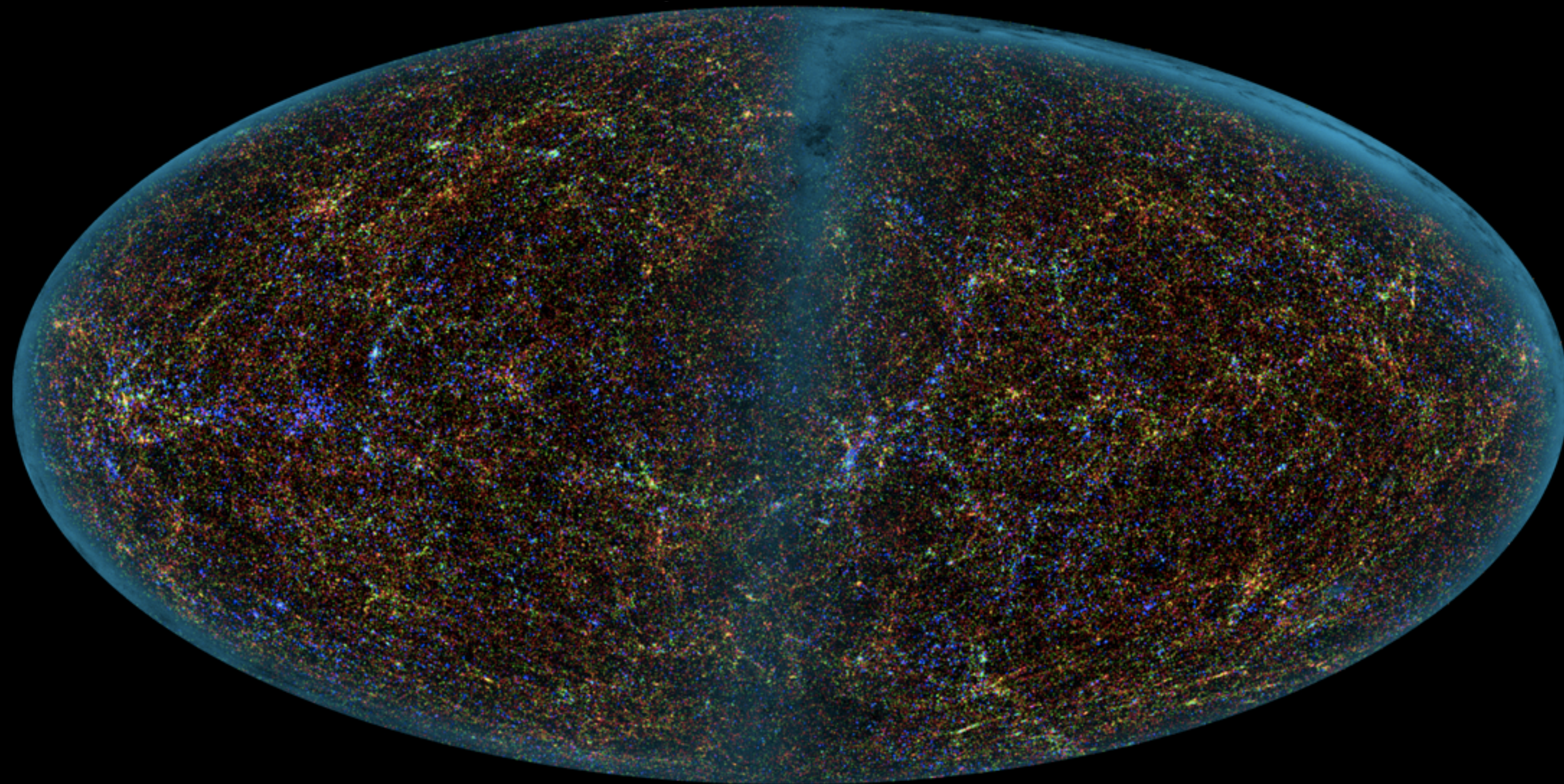


# Cosmology

## and Large Scale Structure



Today  
Distance Scale

next homework due 10/13

# Distance Scale

So why do we need to get this right?

*Astrophysics:*

- turn observed properties of objects (apparent magnitude, angular size) into intrinsic properties of objects (luminosity, physical size)

*Measure  $H_0$ :*

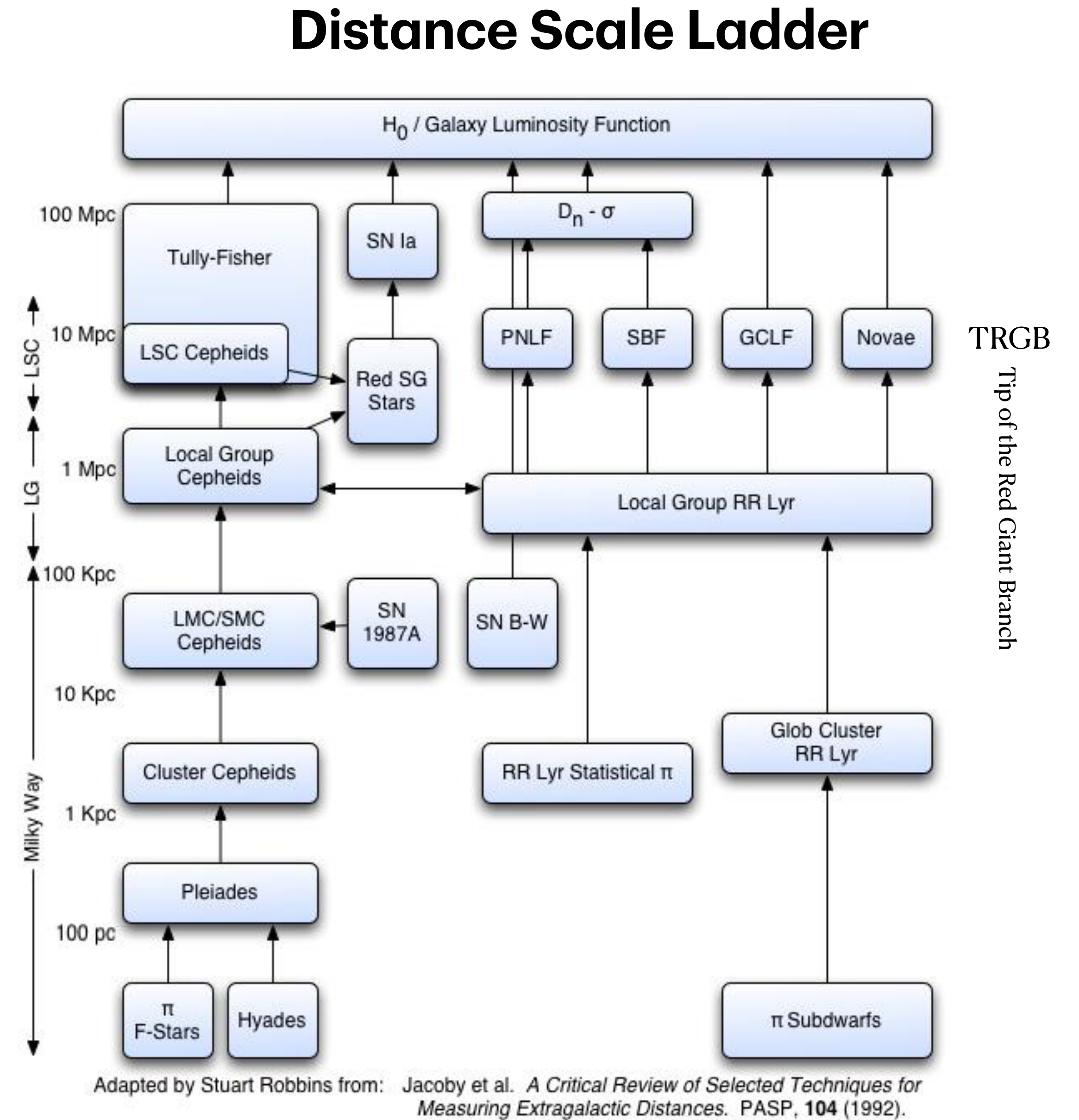
- Cosmological parameter, want local, independent confirmation of cosmological measurements at high redshift.
- Once measured, can use it as a distance indicator (Hubble distance:  $d=v/H_0$ )

