Midterm Notes review

Cosmologies

Finite	Indefinite	Infinite
geocentric	Milky-Way is all	no center
Olber's paradox: finite so no problem	extent of MW large but finite no problem	problem solved by cosmic expansion age finite; light redshift

PRINCIPLES

Great Debate (1920)

Shapley	Curtis
✓ MW big we are not at the center	X MW modest we happen to be near its center
X Spiral nebulae are gas clouds within MW	✓ Spiral nebulae are other "Island Universes" like the M

COPERNICAN PRINCIPLE

Hubble (1929)

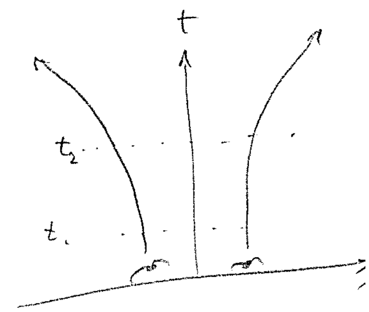
- Spirals (like Andromeda) are exterior to the MW
- The universe is expanding
- Galaxies are the building blocks of the universe

Expansion solves stability problem that bedeviled both Newton & Einstein

- Copernican Principle: nothing special about where we are
- Cosmological Principle: the universe is homogeneous & isotropic (extension of Copernican Prin.)
- Perfect Cosmological Principle: regular CP plus time – the universe looks the same to every observer anywhere at any time

3 Empirical Pillars of the Hot Big Bang

- Hubble Expansion
- Cosmic Microwave Background
- Big Bang Nucleosynthesis



Expansion $\quad a(t)$

separation between galaxies grows with time

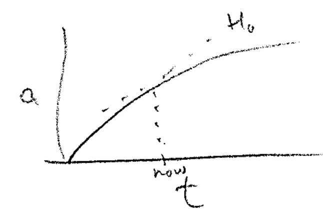
$$d(t_2) = a\, d(t_1) \qquad a = \frac{1}{1+z}$$

Gives linear Hubble law between apparent recession velocity and distance

$$v = H_0 d \qquad H = \frac{\dot{a}}{a}$$

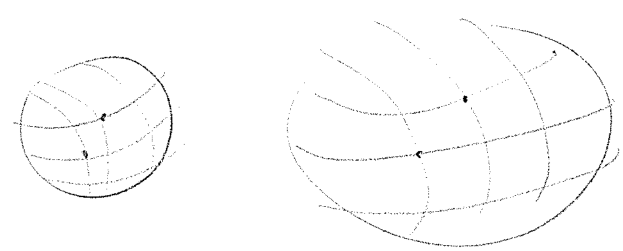
Hubble parameter is slope of expansion fcn $a(t)$

H_0 sets the scale of the U:



Hubble time $\frac{1}{H_0} \approx 13.5$ Gyr

Hubble distance $\frac{c}{H_0} \approx 4$ Gpc



Define comoving coordinates as the set of coordinates that expand along with the universe so that at any given time,

$$d = a(t)\, r$$

distance at time t

comoving separation

By convention, the current expansion is usually used to define comoving distance.

Since the universe is expanding, the light that reaches us from a remote cosmic source has been redshifted by the stretching of space, and the distance it has had to travel is greater than when it was emitted. This makes the effective distance for the purpose of dimming different from the current proper distance.

It is thus useful to define

- Luminosity distance: $d_L = (1+z)\, d_p$

  Has the property that $f = \dfrac{L}{4\pi d_L^2}$

- Angular size distance: $d_A = \dfrac{d_p}{(1+z)}$

  Has the property that $\theta = \dfrac{\ell}{d_A}$

as in Euclidean space

Asside: Astronomical magnitude system

- apparent ~~magnitide~~ magnitude $m = -2.5 \log f + \xi$ is a logarithmic measure of flux
- absolute magnitude $M - M_\odot = -2.5 \log\left(\dfrac{L}{L_\odot}\right)$ is a measure of intrinsic luminosity (power)
- distance modulus $m - M = 5 \log d_L - 5$ is a measure of distance in parsecs

The absolute magnitude is defined to be the apparent magnitude an object would have if it were at a distance of 10 pc — less than the Kessel run

Governing Equations -

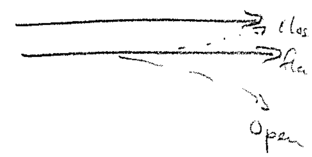
Friedmann equation
Acceleration equation
Robertson-Walker metric

Geometry is not Euclidean, only appears Euclidean in the limit of small sizes $r \to 0$

Robertson-Walker metric :

$$ds^2 = c^2 dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega\right] \qquad k = \begin{cases} +1 & \text{closed} \\ 0 & \text{flat} \\ -1 & \text{open} \end{cases}$$

Initially parallel light rays remain parallel in a flat ($k=0$) geometry. They converge in a closed geometry and diverge in an open geometry



Photons travel at the speed of light, so the events of emission and observation of a photon have a light-like separation : $ds = 0$.

