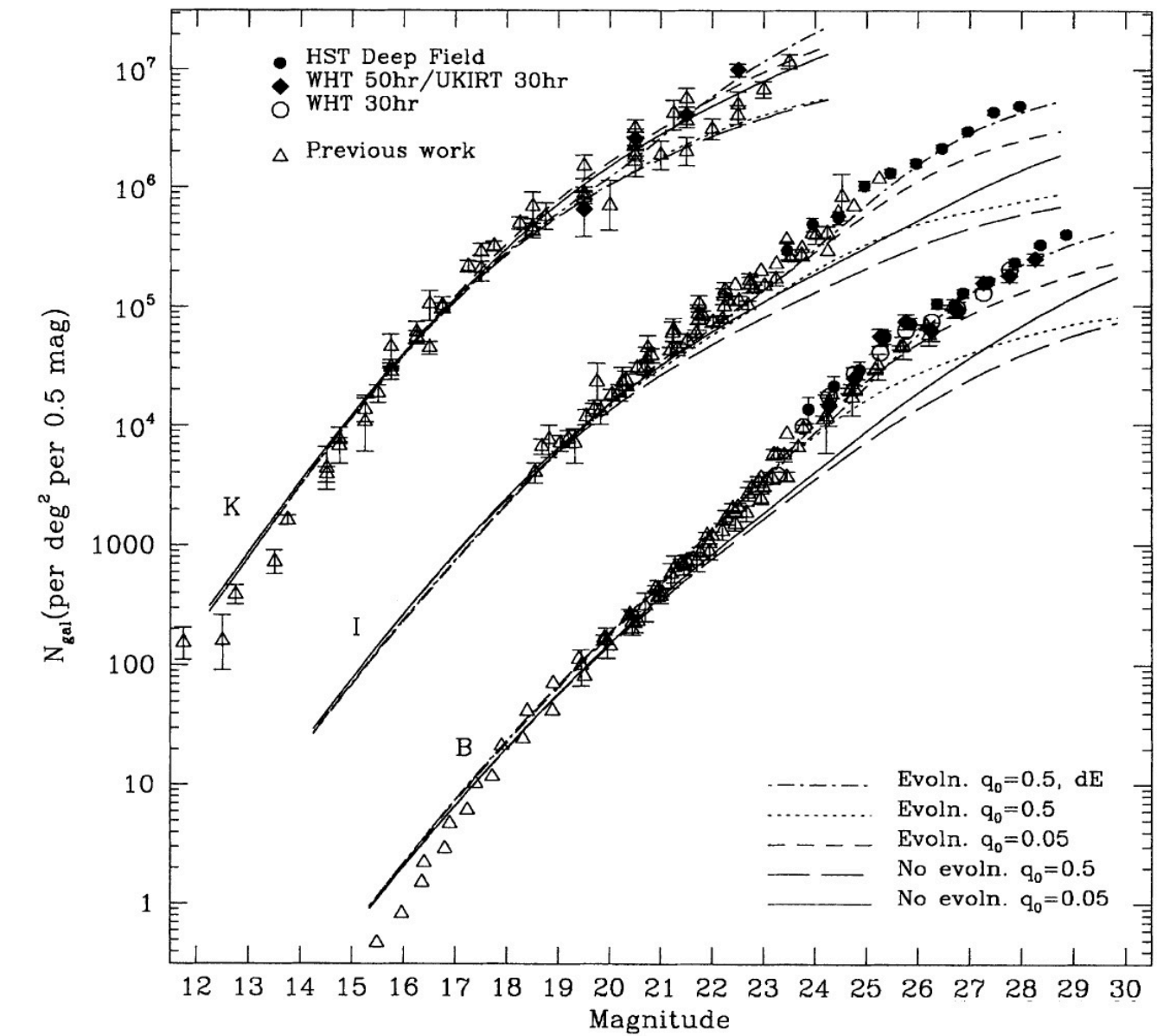
$$A(m, T) = A_0 \int_0^z D(z, T) \Phi(M, T) dV(z, q_0)$$

↗ Density distribution (e.g., non-uniform large scale structure)
↑ Luminosity function $n(L) \leftrightarrow \Phi(M)$
↖ Volume element (cosmological)

$$N(m, T) = \int_T \int_0^m A(m, T) dT dm$$

For sources of type T and magnitude m .

Metcalfe et al. (1996)



We can only get at the volume element if we understand the other terms and their redshift evolution.