*Measure peculiar motions in the universe:*

- $v_{\text{obs}} = H_0 d + v_{\text{pec}}$
- if we know distance *independent* of redshift, we can look for large scale velocity structure in the universe, test the assumption of isotropy, and measure  $\Omega_m$ .

**Important Complications:**

- An accurate measure of  $H_0$  means getting out to a distance where  $v_{\text{pec}} \ll H_0 d$ .
- Local galaxies do *not* have useful Hubble distances, due to [peculiar motions](#) and [Virgo-centric flow](#).
- Distances *within* clusters (ie with accuracies of +/- few Mpc) are *not knowable* via Hubble's law.
- Need *several* distance estimators to reduce systematic errors between methods.

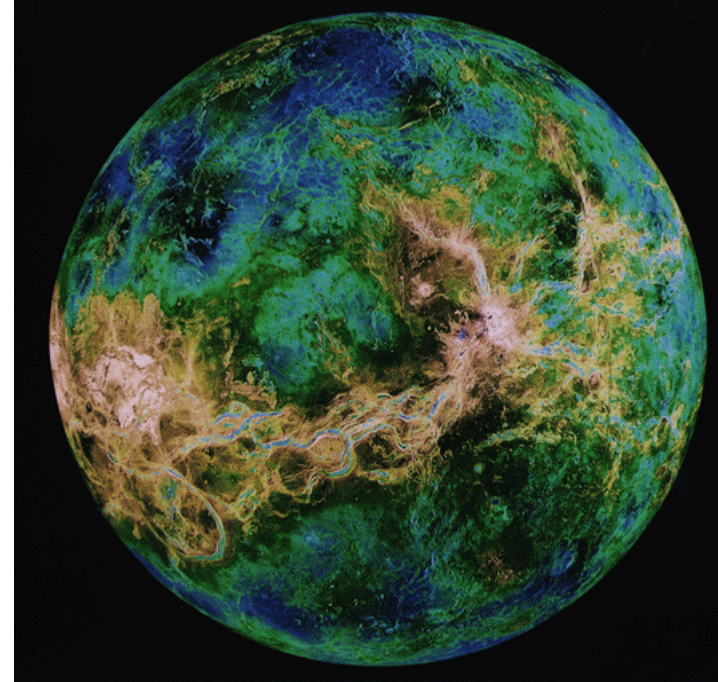


# Distance Scale

- Solar System
  - earth-sun distance
- Trigonometric Parallax
  - statistical & secular parallax; moving clusters
- Main Sequence Fitting
- Bright Star Standard Candles
  - Cepheids, RR Lyraes, TRGB
- Secondary Distance Indicators
  - Type Ia SN, Tully-Fisher, Fundamental Plane, SB Fluctuations
- Absolute Methods
  - Gravitational lens time delay, SZ effect, water masers
- Trigonometric methods absolute
  - same as land surveys - use Pythagoras!
- Secondary Distance Indicators
  - Generally relate a distance dependent quantity (luminosity or size) to a distance independent quantity that is correlated with it.
  - e.g., Cepheid P-L relation: the period P is used as an indicator of the luminosity L
- Absolute Methods
  - make use of physics that is distance-independent
  - e.g., the speed of light is constant, but light must traverse a different path for each image in a gravitational lens, so measuring the time delay between images constrains the distance through  $c\Delta t$ .

# Distance Scale

1 AU = 149597870.7 km (IAU definition, 2012)



Radar map of Venus

Prior to the Magellan mission, the biggest uncertainty in the AU was the thickness of Venus's atmosphere: one measured the distance via radar reflection off the ground, but the angular size from the reflection of clouds high up in the atmosphere.

- Solar System

- earth-sun distance
- measure
  - sun-venus angular separation  $\theta$  at maximum elongation (45 - 47°; varies due to eccentricity)
    - known with great accuracy via orbital periods
  - earth-venus distance  $d_{EV}$ 
    - measure via radar reflection
- solve for earth-sun distance (1 AU)
- Historically, use period ratio to relate  $\sin \theta$  to observed  $\theta$ .
  - Gauss's gravitational constant extremely well measured