Hence $\qquad c\,dt = a(t) \cdot f^{1/2}(r)$

If we know the cosmic expansion history $a(t)$, we can integrate over it to find the path-length travelled by a photon between two points of comoving separation r

$$c\int_{t_e}^{t_0} \frac{dt}{a(t)} = \int_0^r f^{1/2}(r)\,dr \qquad \left[\; = r \quad \text{in Ryden's notation} \;\right]$$

Acceleration equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P) + \frac{1}{3}\Lambda$$

mass density

Pressure – usually the energy-density of relativistic stuff

Eqn of state : $P = w\rho$

$w = 0$	matter
$1/3$	radiation
-1	Λ

Friedmann eqn

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\left(\rho_m + \frac{\varepsilon_r}{c^2}\right) - \frac{kc^2}{(aR_0)^2} + \frac{c^2}{3}\Lambda \qquad \varepsilon_r = \alpha T_r^4$$

critical density – over/under between collapse and eternal expansion

$$\rho_{crit} = \frac{3H_0^2}{8\pi G} \qquad \Omega_i = \frac{\rho_i}{\rho_{crit}}$$

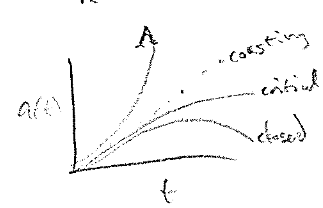
so defined $\qquad \Sigma\,\Omega_i \equiv 1$

recall $H = \frac{\dot{a}}{a}$ and $a = \frac{1}{1+z}$

$$\left(\frac{H}{H_0}\right)^2 = \Omega_m a^{-3} + \Omega_r a^{-4} + \Omega_k a^{-2} + \Omega_\Lambda$$

$$= \Omega_m (1+z)^3 + \Omega_r (1+z)^4 + \Omega_k (1+z)^2 + \Omega_\Lambda$$

$$= E^2(z)$$



Five Classic Tests

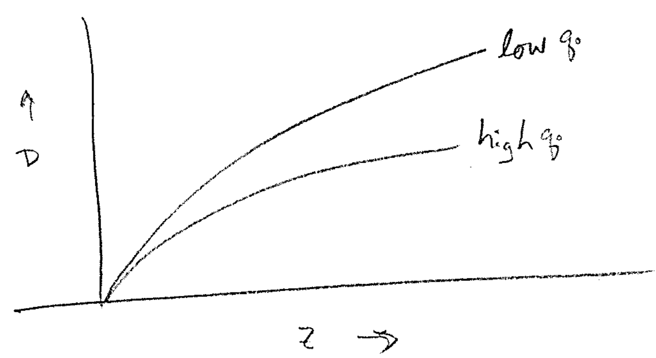
$D_L - z$	standard candles
$D_A - z$	standard rods
$N(z)$	number counts with redshift
$N(m)$	number counts with magnitude
Tolman test	cosmological dimming

Surface brightness is distance-independent in Euclidean geometry, but suffers strong dimming cosmologically:

$$\Sigma \sim \frac{f}{\theta^2} \sim \frac{D_L^{-2}}{D_A^{-2}} \sim (1+z)^{-4} \quad !$$

This Tolman test does not distinguish between cosmologies, it only ~~tests~~ that the geometry is non-Euclidean as expected.

The other four can distinguish between different q_0, always in the sense that cosmologies the decelerate a lot (high q_0) have expanded less than those which don't (low q_0). So distances and volumes are always bigger when q_0 is smaller



Early universe basics

$\rho_m \sim a^{-3} \sim (1+z)^3$

$\varepsilon_r \sim a^{-4} \sim (1+z)^4$

So the very early universe ($t \lesssim 10^5$ yr) is radiation dominated when $(1+z)^4$ wins over $(1+z)^3$

$\varepsilon_r > \rho_m c^2$!

$a(t) \sim t^{1/2}$ $\quad T^2 t =$ constant (Peebles 6.2)

Sequence of events

physics	t	
	very early	particles exist in equilibrium with their anti-particles and the radiation field
proton mass	$\sim 10^{-6}$ s	protons & antiprotons annihilate for last time leaving relic radiation field $T \sim 10^{13}$ K, now 2.7 K
weak force	~ 1 s	neutrons & protons freeze out with $\frac{n_n}{n_p} = 0.2$ neutrinos decouple at this time (weak force time scale) $n_\nu = \frac{9}{11} n_\gamma$ just from counting available quantum state
nuclear reactions	1s – 5 min	BBN. Neutrons fuse with protons to form deuterium and other isotopes of the light elements. They also decay as unstable free particles during this time (half light ~10 min). Any surviving neutrons wind up incorporated into helium after 5 min. when the universe becomes too diffuse & cool to sustain nuclear reaction.
	$\sim 10^5$ yr	Matter – radiation equality
hydrogen atom (1 Ry)	$\sim 4 \times 10^{10}$ yr	Recombination. Creation of hydrogen Liberation of photons
	...	Dark Ages
	—	Structure formation