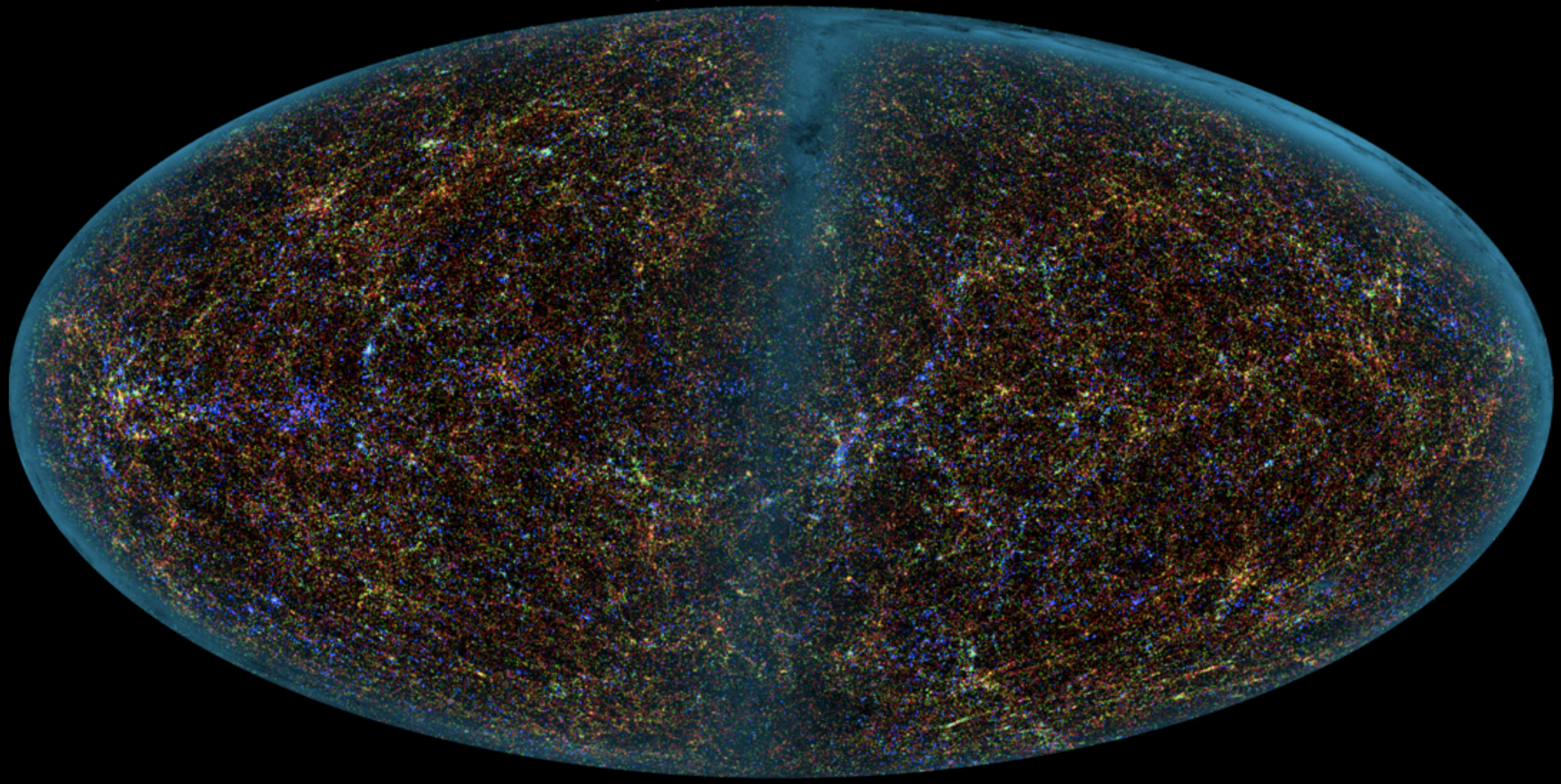
Cosmology

and Large Scale Structure



Today

Large Scale Structure

The Power Spectrum

Measurements of Ω_m

9 November 2020

http://astroweb.case.edu/ssm/astr328/

Large Scale Structure

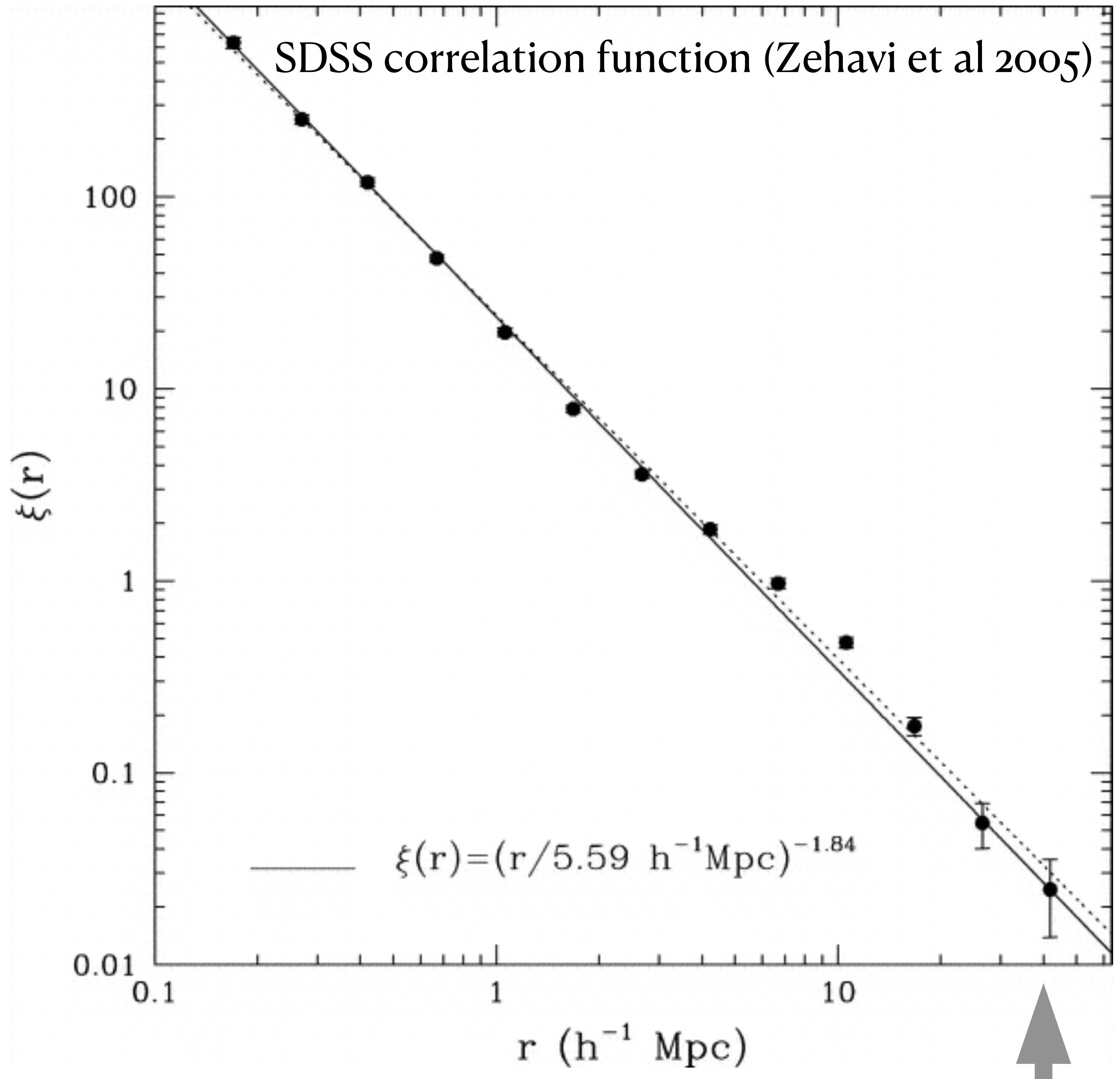
Quantified with the **correlation function** $\xi(r)$ which is the Fourier transform of the **power spectrum** $P(k)$.

The correlation function is the excess probability of finding a galaxy near another galaxy over that in a random distribution.

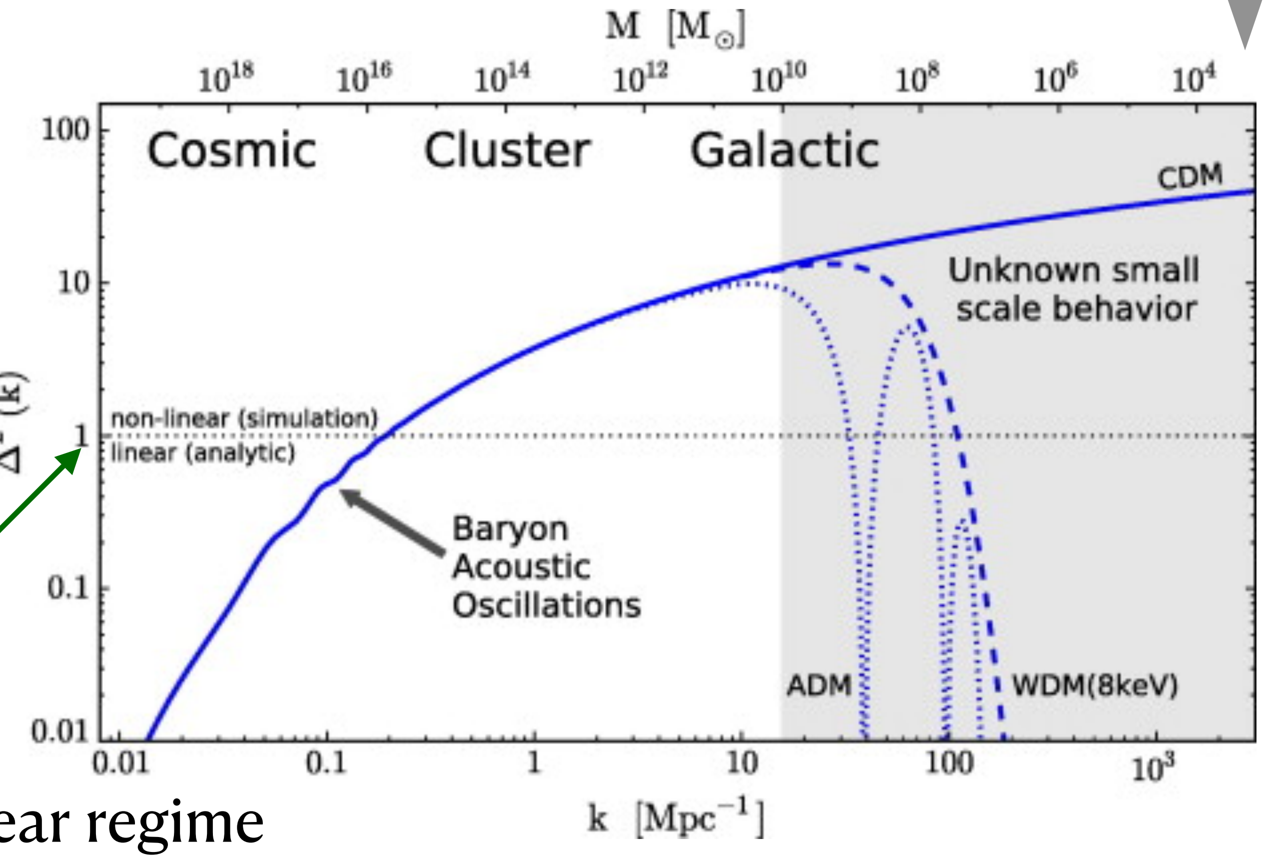
$$\frac{dN}{N} = [1 + \xi(r)]dV \qquad \xi(r) = \frac{V}{(2\pi)^3}\int P(k)e^{-\vec{k}\cdot\vec{r}}\, d^3k$$

$$P(k) \propto |\delta(k)|^2 \propto k^n \qquad \xi(r) \propto r^{-(n+3)}$$

Harrison-Zeldovich spectrum has $n = 1$, which is a Gaussian random field. Inflation predicts $n \approx 1$, but different flavors of Inflationary theory predict slightly different values depending on the shape of the Inflationary potential (the Inflaton). Planck measures $n = 0.965 \pm 0.004$


SDSS correlation function (Zehavi et al 2005)

Power Spectrum



$\delta > 1$ marks the transition to the non-linear regime where perturbation theory no longer applies.

• Power spectrum of galaxies

$$\delta \equiv \frac{\delta\rho}{\rho}$$

The power spectrum is commonly used to quantify large scale structure. It is the related to the 2 point correlation function via Fourier transform.

2 point correlation function: $$\xi(r) = \langle \delta(\vec{x}) \cdot \delta(\vec{x} + \vec{r}) \rangle$$

The 2 point correlation function is the probability of finding one galaxy near another in excess over a random distribution.