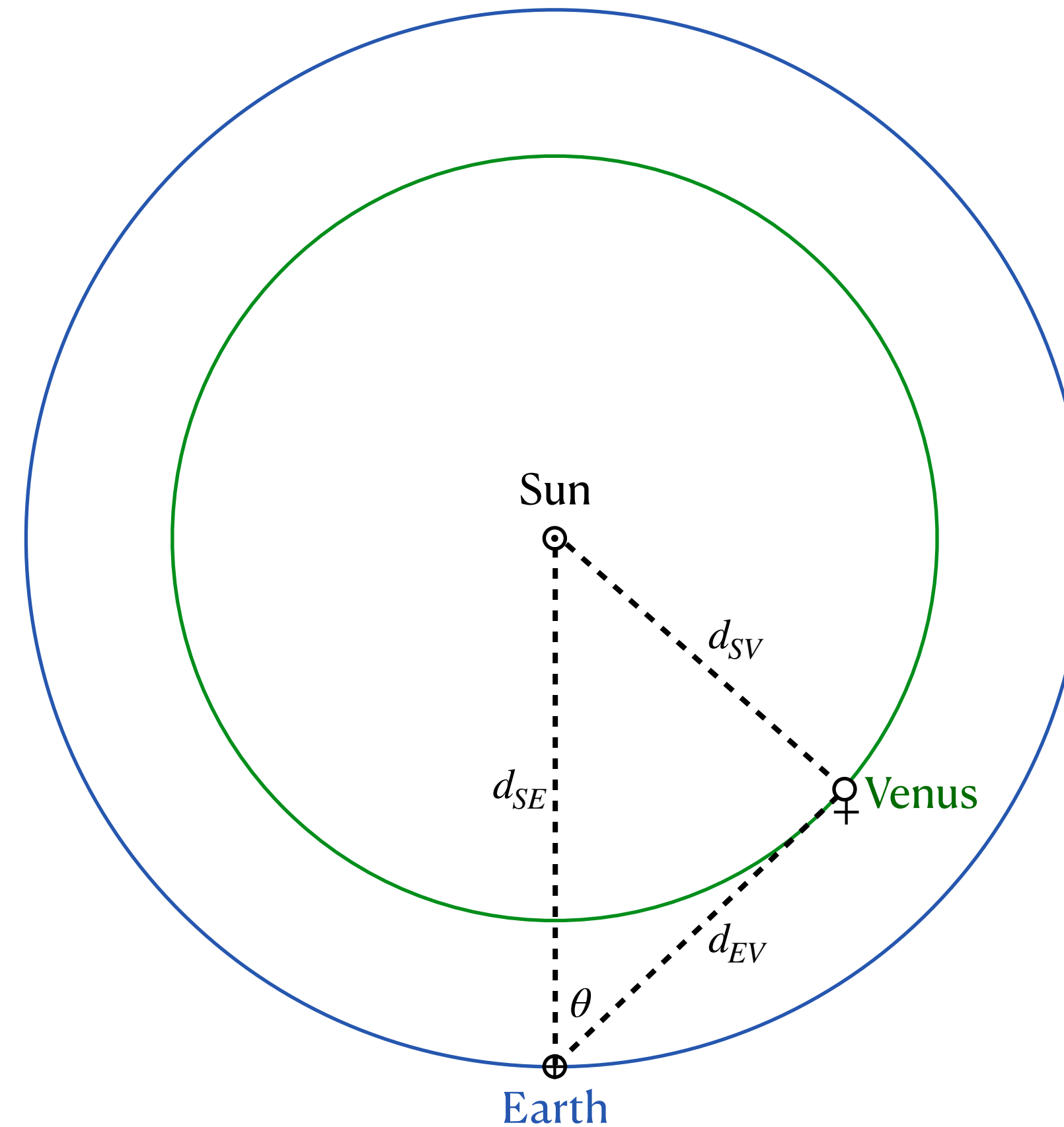
- $k = \frac{2\pi}{P(aM)^{1/2}} = 0.01720209895 \text{ rad/day}$

- in modern parsing,

- $GM_{\odot} = 1.32712440018(9) \times 10^{20} \text{ m}^3 \text{ s}^{-2}$

Experimental measurements of  $G$  alone are considerably less accurate:

$$G = 6.67430(15) \times 10^{-11} \text{ kg}^{-1} \text{ m}^3 \text{ s}^{-2}$$



$$\cos \theta = \frac{d_{EV}}{d_{SE}} \qquad \sin \theta = \frac{d_{SV}}{d_{SE}} = \left( \frac{P_V}{P_E} \right)^{2/3}$$

Thanks, Kepler!

# Distance Scale

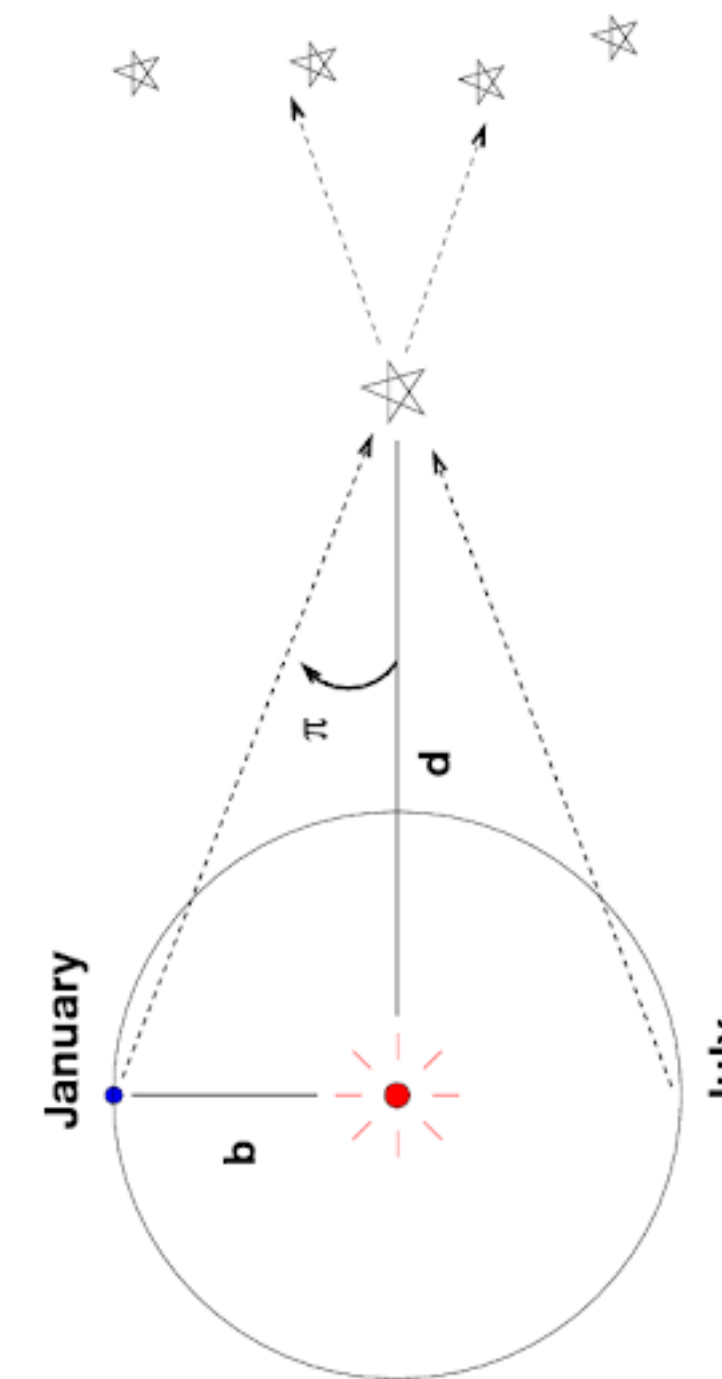
- Trigonometric Parallax
  - use Earth's orbit as baseline
  - measure angular shift in position of a star relative to background stars