Schechter function

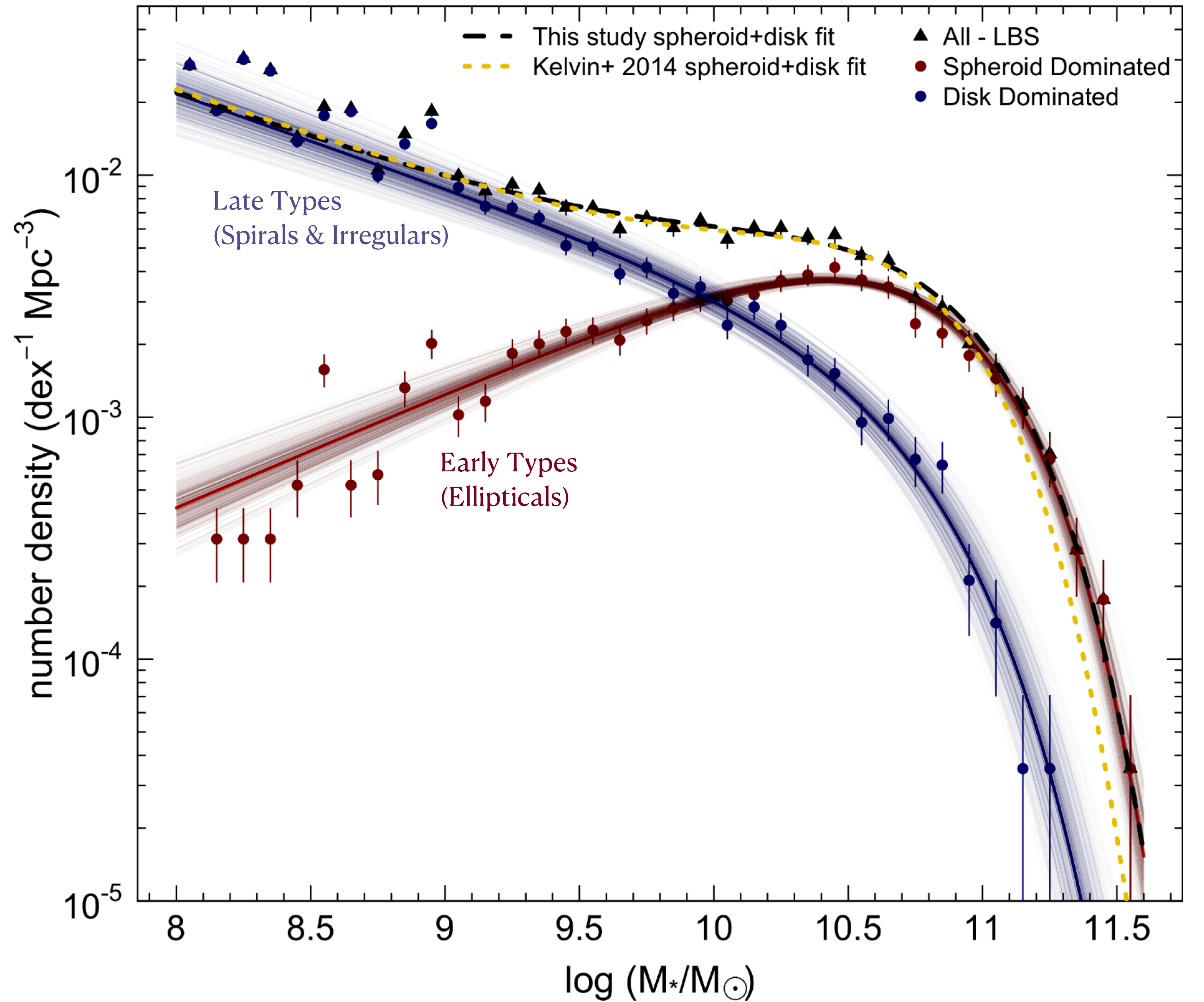
$$\Phi(L) = \Phi^* \left(\frac{L}{L^*} \right)^{-\alpha} e^{-L/L^*}$$

L^* Characteristic luminosity

Φ^* Characteristic number density

α Faint end slope

Population	$\log(M^* h_{0.72}/M_\odot)$	α	$\phi^*/10^{-3}$ ($\text{dex}^{-1} \text{Mpc}^{-3} h_{0.73}$)
Early Type	10.74 ± 0.026	0.525 ± 0.029	3.67 +0.20
Late Type	10.70 ± 0.049	1.39 ± 0.021	0.855 +0.10