Power spectrum: $$P(k) = \langle |\delta_k|^2 \rangle$$ where $$k = \frac{2\pi}{\lambda}$$

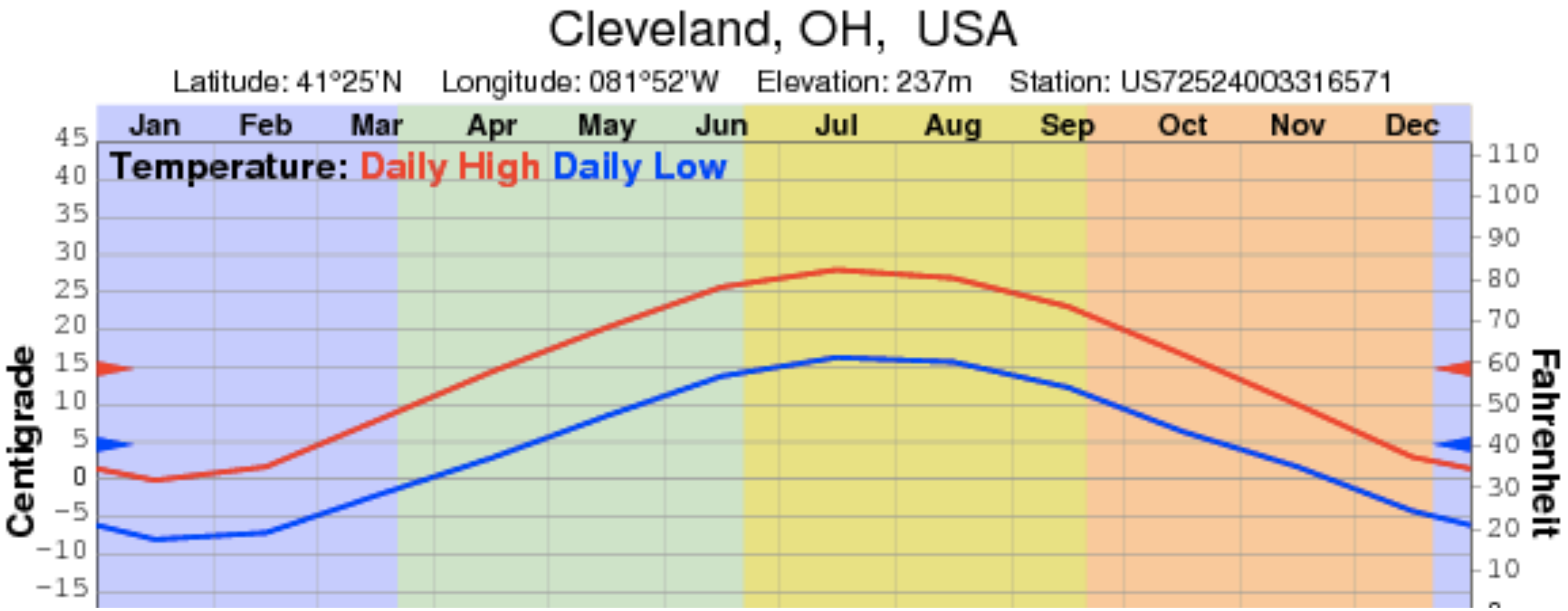
where k is the wavenumber corresponding to the scale λ

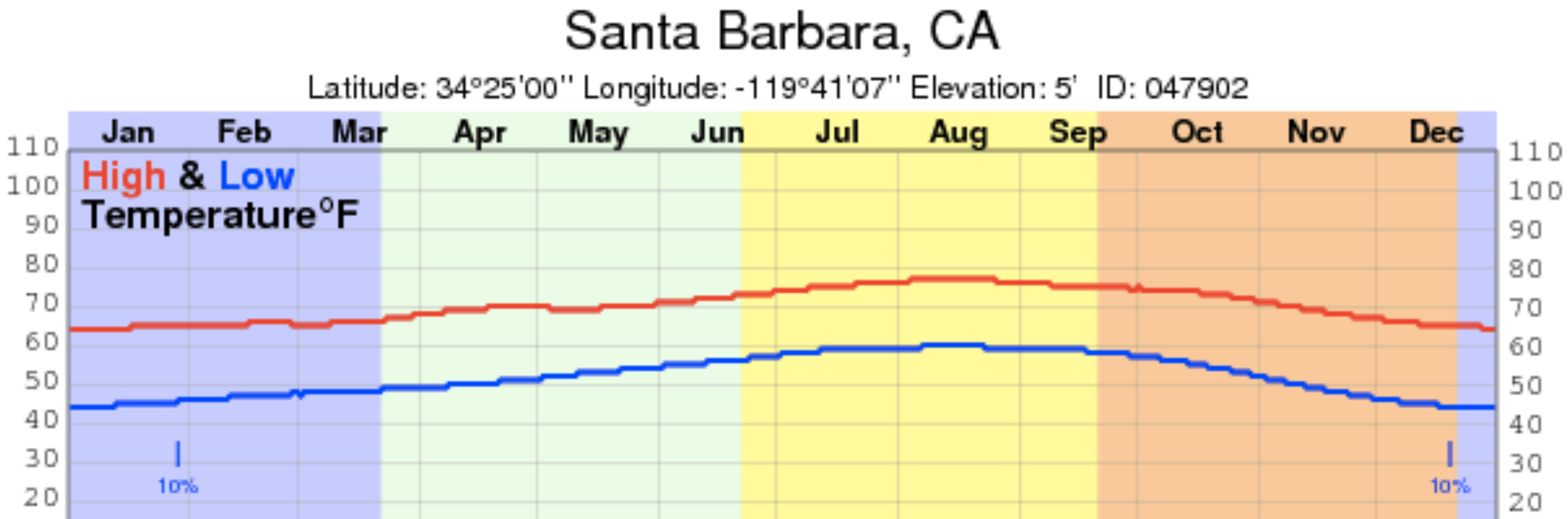
Fourier transform:

$$\xi(\vec{r}) = \frac{V}{(2\pi)^3} \int \underset{\substack{\uparrow \\ P(k)}}{|\delta_k|^2} e^{-i\vec{k}\cdot\vec{r}d^3k}$$ averaged over volume V

Power Spectrum

Example: weather in Cleveland and Santa Barbara
More power on long time scales in Cleveland (seasonal variation)

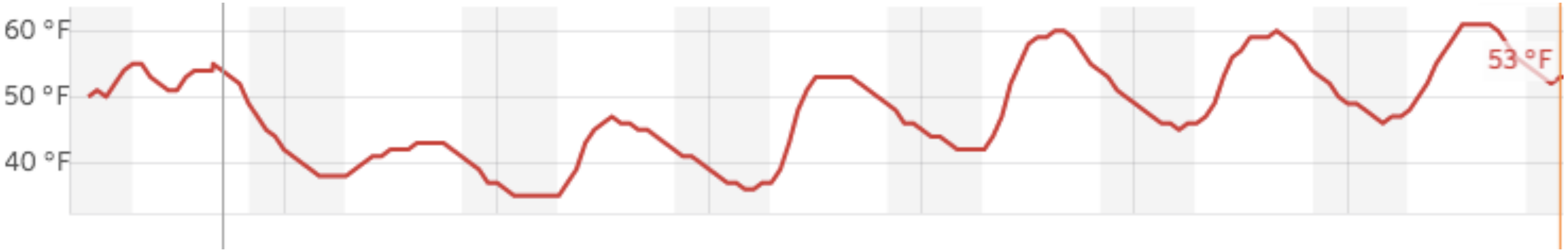




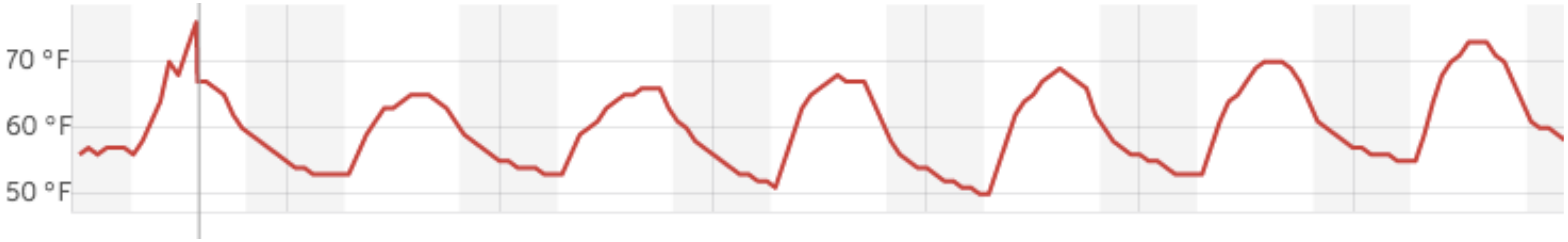
Power Spectrum

Example: weather in Cleveland and Santa Barbara
Similar power on short time scales in Santa Barbara (diurnal variation)

Cleveland forecast



Santa Barbara forecast



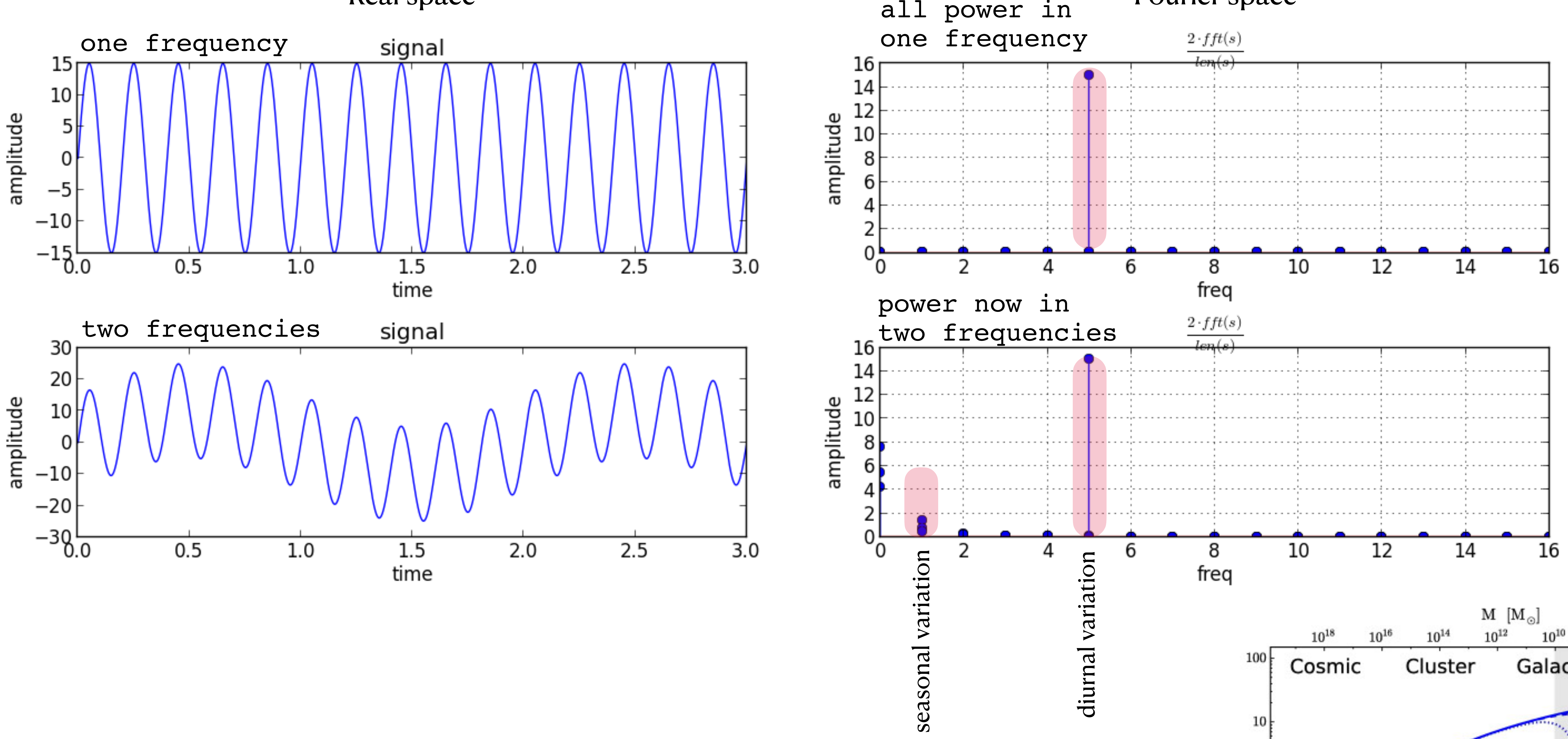
A power spectrum is a Fourier transform that quantifies
the relative variability on different scales

Superposition of two sinusoids

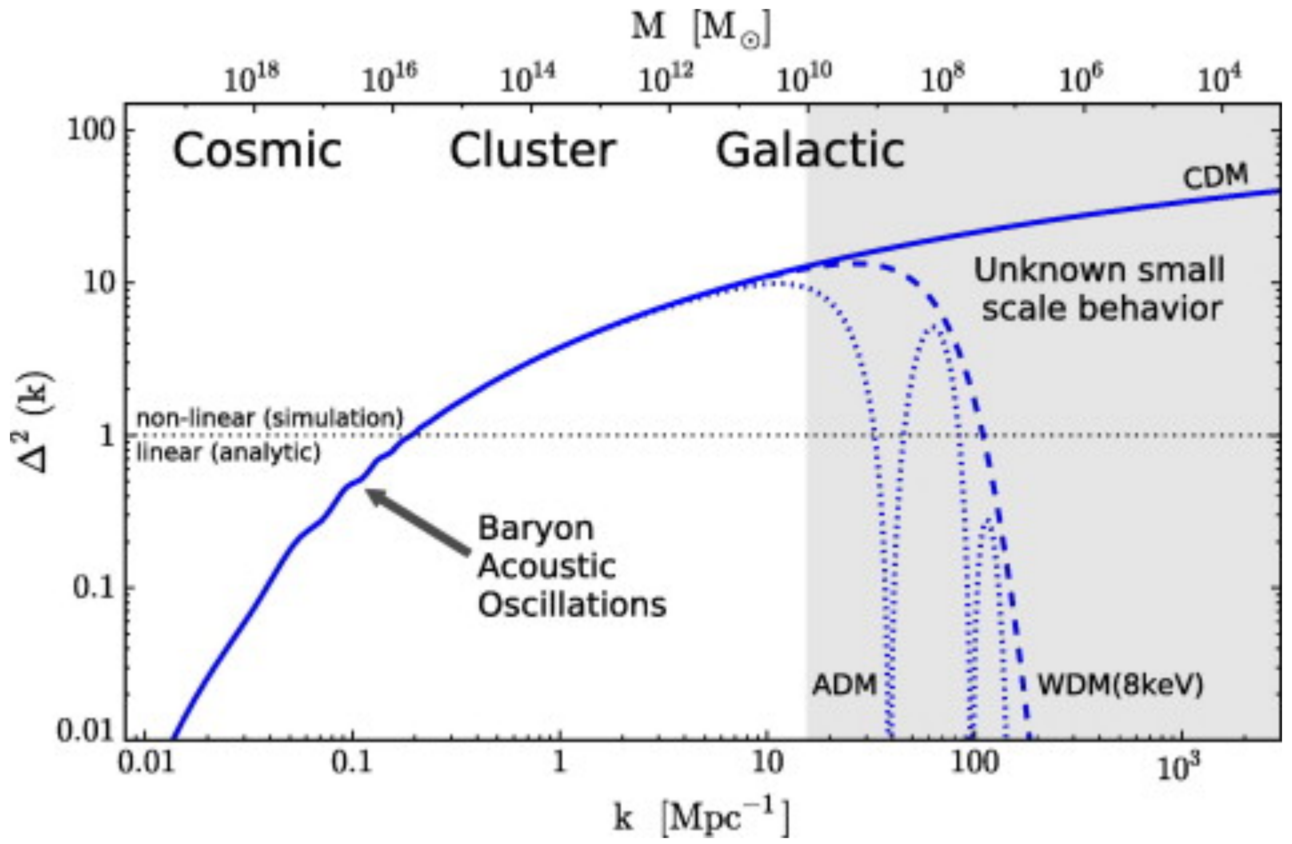
(e.g., diurnal and annual temperature variation)

Real space

Fourier space



So a smooth power spectrum has contributions from all frequencies, but also picks out which are more common.