$$d_* = \frac{1}{\pi}$$

$d$  in pc for  $\pi$  in arcseconds  
(1 pc is defined by a parallax angle of 1")

206,265 arcseconds in one radian, so  
206,265 AU in one pc

$$1 \text{ pc} = 3.086 \times 10^{13} \text{ km}$$



$$\pi \approx \tan \pi = \frac{b}{d_*}$$

$$b = d_{SE} = 1 \text{ AU}$$

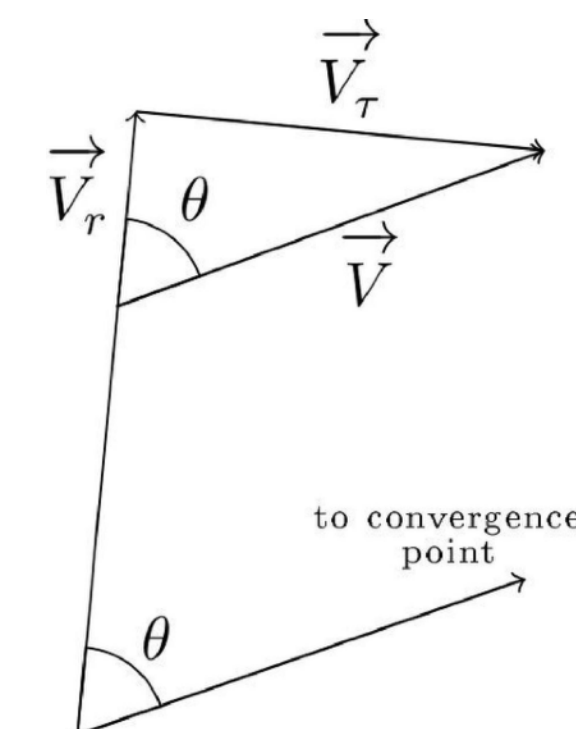
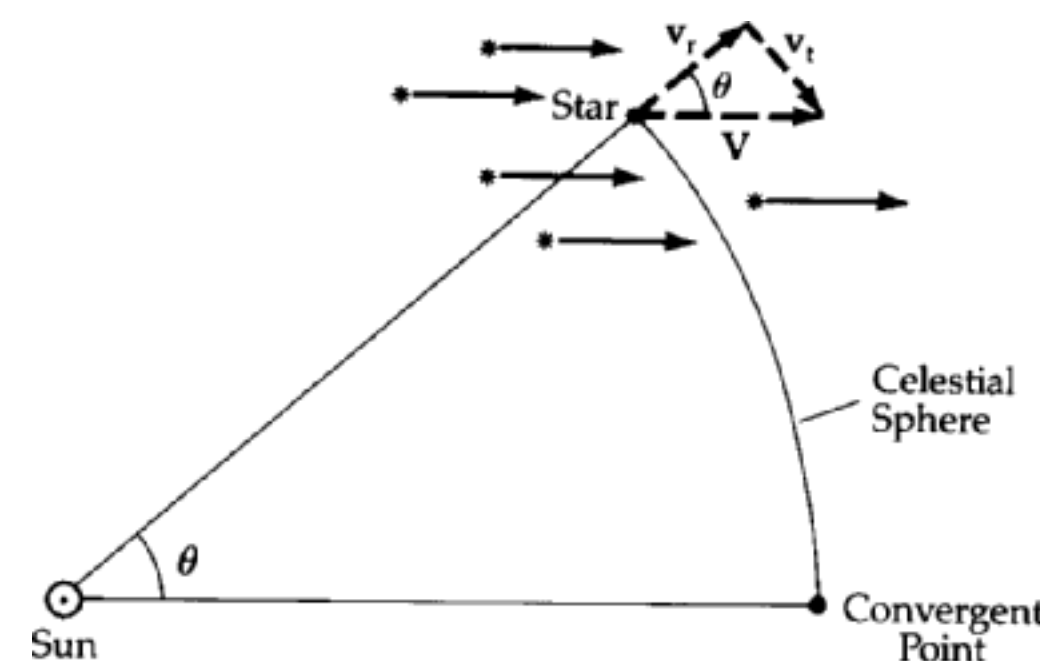
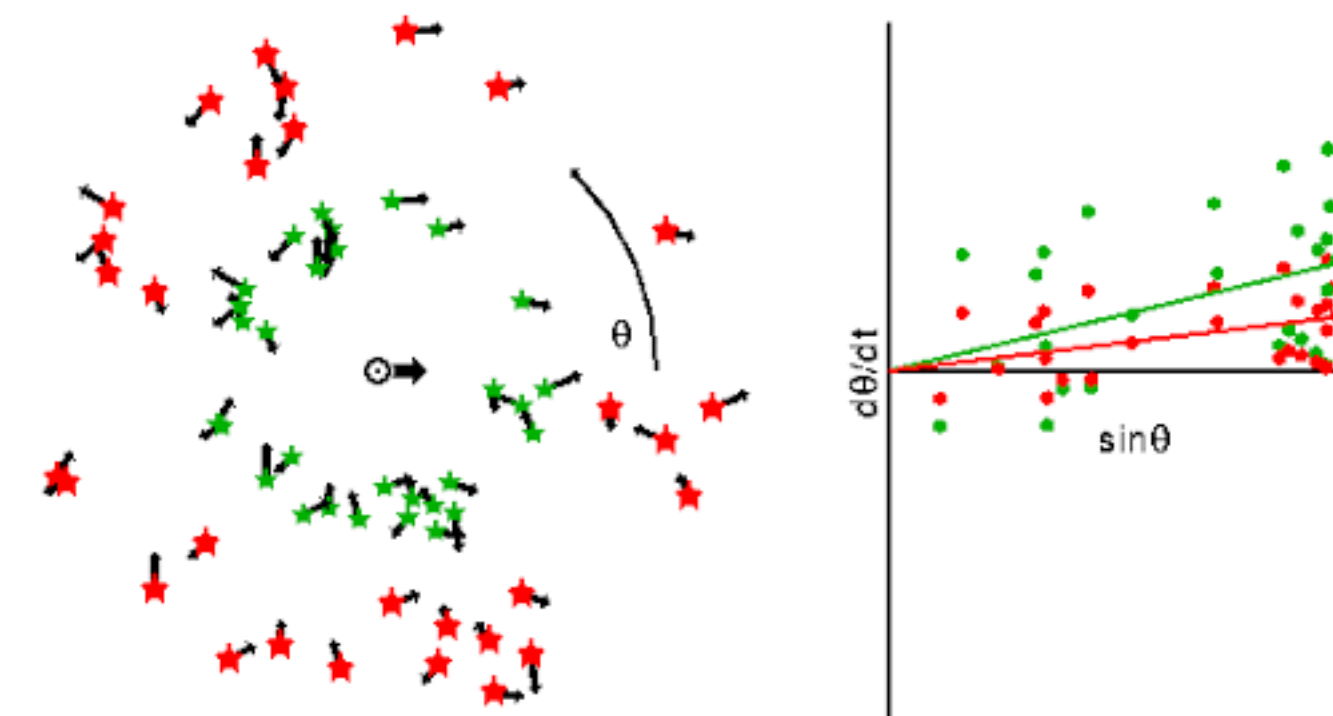
small angle approximation excellent here