Since the volume depends on curvature, source counts provide a test

For sources of luminosities L and constant comoving number density $\Phi(L)$,

Number-redshift:

$$N(< z) = \frac{4\pi}{3H_0^2} z^3 \int_0^\infty \Phi(L) \left[1 + \frac{3}{2} z(1 + q_0) \right] dL$$

Number-magnitude:

$$N(< f) = \frac{4\pi}{3} (4\pi f)^{-3/2} \int_0^\infty \Phi(L) \left[1 - 3H_0 \left(\frac{L}{4\pi f} \right)^{1/2} \right] L^{3/2} dL$$

Historically, radio source counts in the 1960s played an important role in excluding the Steady State cosmology.

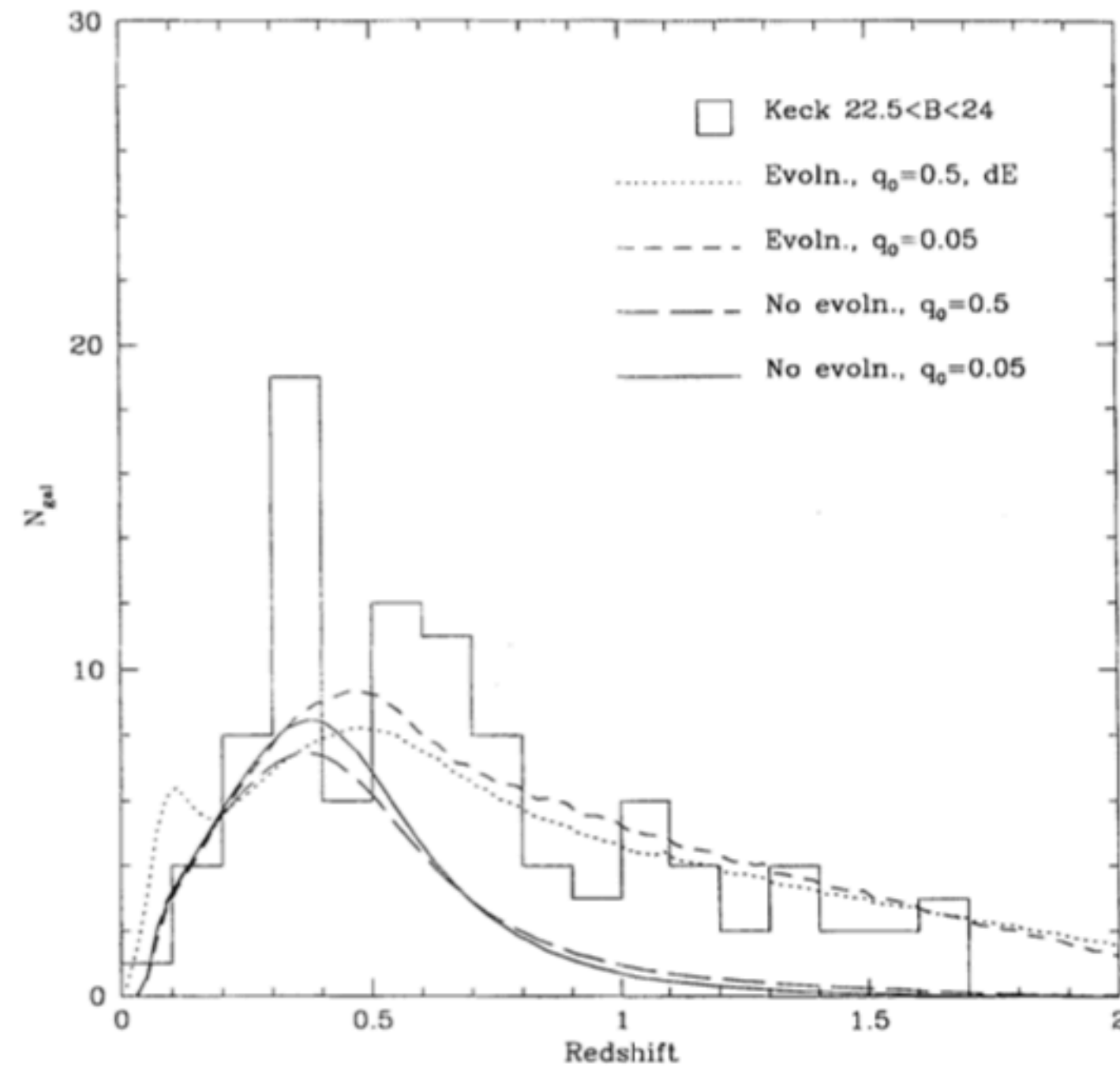


FIG. 2 The galaxy number-redshift distribution, $n(z)$, for $22.5 \text{ mag} < B < 24 \text{ mag}$ implied by new redshift data acquired on the Keck Telescope (refs. 6, 7). The observed $n(z)$ is clearly more extended than the non-evolving models with either $q_0 = 0.05$ or $q_0 = 0.5$. The extended redshift distribution is well fitted by our evolutionary models whose parameters are described in Fig. 1 legend.

Large Scale Structure is apparent in the non-smoothness of $N(z)$.

Galaxies evolve!

Certainly in luminosity, probably also in number.

So a single Schechter fcn doesn't suffice. All of these terms matter.