- Power spectrum of galaxies $\delta \equiv \frac{\delta\rho}{\rho}$ $k = \frac{2\pi}{\lambda}$

Power law power spectrum: $$P(k) = \langle |\delta_k|^2 \rangle \propto k^n$$

where n = 1 is scale free, with the same power on all scales.
This is observed to be nearly the case on large scales that have not yet collapsed. It is modulated on small scales by structure formation.

One way to think of it is the rms variation at each scale λ

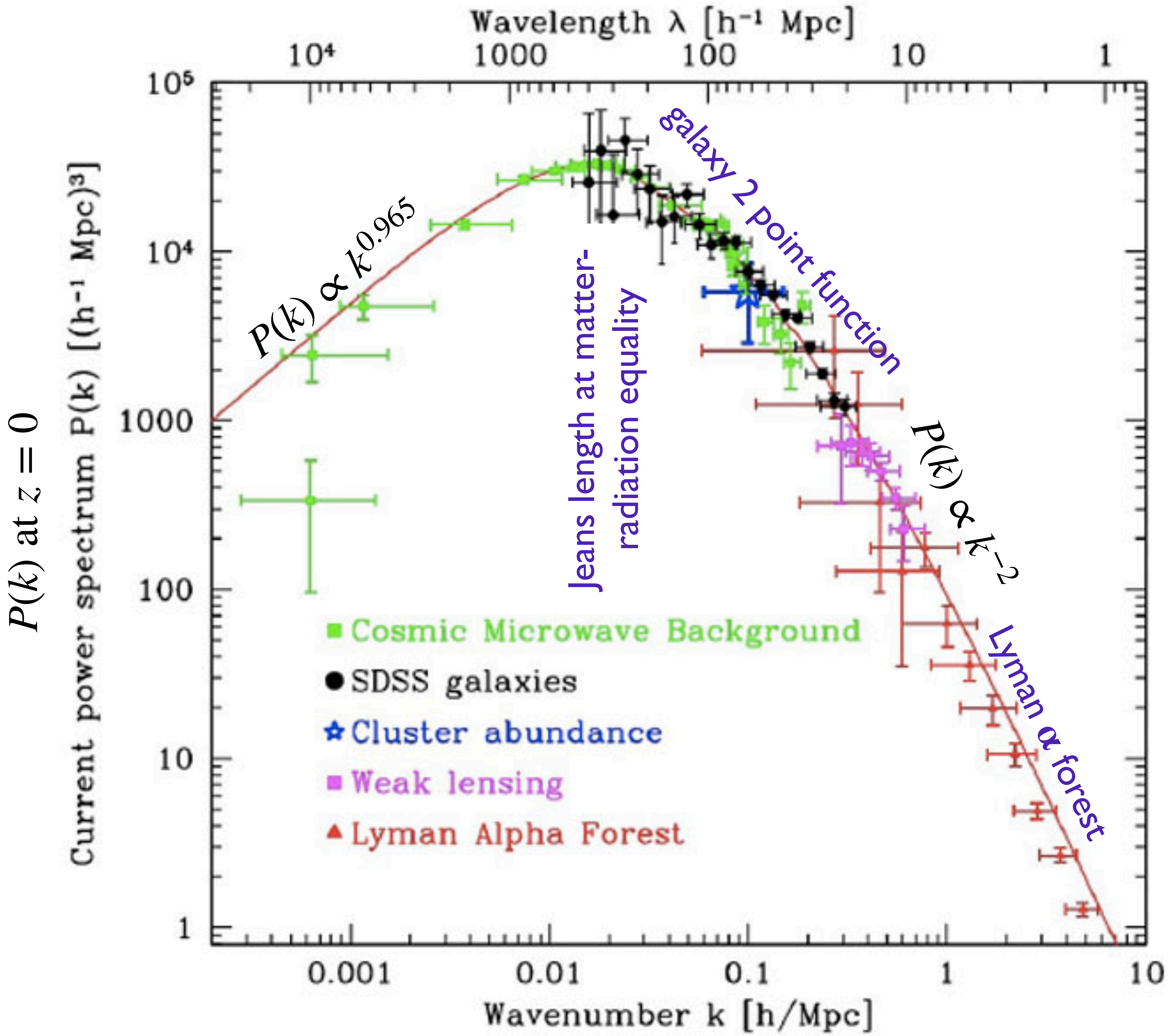
$$M \sim \lambda^3 \qquad \delta_{\mathrm{rms}} \propto M^{-(n+3)/6}$$

There is more rms variance on small scales, so more power there.
[On very large scales, the universe is homogeneous, so no variance.]

By convention, the normalization is set on a scale of 8 Mpc, where

$$\frac{\delta N_{gal}}{N_{gal}} = 1$$ with corresponding mass variance σ_8

Planck estimates: $$n = 0.965 \pm 0.004$$ $$\sigma_8 = 0.811 \pm 0.006$$



Jeans length at matter-radiation equality

$$\lambda_J = c_s \sqrt{\frac{\pi}{G\rho}}$$

sound speed of photon-baryon fluid

$$c_s^2 = \frac{\partial P}{\partial \rho} = \frac{1}{3}c^2$$

For photons,

$$P = \frac{1}{3}\rho c^2$$

imprints standard rod on surface of last scattering.

at smaller scales, things go non-linear from gravitational collapse, pressure, dissipation, feedback, etc. Described by a Transfer function

$$T(k) \equiv \frac{\delta_k(z=0)}{D(z)\delta_k(z)}$$

where D(z) is the linear growth factor - what it would have been without all these nasty non-linear effects.

Large Scale Structure

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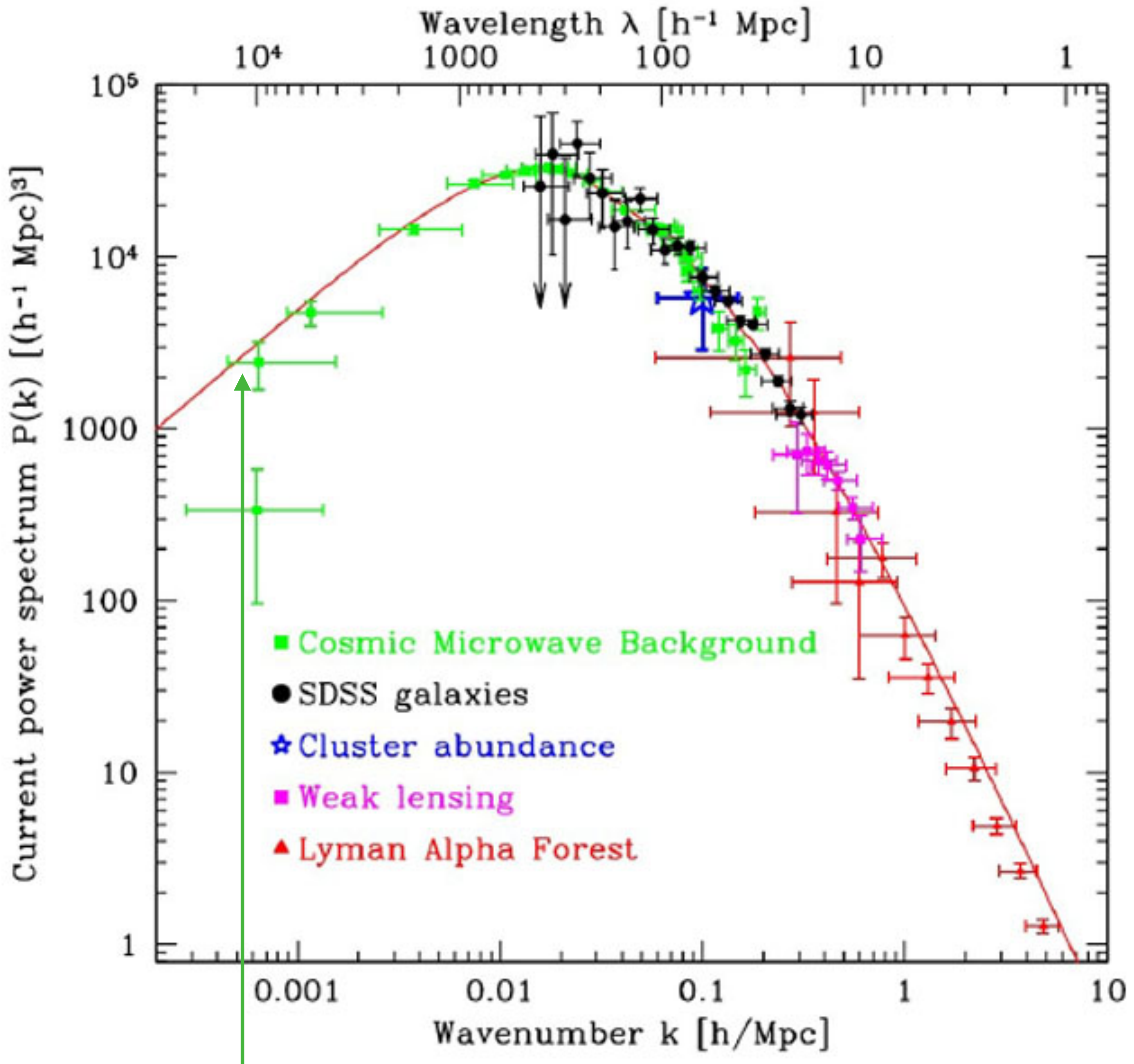
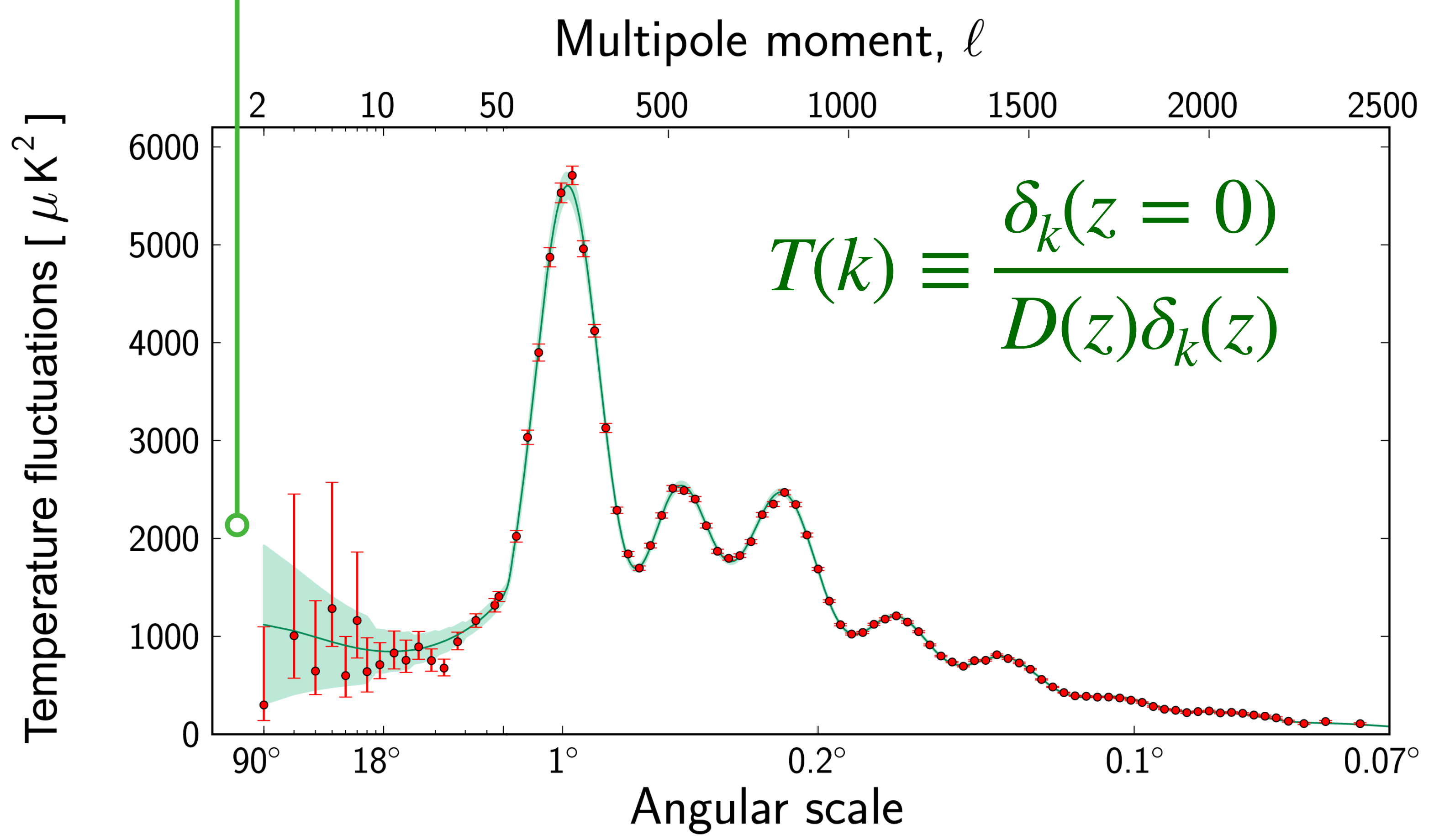
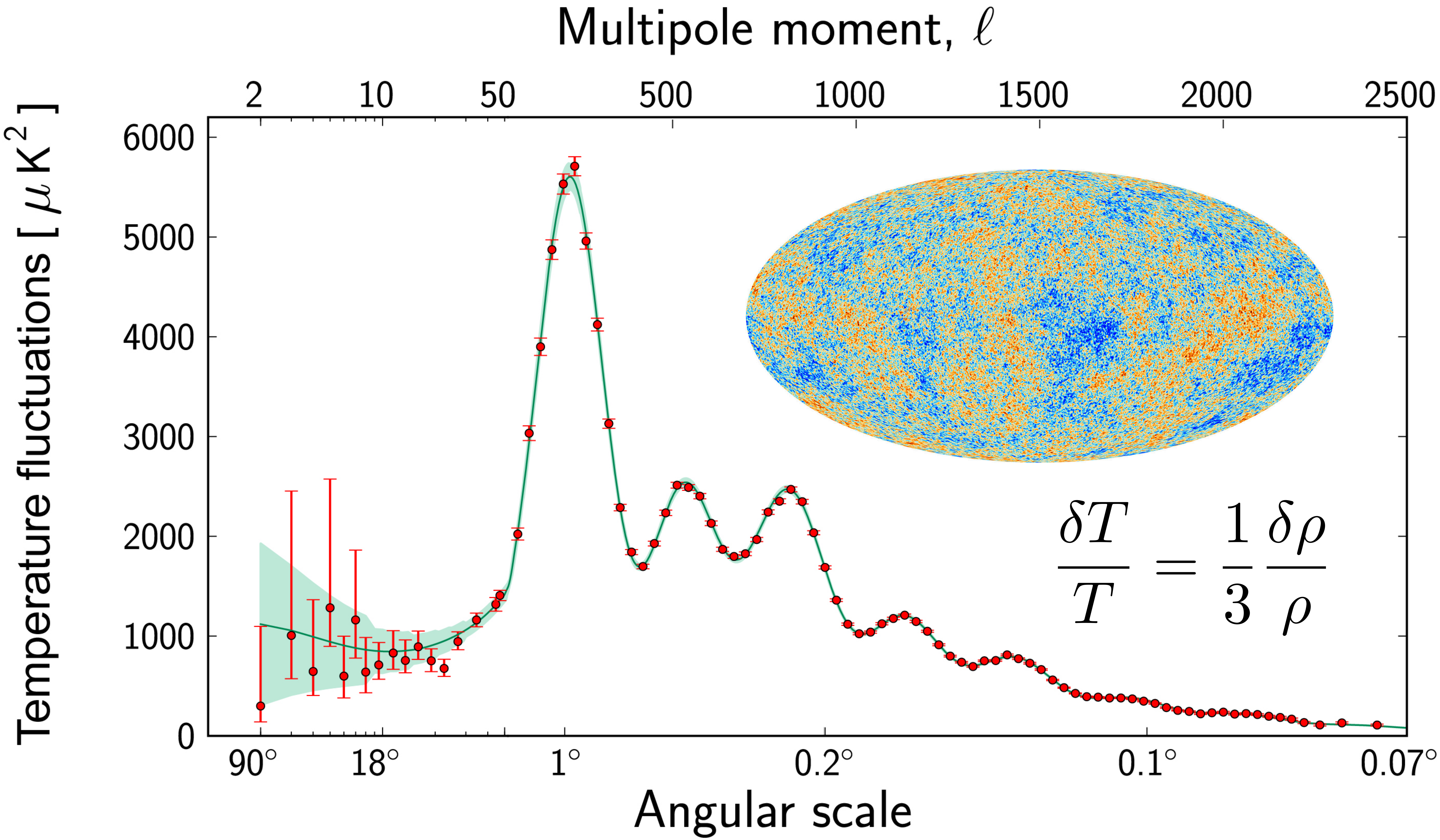