# Distance Scale

- Statistical Parallax
  - Stars move.
  - Can determine mean baseline from the observed proper motion for a specified stellar type.

- Secular Parallax
  - The Sun moves wrt the Local Standard of Rest
  - Motion of the sun provides a baseline

- Moving Clusters
  - convergent point method



Moving cluster method

# Distance Scale

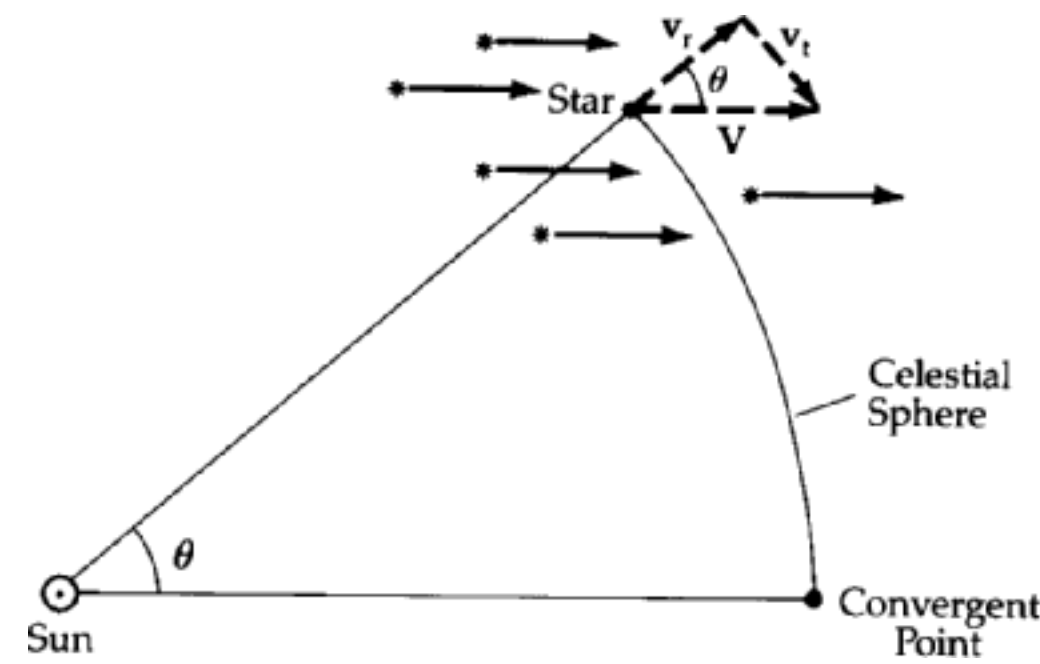
- Moving Clusters
  - convergent point method