$$A(m, T) = A_0 \int_0^z D(z, T) \Phi(M, T) dV(z, q_0)$$

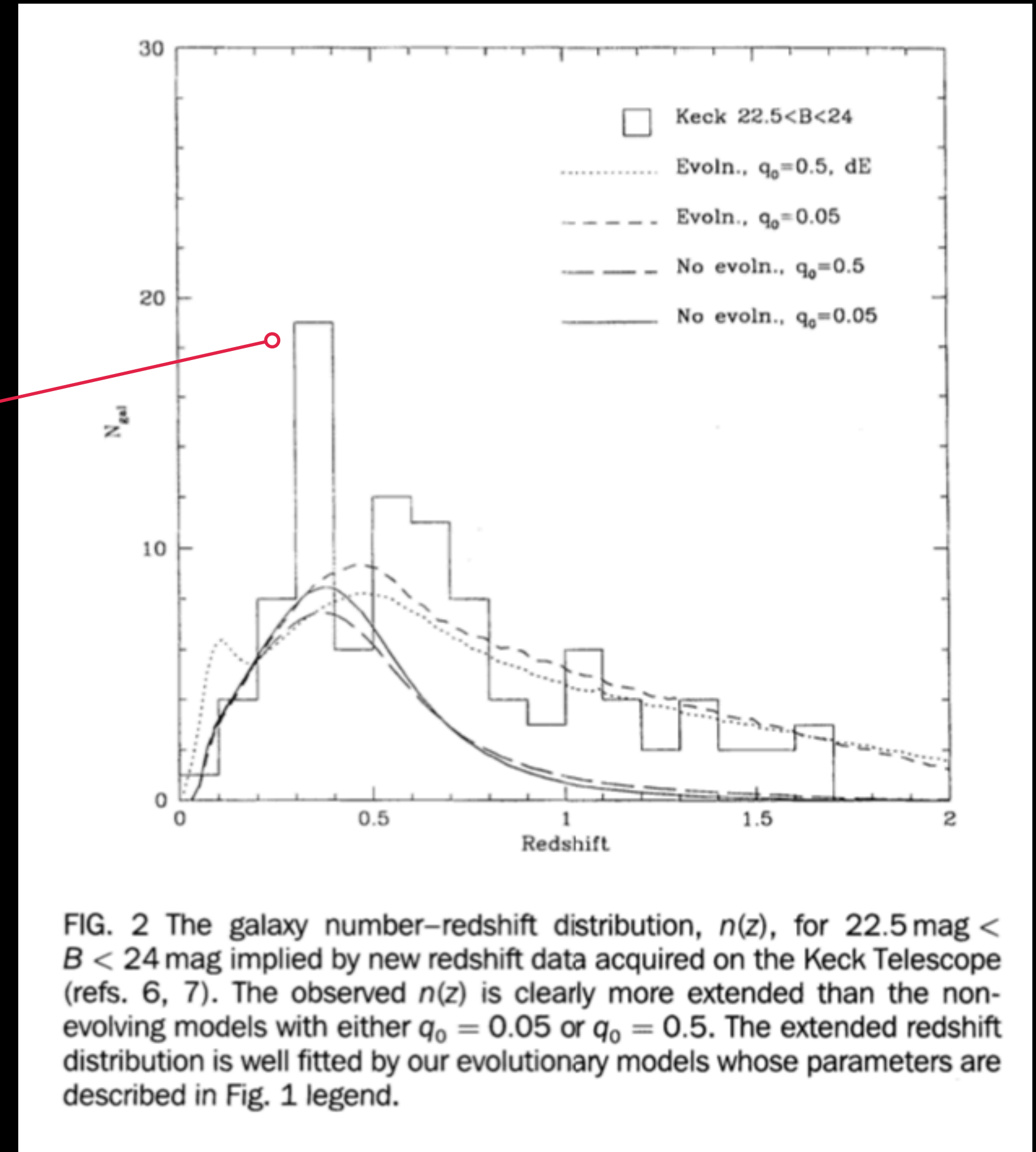
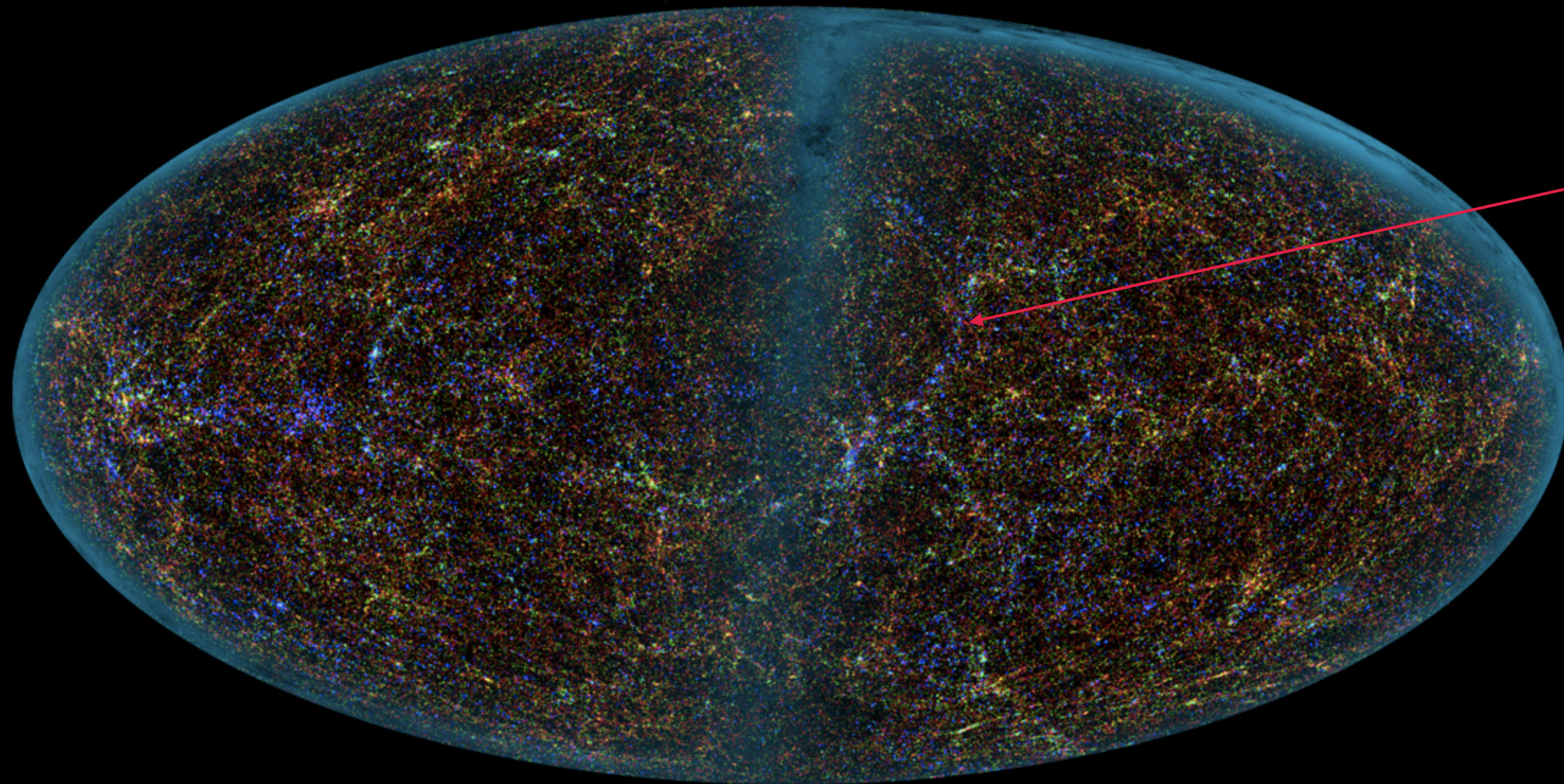
↗ ↑ ↖