CMB power spectrum at z = 1000 in linear regime mapped to galaxy power spectrum at z=0 by calculation of $D(z)$



$$T(k) \equiv \frac{\delta_k(z=0)}{D(z)\delta_k(z)}$$

Detailed shape of the acoustic power spectrum depends sensitively on cosmic parameters. First and foremost, the location of the first peak measures the angular diameter distance to the surface of last scattering. This is the best evidence that the universe is very nearly flat: $\Omega_k = -0.011 \pm 0.006$ (Planck X 2018)



$$\frac{\delta T}{T} = \frac{1}{3}\frac{\delta\rho}{\rho}$$

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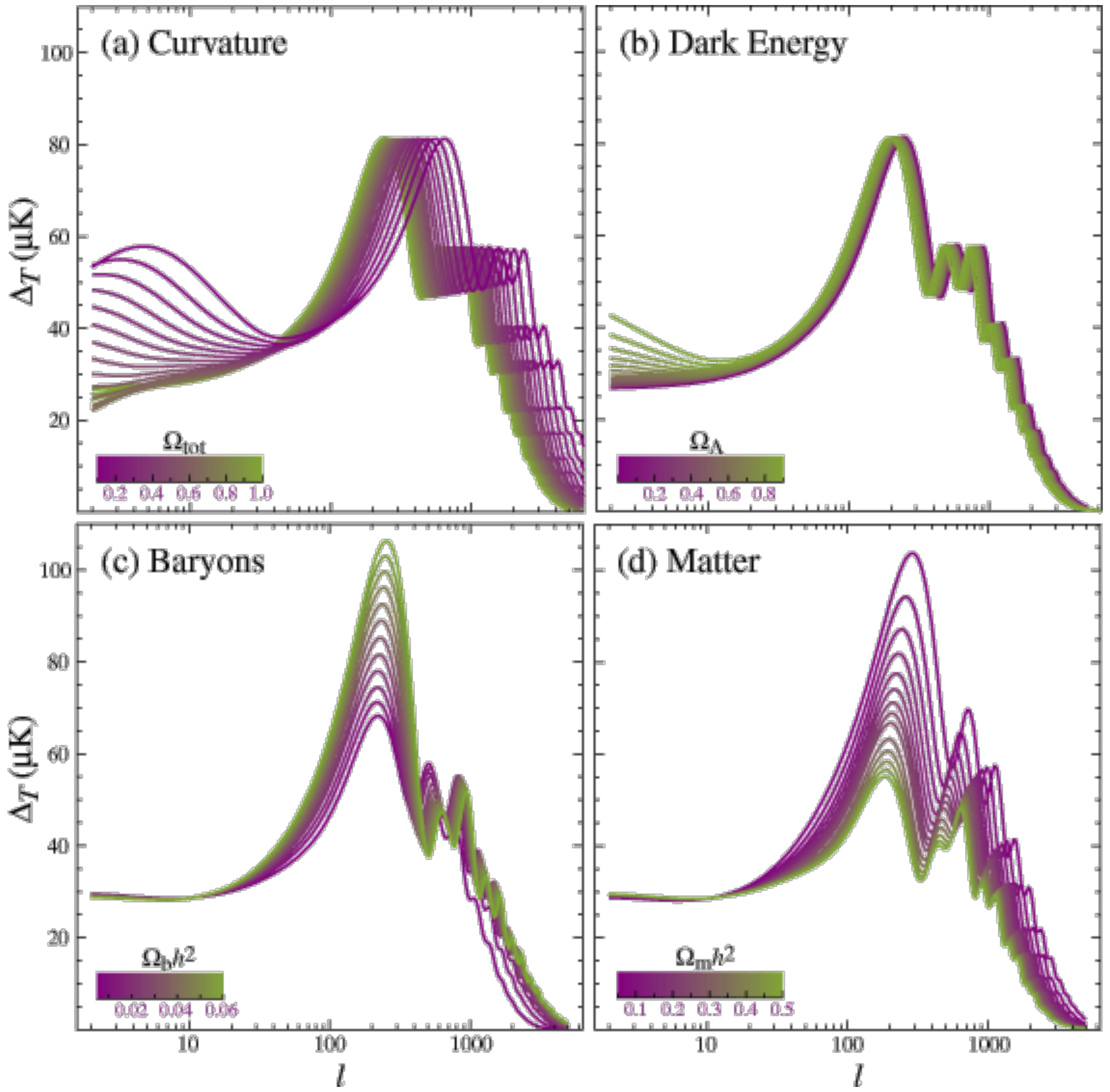


Best-fit cosmology obtained from multi-parameter fit. Well constrained, but not unique - lots of parameter degeneracy.

Compression and rarefaction nearly cancel out, but don't quite. Left with

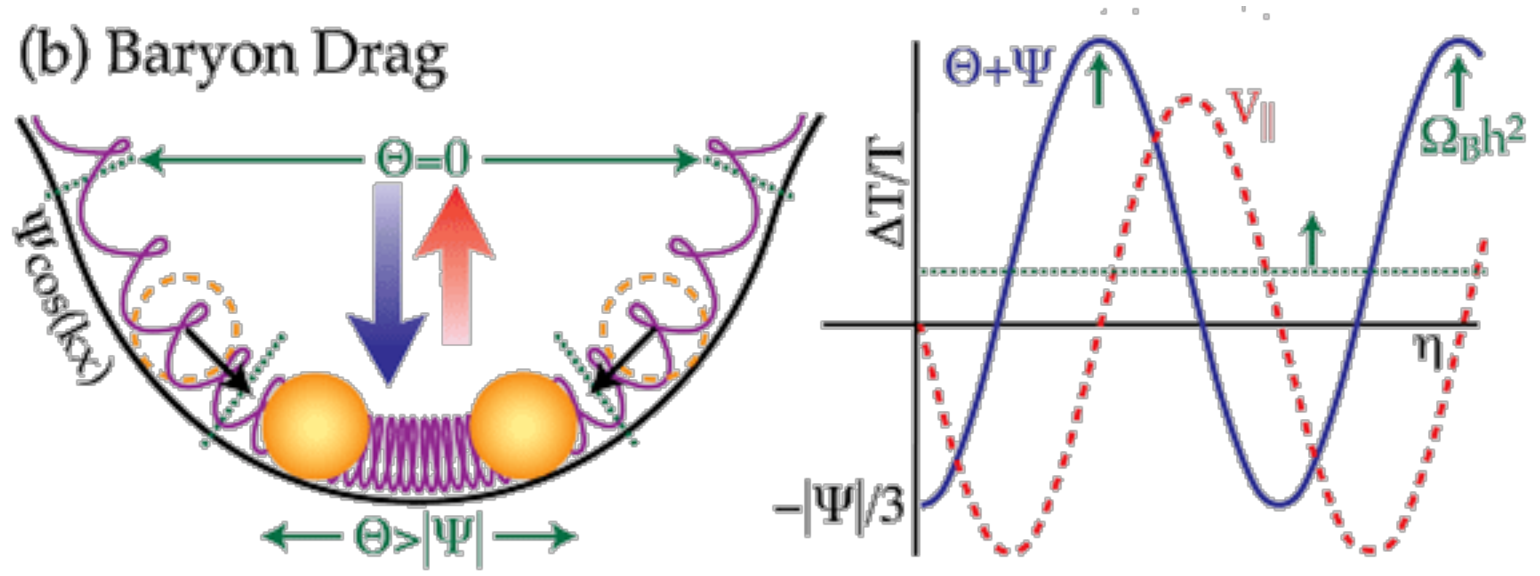
$$\frac{\delta T}{T} = \frac{1}{3}\frac{\delta\rho}{\rho}$$

Damped and driven oscillator

Baryons damp oscillations, like a kid dragging his feet on a swing.

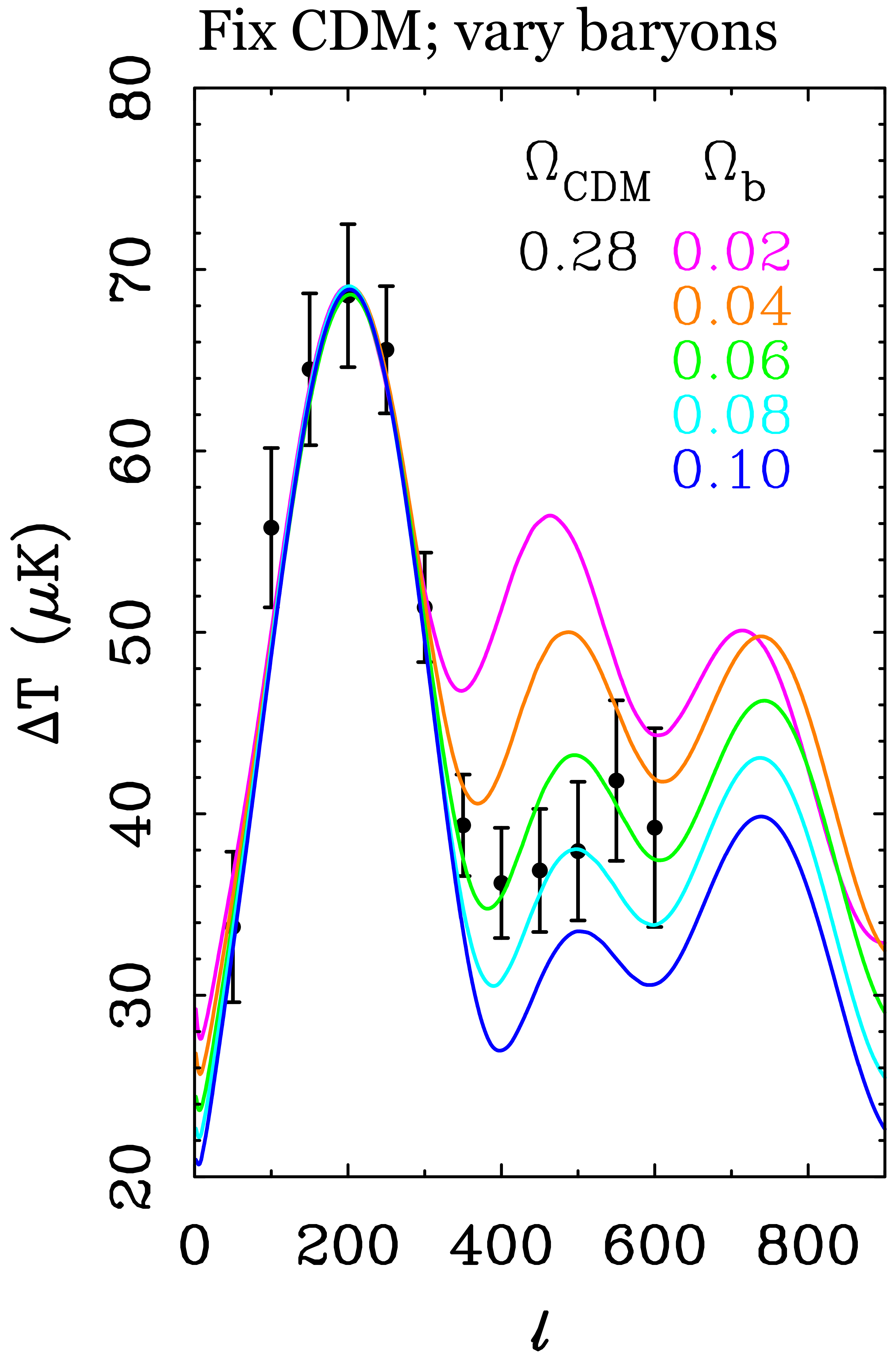
pure damping spectrum in limit of all baryons