$$1 \text{ AU/yr} = 4.74 \text{ km/s}$$

$$V_{\tau} = 4.74 \frac{\mu}{\pi}$$

$$V = \sqrt{V_r^2 + V_{\tau}^2}$$

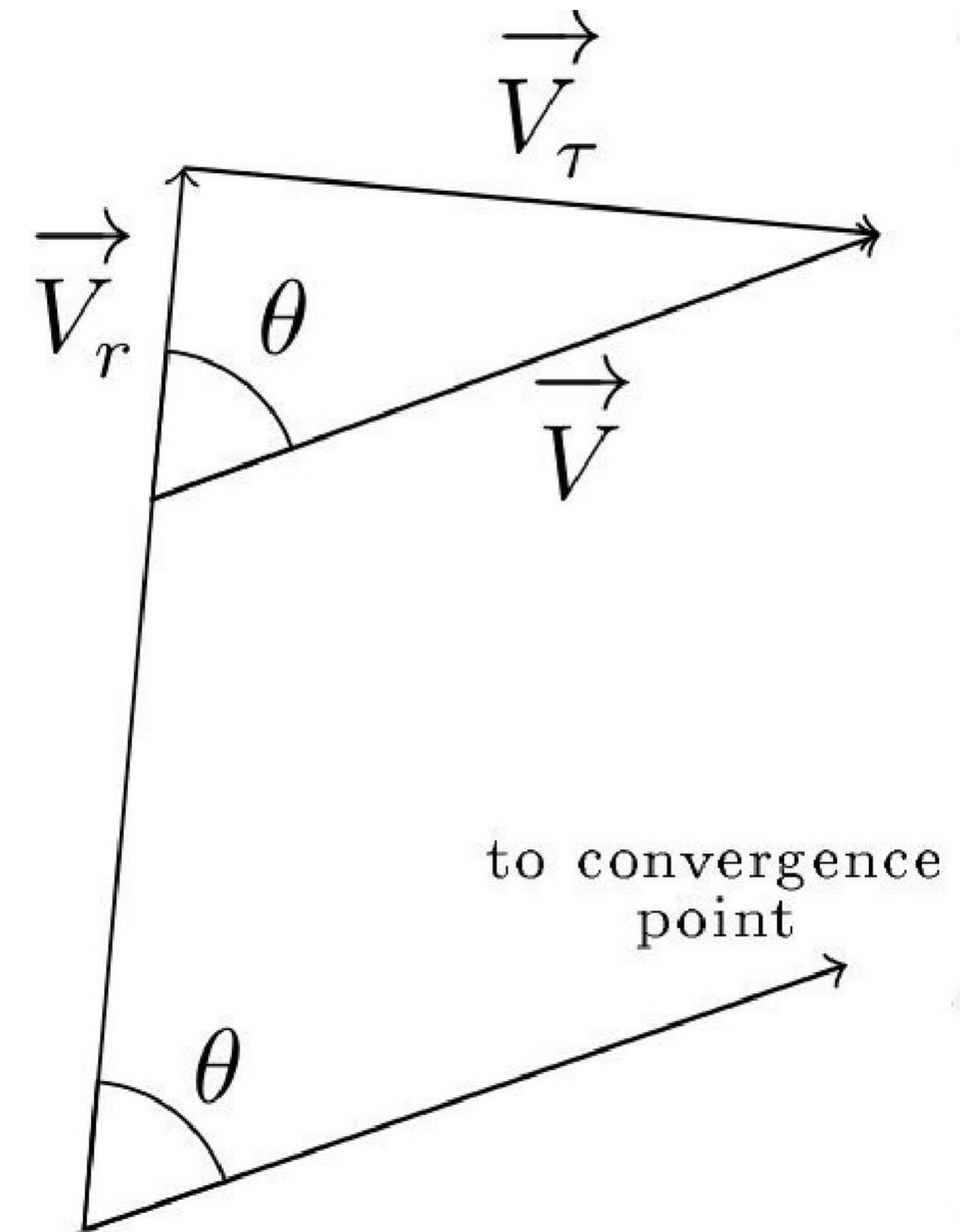
$$\frac{1}{d} = \pi = \frac{4.74\mu}{V \tan \theta}$$



$\mu$  is the proper motion (arcsec/yr)  
 $\pi$  is the parallax (arcseconds)

$$V_r = V \cos \theta$$

$$V_{\tau} = V \sin \theta = 4.74 \frac{\mu}{\pi}$$



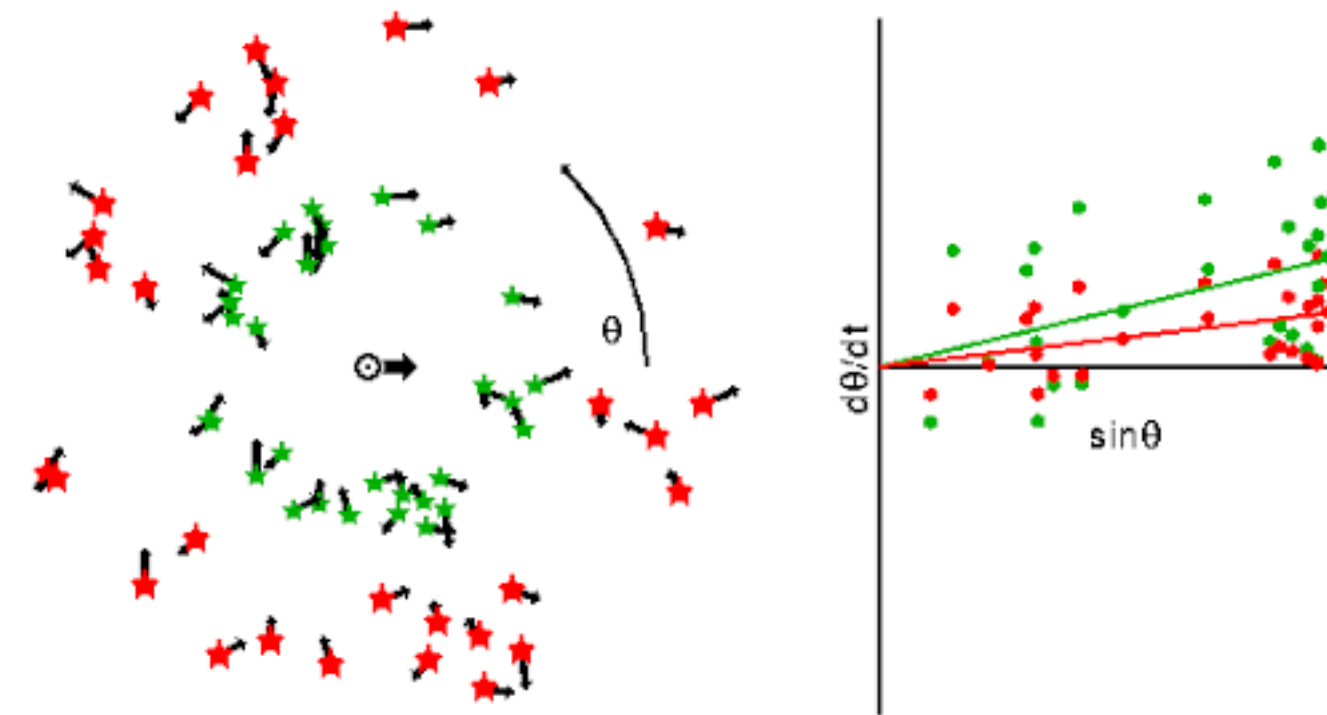
Works on clusters of stars where it is possible to perceive their joint motion on the sky

# Distance Scale

- Secular Parallax
  - The Sun moves wrt the Local Standard of Rest
  - Motion of the sun provides a baseline

$$d = \frac{V_{\odot}}{m} = \frac{4.16}{m}$$

where the odd constant 4.16 is the Solar motion in au/yr.