Density distribution
(e.g., non-uniform
large scale structure)

Luminosity function
 $n(L) \leftrightarrow \Phi(M)$

Volume element
(cosmological)

Spikes in $N(z)$ correspond to filament & voids along the line of sight



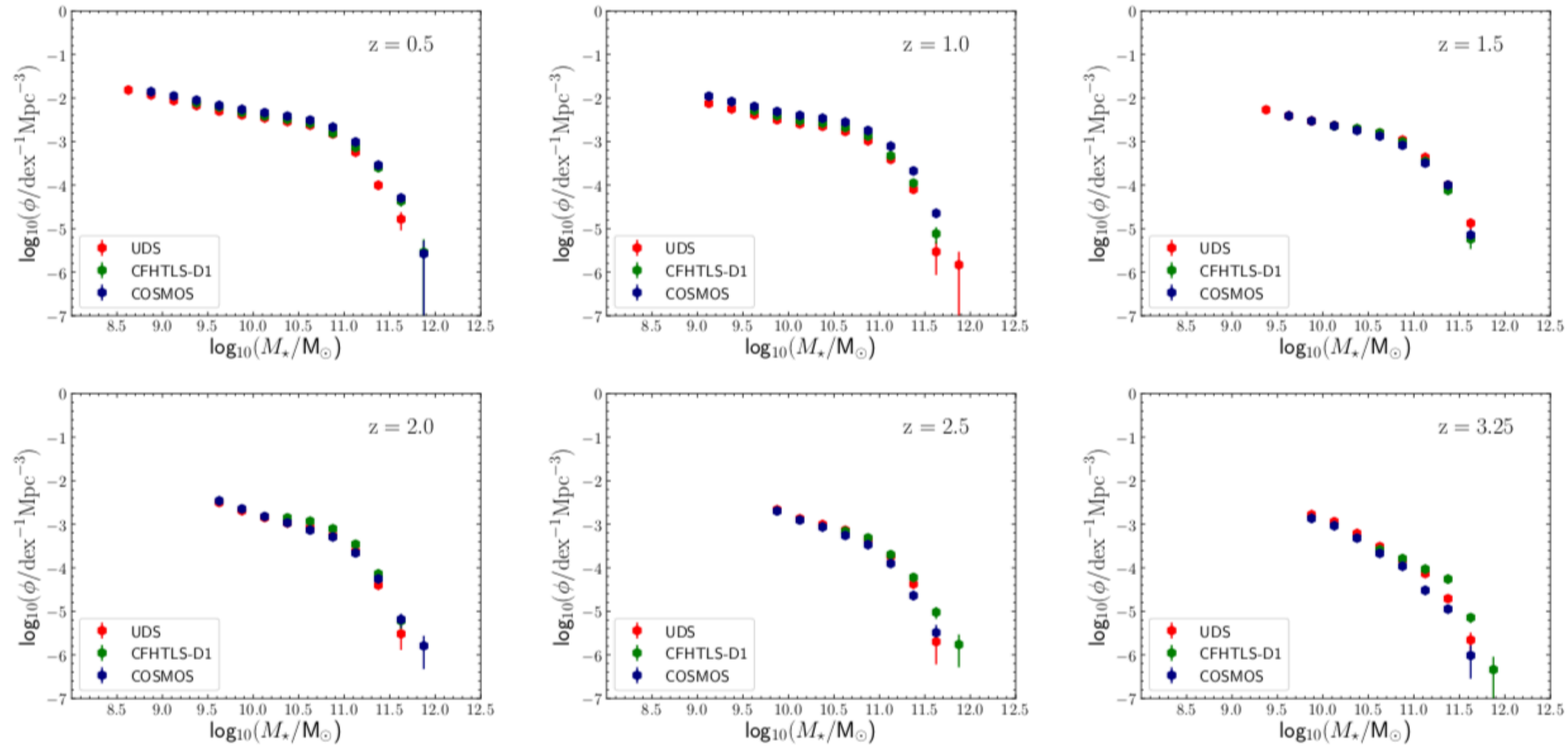


Figure 3. A comparison of how the observed GSMF evolves as a function of redshift in our three, degree-scale, survey fields. In this plot the number density uncertainties are simply the Poissonian counting errors. The availability of three, non-contiguous, degree-scale survey fields allows an empirical measurement of the level of cosmic variance in the high-mass end of the GSMF (see text for discussion).