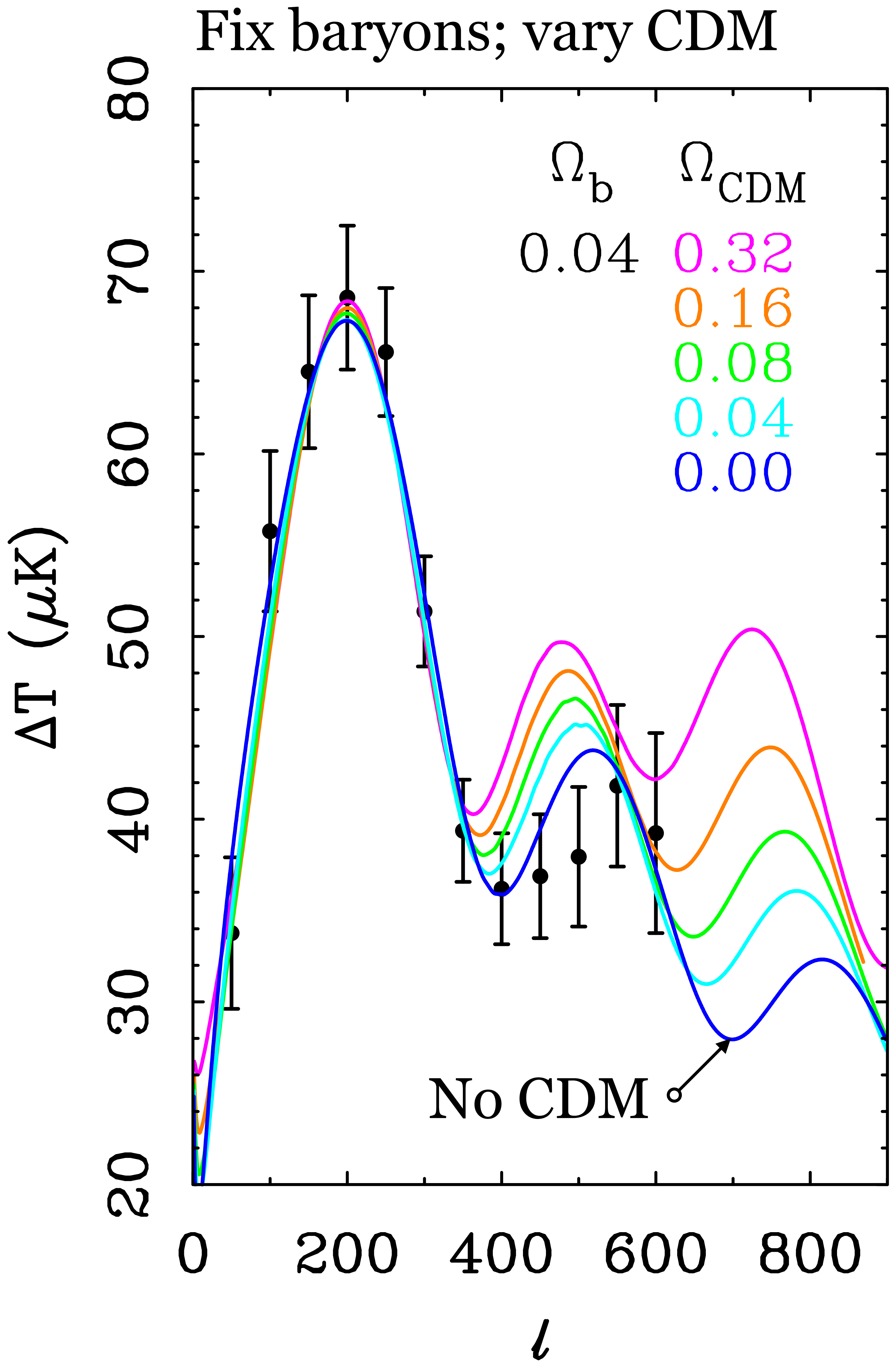
Dark matter helps drive oscillations, like a parent pushing the kid.



Wayne Hu provides a nice CMB tutorial at http://background.uchicago.edu/index.html

CMB dependence on the density of baryonic and non-baryonic matter


Fix CDM; vary baryons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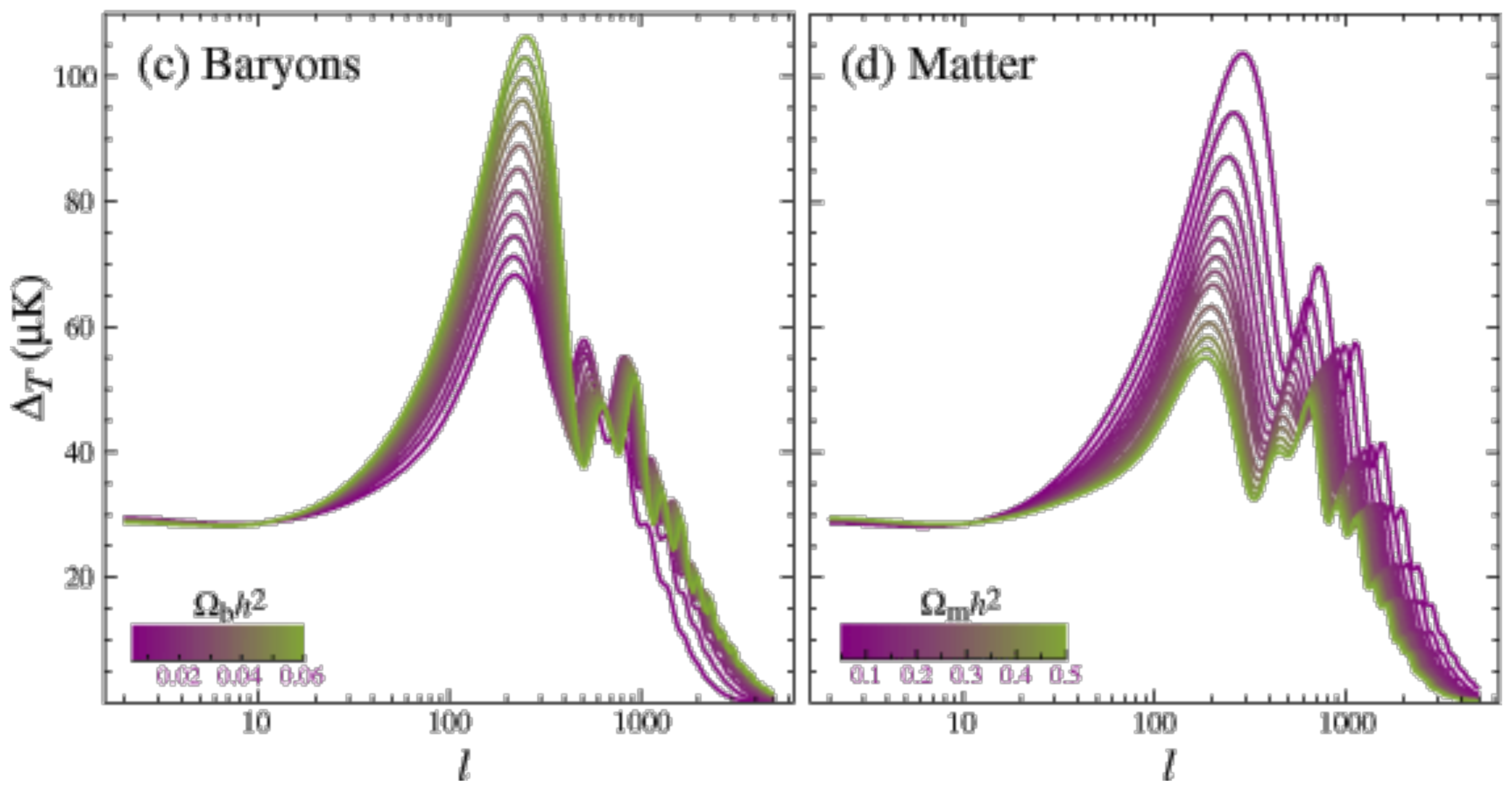
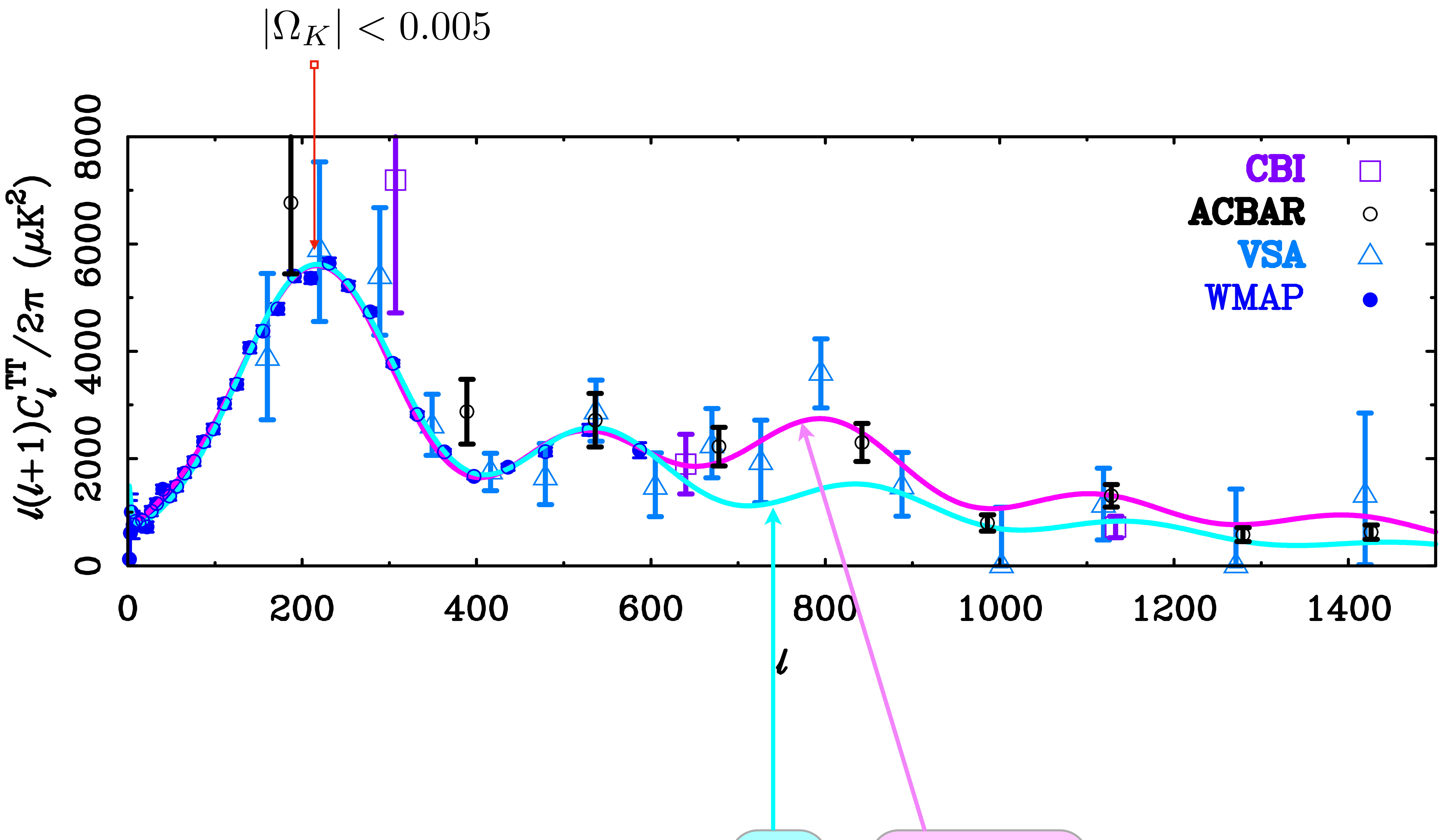
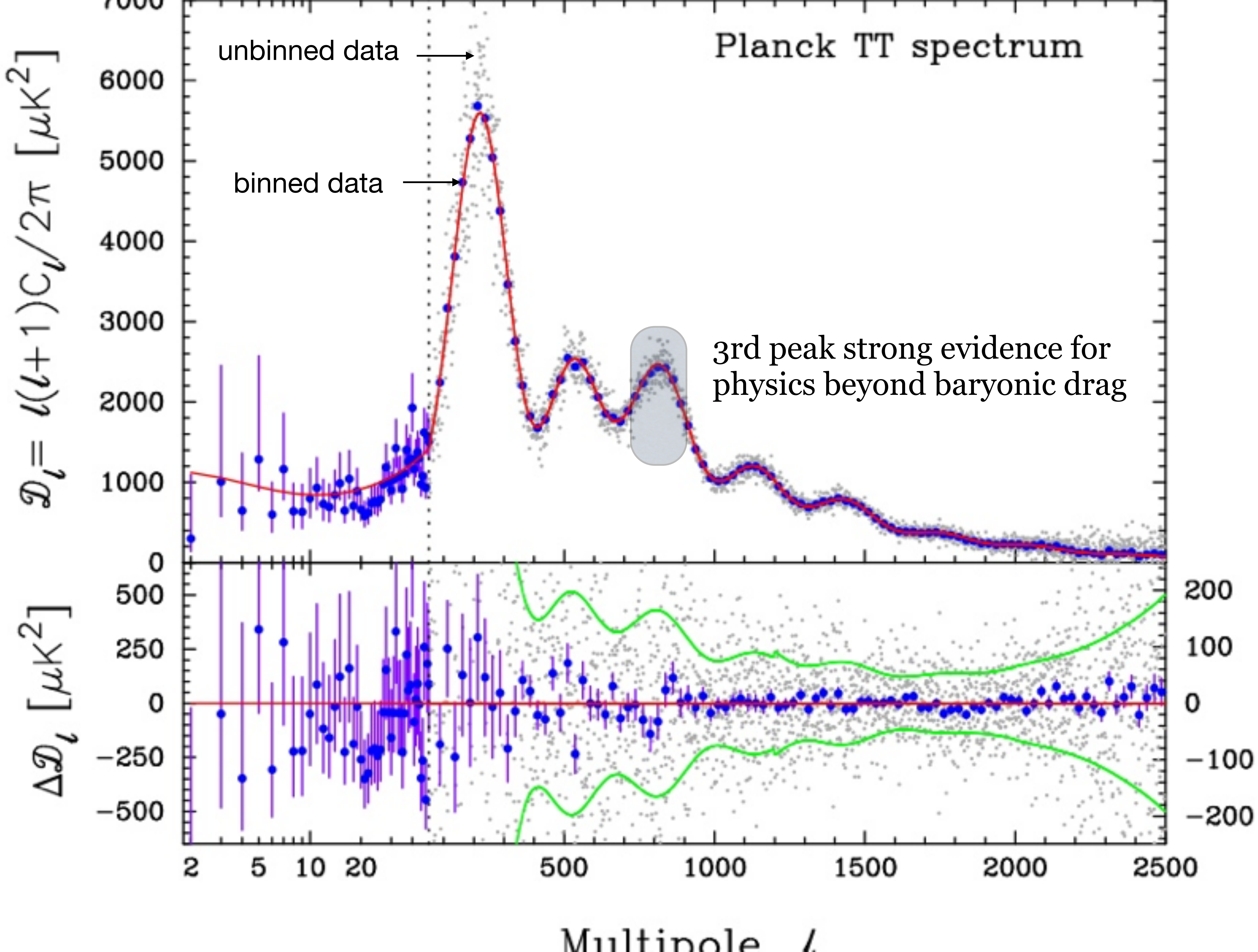