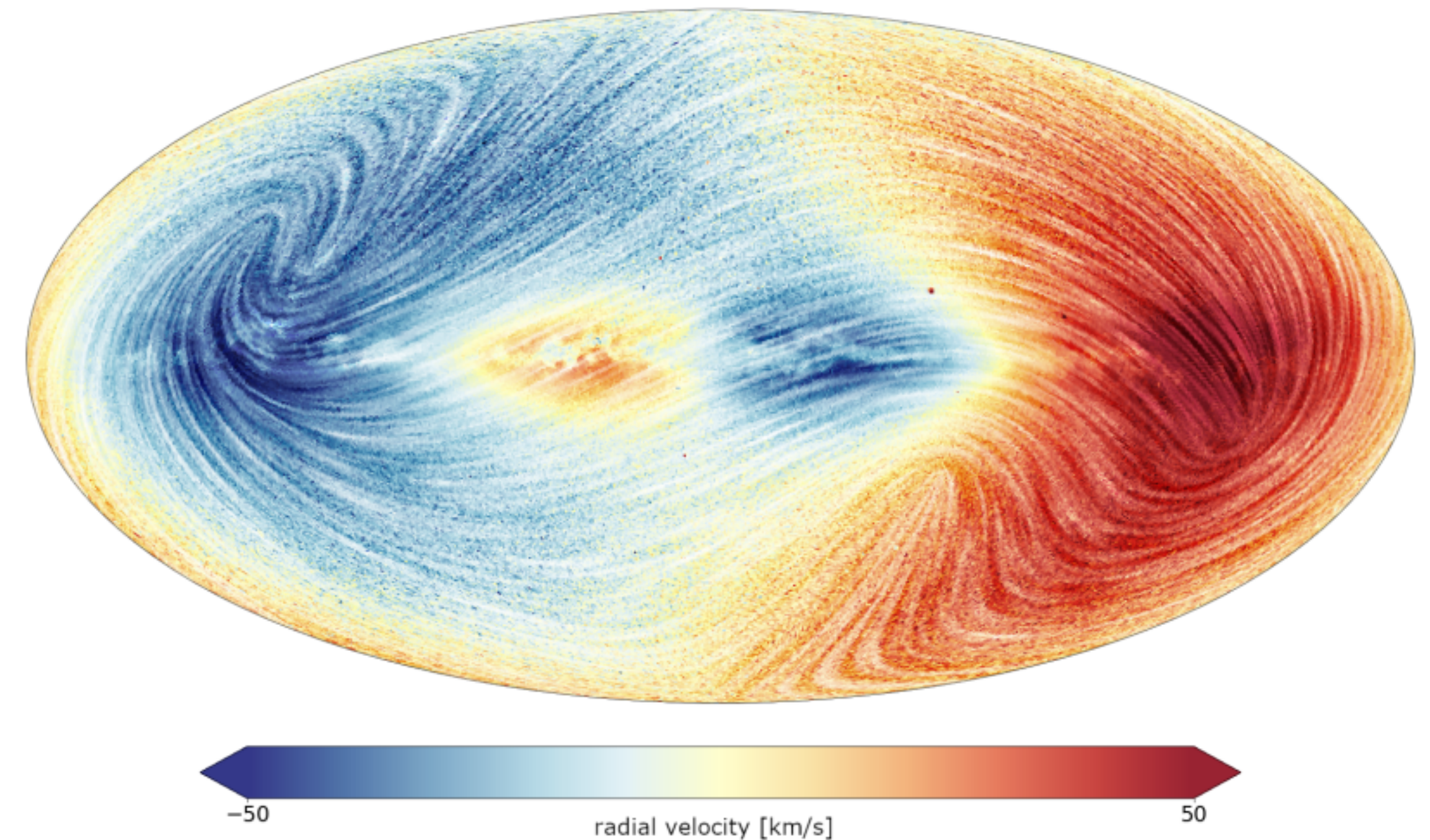
The diagram above shows two sets of stars, with two mean distances. The green stars show a small mean distance, while the red stars show a large mean distance. Because of the Solar motion (20 km/sec relative to the average of nearby stars) there will be an average proper motion away from the point of the sky the Solar System is moving towards. This point is known as the *apex*. Let the angle to the apex be  $\theta$ . Then the proper motion  $\mu = d\theta/dt$  will have a mean component proportional to  $\sin \theta$ , shown by the lines in the plot of  $d\theta/dt$  vs  $\sin \theta$ . The slope of this line is  $m$ .



# Distance Scale

- Statistical Parallax
  - Stars move.
  - Can determine mean baseline for a specified stellar type.
  - Assuming motion is random, so proper motion and radial motion are on average the same,

$$d = \frac{\langle V_r \rangle}{\langle \mu \rangle} = \frac{\text{scatter in radial velocities}}{\text{scatter in proper motions}}$$



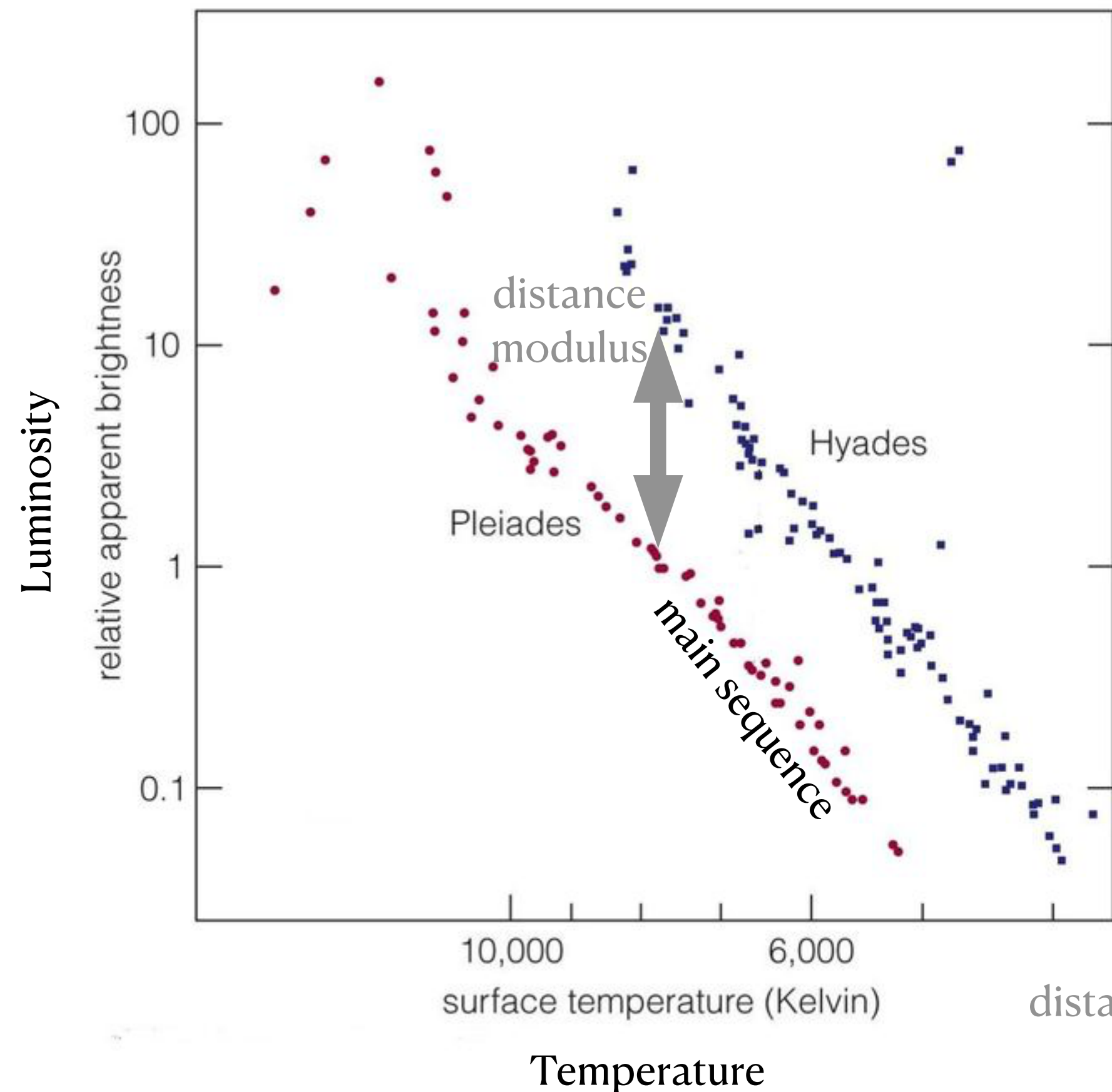
This sky map shows the velocity field of the Milky Way for ~26 million stars. The colours show the radial velocities of stars along the line-of-sight. Blue shows the parts of the sky where the average motion of stars is towards us and red shows the regions where the average motion is away from us. The lines visible in the figure trace out the motion of stars projected on the sky (proper motion).