$|\Omega_K| < 0.005$

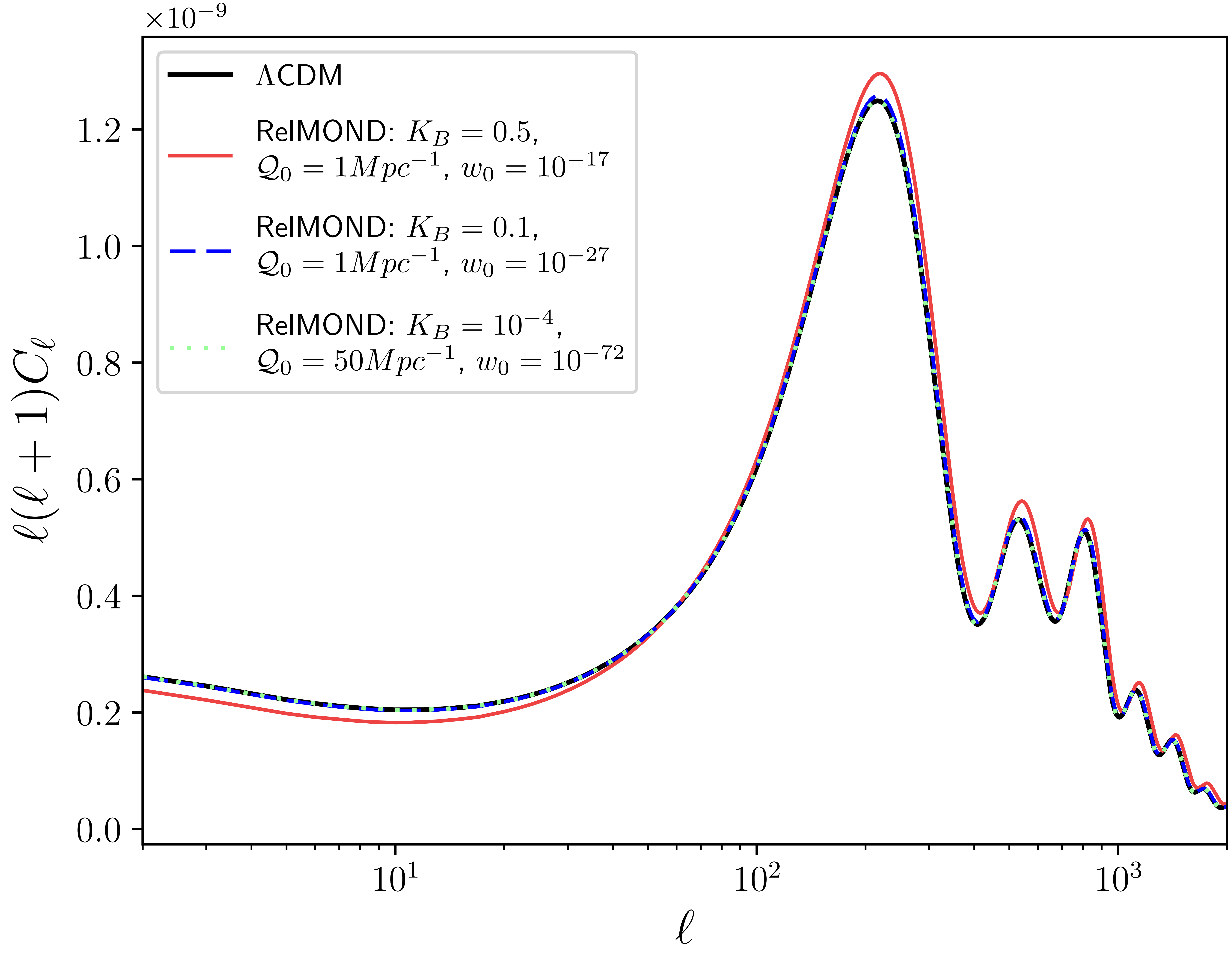
$$k^2\Phi = 4\pi G(\rho_b\delta_b + \boxed{\rho_\gamma\delta_\gamma} + \boxed{\rho_{CDM}\delta_{CDM}})$$

baryons a net drag

CDM a net forcing term



3rd peak strong evidence for physics beyond baryonic drag

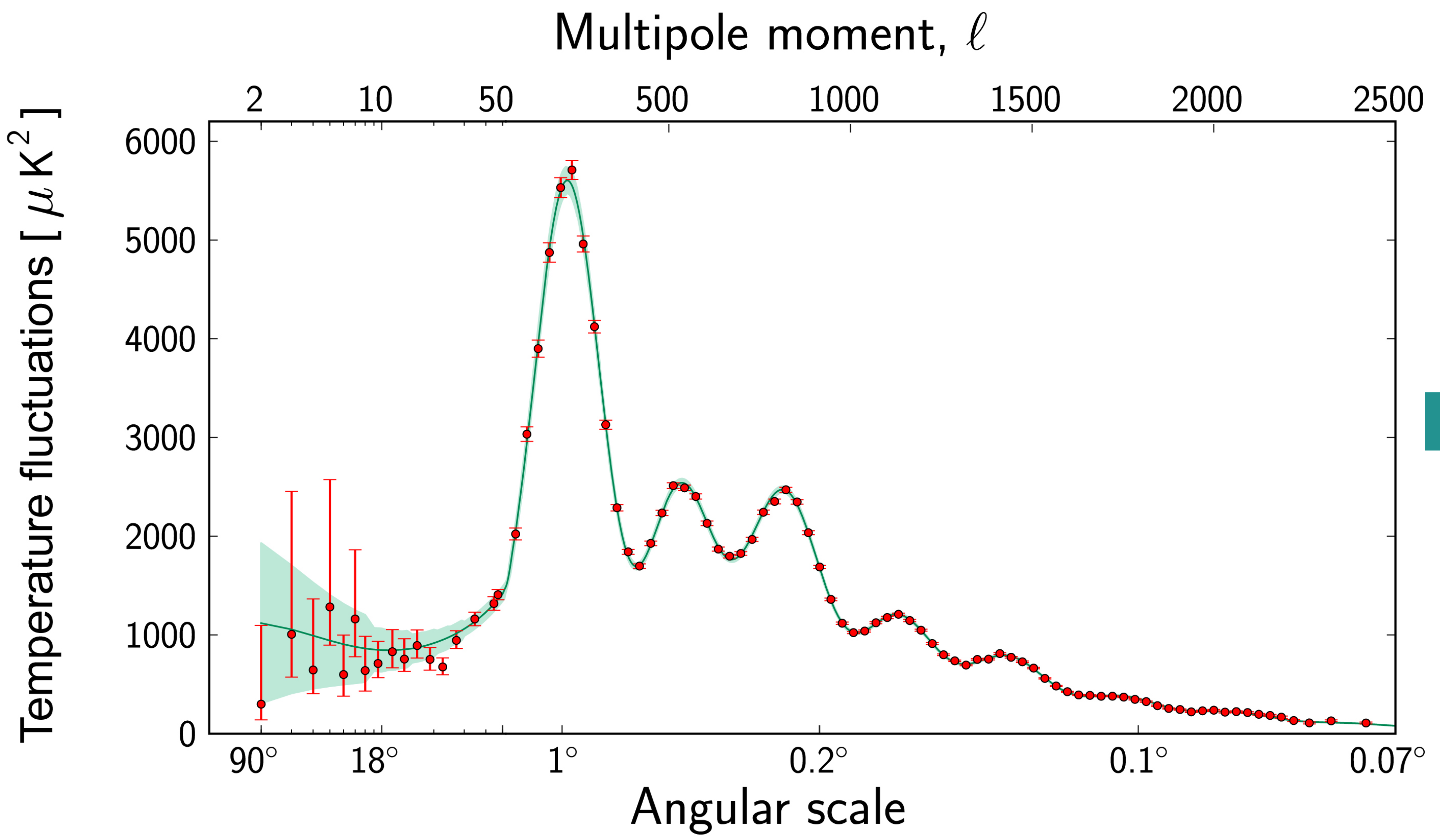


3rd peak strong evidence for physics beyond baryonic drag.

This is usually interpreted to require the existence of non-baryonic cold dark matter, which Planck requires at over 50 σ:
$\Omega_{\rm CDM}h^2 = 0.1206 \pm 0.0021$

However, the interpretation remains ambiguous - could also be a modification of gravity (e.g., RelMOND: arXiv: 2007.00082 gives an identical power spectrum.)

Detailed shape of the acoustic power spectrum depends sensitively on cosmic parameters.



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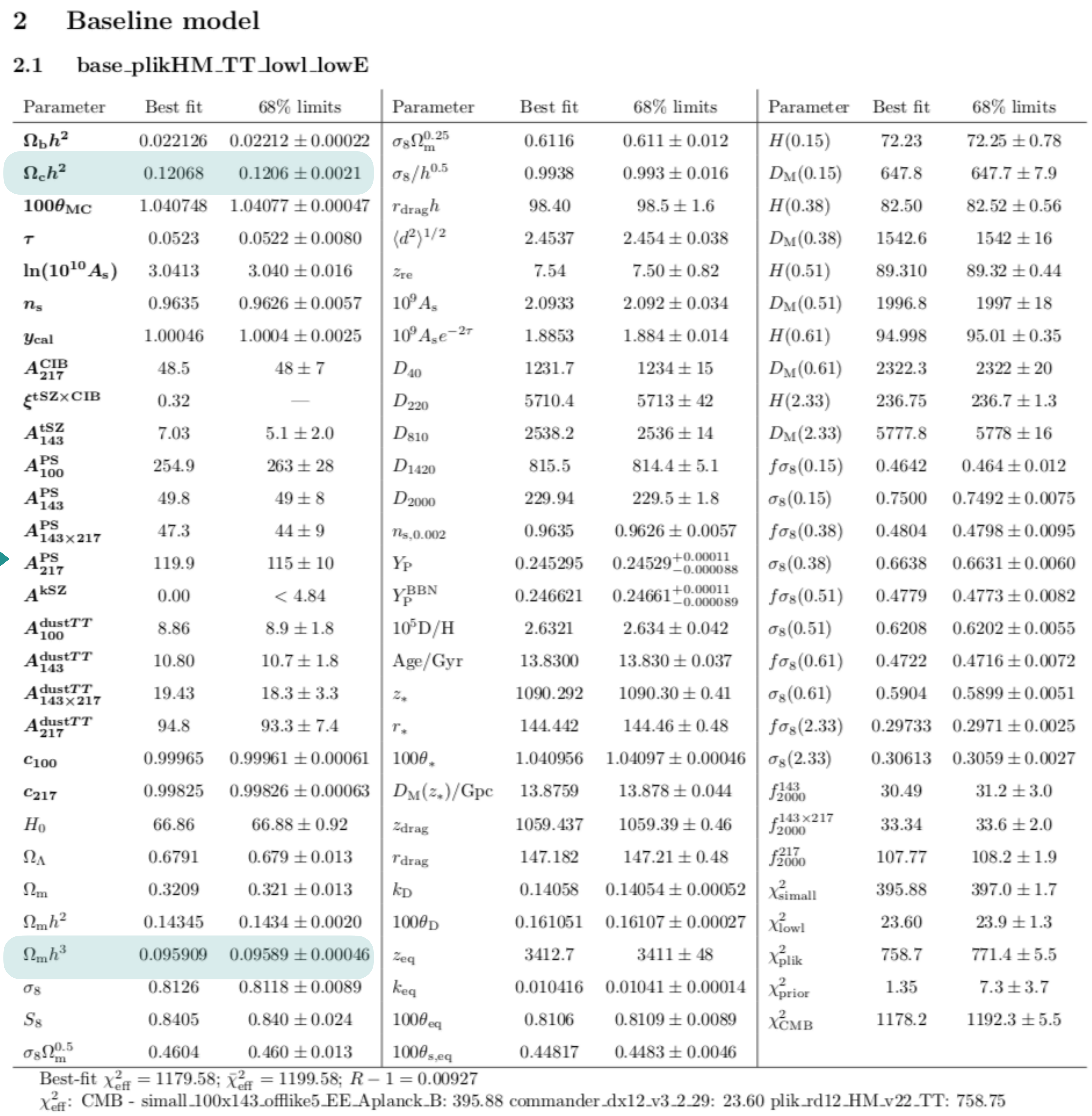


2 Baseline model

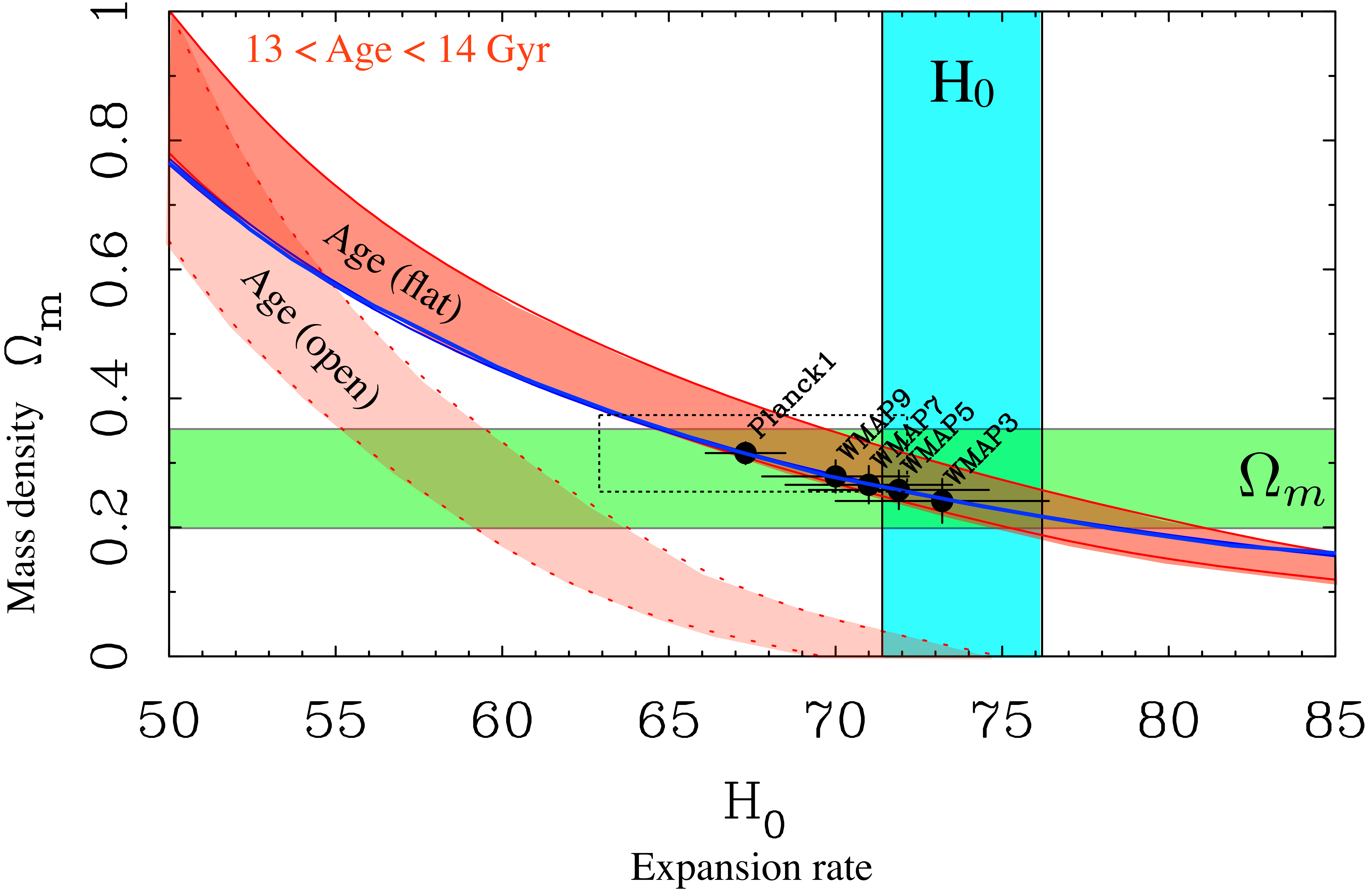
2.1 base_plikHM_TT_lowl_lowE

Parameter	Best fit	68% limits	Parameter	Best fit	68% limits	Parameter	Best fit	68% limits
$\boldsymbol{\Omega_b h^2}$	0.022126	0.02212 ± 0.00022	$\sigma_8\Omega_m^{0.25}$	0.6116	0.611 ± 0.012	$H(0.15)$	72.23	72.25 ± 0.78
$\boldsymbol{\Omega_c h^2}$	0.12068	0.1206 ± 0.0021	$\sigma_8/h^{0.5}$	0.9938	0.993 ± 0.016	$D_M(0.15)$	647.8	647.7 ± 7.9
$\boldsymbol{100\theta_{MC}}$	1.040748	1.04077 ± 0.00047	$r_{drag}h$	98.40	98.5 ± 1.6	$H(0.38)$	82.50	82.52 ± 0.56
$\boldsymbol{\tau}$	0.0523	0.0522 ± 0.0080	$\langle d^2\rangle^{1/2}$	2.4537	2.454 ± 0.038	$D_M(0.38)$	1542.6	1542 ± 16
$\boldsymbol{\ln(10^{10}A_s)}$	3.0413	3.040 ± 0.016	z_{re}	7.54	7.50 ± 0.82	$H(0.51)$	89.310	89.32 ± 0.44
$\boldsymbol{n_s}$	0.9635	0.9626 ± 0.0057	10^9A_s	2.0933	2.092 ± 0.034	$D_M(0.51)$	1996.8	1997 ± 18
$\boldsymbol{y_{cal}}$	1.00046	1.0004 ± 0.0025	$10^9A_s e^{-2\tau}$	1.8853	1.884 ± 0.014	$H(0.61)$	94.998	95.01 ± 0.35
$\boldsymbol{A_{217}^{CIB}}$	48.5	48 ± 7	D_{40}	1231.7	1234 ± 15	$D_M(0.61)$	2322.3	2322 ± 20
$\boldsymbol{\xi^{tSZ\times CIB}}$	0.32	—	D_{220}	5710.4	5713 ± 42	$H(2.33)$	236.75	236.7 ± 1.3
$\boldsymbol{A_{143}^{tSZ}}$	7.03	5.1 ± 2.0	D_{810}	2538.2	2536 ± 14	$D_M(2.33)$	5777.8	5778 ± 16
$\boldsymbol{A_{100}^{PS}}$	254.9	263 ± 28	D_{1420}	815.5	814.4 ± 5.1	$f\sigma_8(0.15)$	0.4642	0.464 ± 0.012
$\boldsymbol{A_{143}^{PS}}$	49.8	49 ± 8	D_{2000}	229.94	229.5 ± 1.8	$\sigma_8(0.15)$	0.7500	0.7492 ± 0.0075
$\boldsymbol{A_{143\times217}^{PS}}$	47.3	44 ± 9	$n_{s,0.002}$	0.9635	0.9626 ± 0.0057	$f\sigma_8(0.38)$	0.4804	0.4798 ± 0.0095
$\boldsymbol{A_{217}^{PS}}$	119.9	115 ± 10	Y_P	0.245295	$0.24529^{+0.00011}_{-0.000088}$	$\sigma_8(0.38)$	0.6638	0.6631 ± 0.0060
$\boldsymbol{A^{kSZ}}$	0.00	< 4.84	Y_P^{BBN}	0.246621	$0.24661^{+0.00011}_{-0.000089}$	$f\sigma_8(0.51)$	0.4779	0.4773 ± 0.0082
$\boldsymbol{A_{100}^{dustTT}}$	8.86	8.9 ± 1.8	$10^5D/H$	2.6321	2.634 ± 0.042	$\sigma_8(0.51)$	0.6208	0.6202 ± 0.0055
$\boldsymbol{A_{143}^{dustTT}}$	10.80	10.7 ± 1.8	Age/Gyr	13.8300	13.830 ± 0.037	$f\sigma_8(0.61)$	0.4722	0.4716 ± 0.0072
$\boldsymbol{A_{143\times217}^{dustTT}}$	19.43	18.3 ± 3.3	z_*	1090.292	1090.30 ± 0.41	$\sigma_8(0.61)$	0.5904	0.5899 ± 0.0051
$\boldsymbol{A_{217}^{dustTT}}$	94.8	93.3 ± 7.4	r_*	144.442	144.46 ± 0.48	$f\sigma_8(2.33)$	0.29733	0.2971 ± 0.0025
$\boldsymbol{c_{100}}$	0.99965	0.99961 ± 0.00061	$100\theta_*$	1.040956	1.04097 ± 0.00046	$\sigma_8(2.33)$	0.30613	0.3059 ± 0.0027
$\boldsymbol{c_{217}}$	0.99825	0.99826 ± 0.00063	$D_M(z_*)/\mathrm{Gpc}$	13.8759	13.878 ± 0.044	f_{2000}^{143}	30.49	31.2 ± 3.0
H_0	66.86	66.88 ± 0.92	z_{drag}	1059.437	1059.39 ± 0.46	$f_{2000}^{143\times217}$	33.34	33.6 ± 2.0
Ω_Λ	0.6791	0.679 ± 0.013	r_{drag}	147.182	147.21 ± 0.48	f_{2000}^{217}	107.77	108.2 ± 1.9
Ω_m	0.3209	0.321 ± 0.013	k_D	0.14058	0.14054 ± 0.00052	χ^2_{simall}	395.88	397.0 ± 1.7
$\Omega_m h^2$	0.14345	0.1434 ± 0.0020	$100\theta_D$	0.161051	0.16107 ± 0.00027	χ^2_{lowl}	23.60	23.9 ± 1.3
$\Omega_m h^3$	0.095909	0.09589 ± 0.00046	z_{eq}	3412.7	3411 ± 48	χ^2_{plik}	758.7	771.4 ± 5.5
σ_8	0.8126	0.8118 ± 0.0089	k_{eq}	0.010416	0.01041 ± 0.00014	χ^2_{prior}	1.35	7.3 ± 3.7
S_8	0.8405	0.840 ± 0.024	$100\theta_{eq}$	0.8106	0.8109 ± 0.0089	χ^2_{CMB}	1178.2	1192.3 ± 5.5
$\sigma_8\Omega_m^{0.5}$	0.4604	0.460 ± 0.013	$100\theta_{s,eq}$	0.44817	0.4483 ± 0.0046			

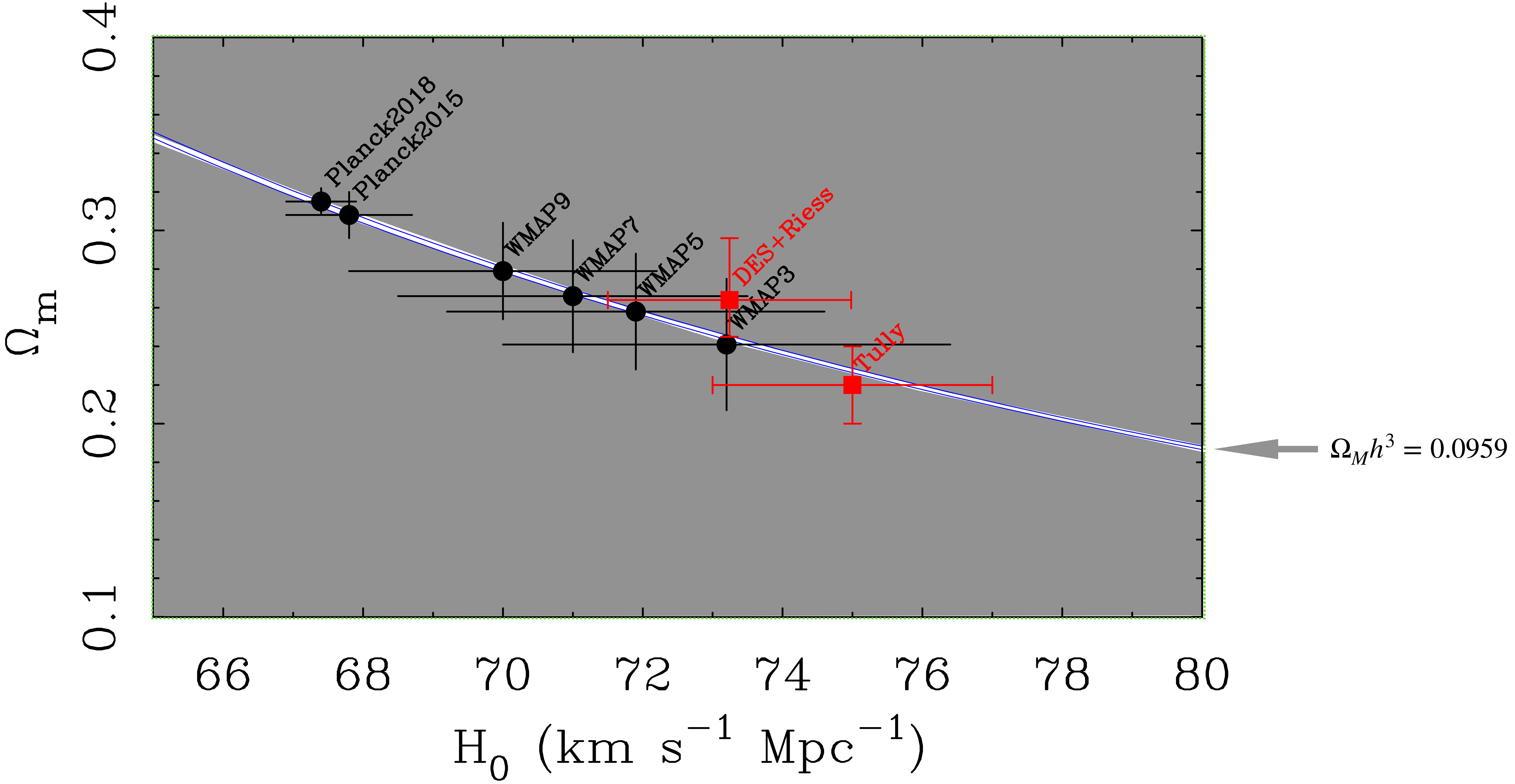
Best-fit $\chi^2_{eff} = 1179.58$; $\bar{\chi}^2_{eff} = 1199.58$; $R - 1 = 0.00927$
χ^2_{eff}: CMB - simall_100x143_offlike5_EE_Aplanck_B: 395.88 commander_dx12_v3_2_29: 23.60 plik_rd12_HM_v22_TT: 758.75

https://wiki.cosmos.esa.int/planck-legacy-archive/images/b/be/Baseline_params_table_2018_68pc.pdf

Planck constraint: $\Omega_m h^3 = 0.0959 \pm 0.0006$



Cosmology today: tension in H_0 …and Ω_m



The CMB best fits have marched away from the original concordance region

Measurements of the gravitating mass density

- Cluster M/L
  - measure M/L of a cluster, combine with measured luminosity density of universe.
- Weak lensing
  - measure shear over large scales
- Peculiar Velocity Field
  - measure deviations from Hubble flow
- Power spectrum of galaxies
- CMB fits

Measurements of the gravitating mass density

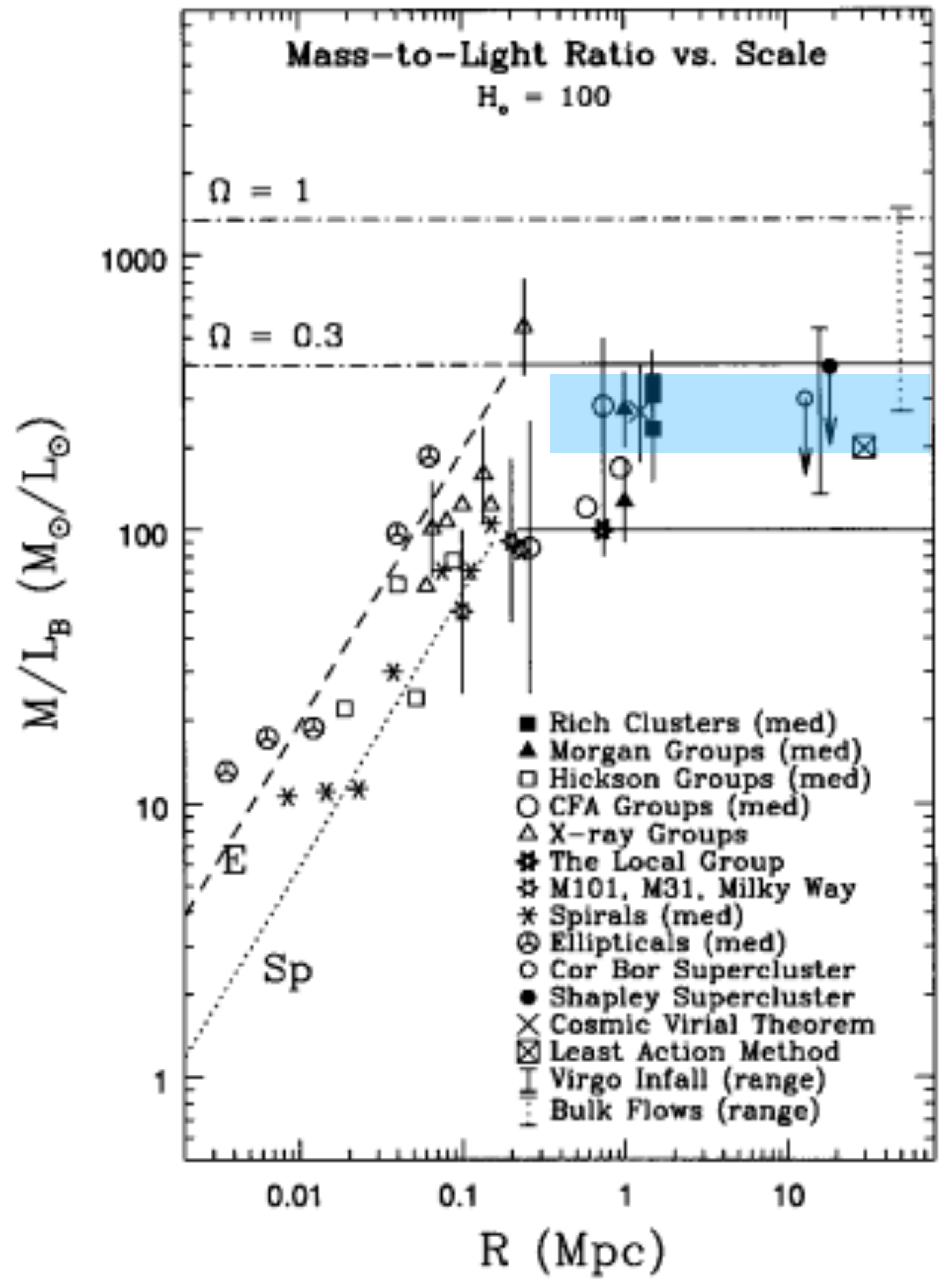
- Cluster M/L
  - measure M/L of a cluster, combine with measured luminosity density of universe.
  - j from integrating the luminosity function of galaxies:

$$\rho_m = \left(\frac{M}{L}\right) j$$

  - Also, cluster baryon fractions:

$$f_b = \frac{M_b}{M_{tot}} \quad \longrightarrow \quad \Omega_m = \frac{\Omega_b}{f_b}$$

  - both assume clusters are representative of the whole.



$$\Omega_m \approx \frac{1}{4}$$

FIG. 2.—Composite mass-to-light ratio of different systems—galaxies, groups, clusters, and superclusters—as a function of scale. The best-fit $M/L_B \propto R$ lines for spirals and ellipticals (from Fig. 1) are shown. We present median values at different scales for the large samples of galaxies, groups and clusters, as well as specific values for some individual galaxies, X-ray groups, and superclusters. Typical 1 σ uncertainties and 1 σ scatter around median values are shown. Also presented, for comparison, are the M/L_B (or equivalently Ω) determinations from the cosmic virial theorem, the least action method, and the *range* of various reported results from the Virgocentric infall and large-scale bulk flows (assuming mass traces light). The M/L_B expected for Ω = 1 and Ω = 0.3 are indicated.

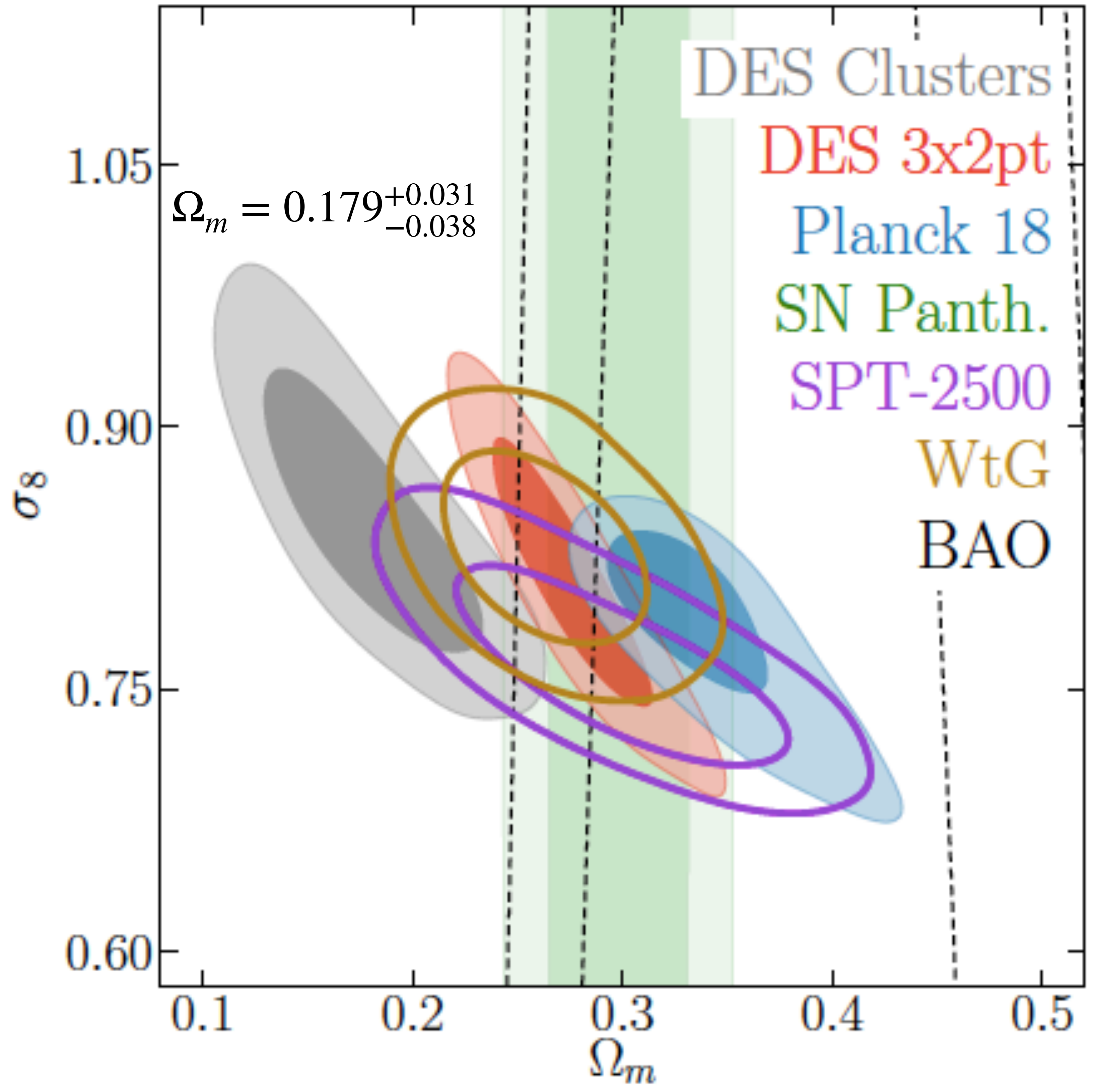
Bahcall, Lubin, & Dorman (1995)

Measurements of the gravitating mass density

- Weak lensing
  - measure shear over large scales

$$\Omega_m \approx 0.18 \pm 0.04$$

Dark Energy Survey
arxiv:2002.11124



Hasn't changed much. Hoekstra et al (2001) state "For … $\Omega_m+\Omega_\Lambda=1$, we obtain $\Omega_m=0.13+/-0.07$."

Measurements of the gravitating mass density

- Peculiar Velocity Field
  - – measure deviations from Hubble flow

in linear regime $\frac{\delta\rho}{\rho} \ll 1$

$$\frac{\delta V}{V} \approx \frac{d\ln H}{d\ln\rho}\frac{\delta\rho}{\rho} \approx -\frac{1}{3}\frac{\Omega_m^{0.6}}{b}\frac{\delta\rho_g}{\rho_g}$$

peculiar velocity

distortion in Hubble flow induced by

mass over-density

bias

BIAS ***b***
relates
galaxy over-densities
to mass over-densities

$$\Omega_m = 0.25 \pm 0.05$$

Tonry & Davis (1981), updated to modern H_0
basically unchanged for nearly 40 years

TONRY AND DAVIS

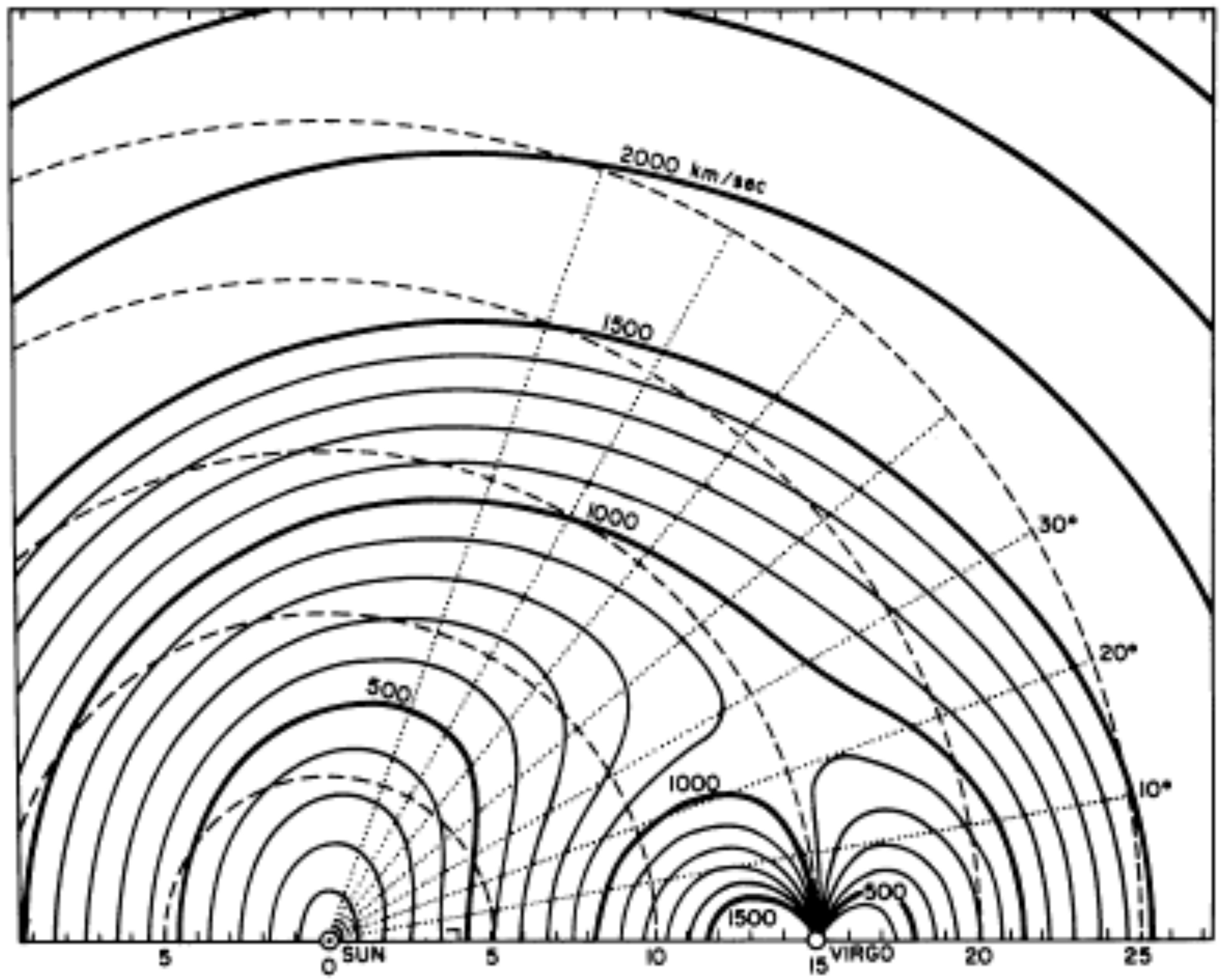


FIG. 1.—On a two-dimensional grid with the Earth and the Virgo cluster on the x axis, redshift contours are plotted for a Hubble flow perturbed by a Virgocentric flow. An infall velocity of 400 km s^{-1} at our position is assumed. A pure Hubble flow would be concentric circles.