# Distance Scale

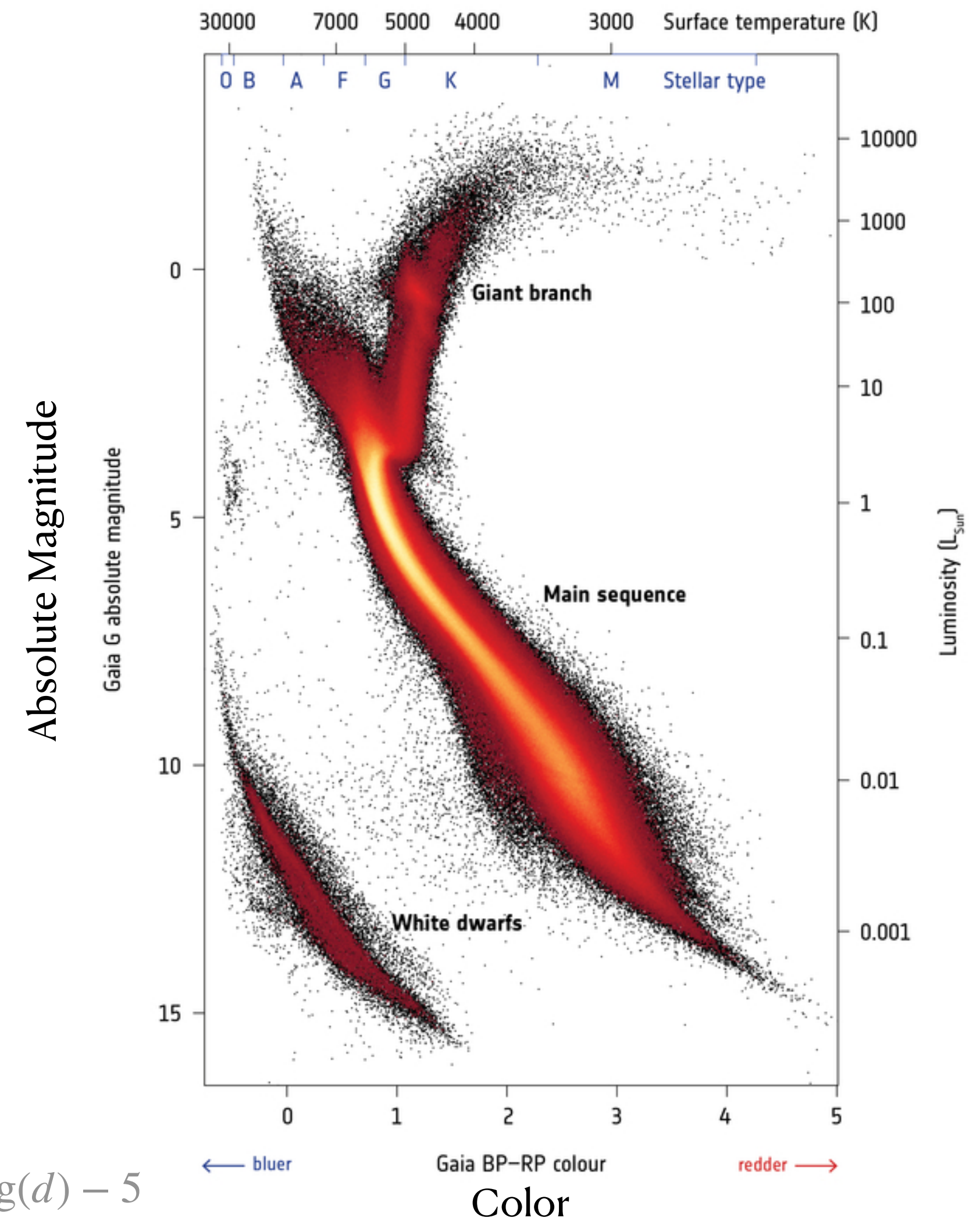
ESA's Gaia mission provides parallax distances for over 4 million stars within 1.5 kpc

→ **GAIA'S HERTZSPRUNG-RUSSELL DIAGRAM**  
aka HR diagram, color-magnitude diagram

- Main Sequence Fitting
  - absolute calibration by parallax
  - apply to more distant clusters



distance modulus  $m - M = 5 \log(d) - 5$



Most stars are main sequence, but other types are well represented (35,000 white dwarfs!)

# Distance Scale

- Bright Star Standard Candles
  - Cepheids, RR Lyraes
  - calibrate by
    - parallax
    - main sequence fitting of clusters containing these stars

Luminosity of variable stars correlate with oscillation period

$$L = 4\pi R^2 \sigma T_e^4$$

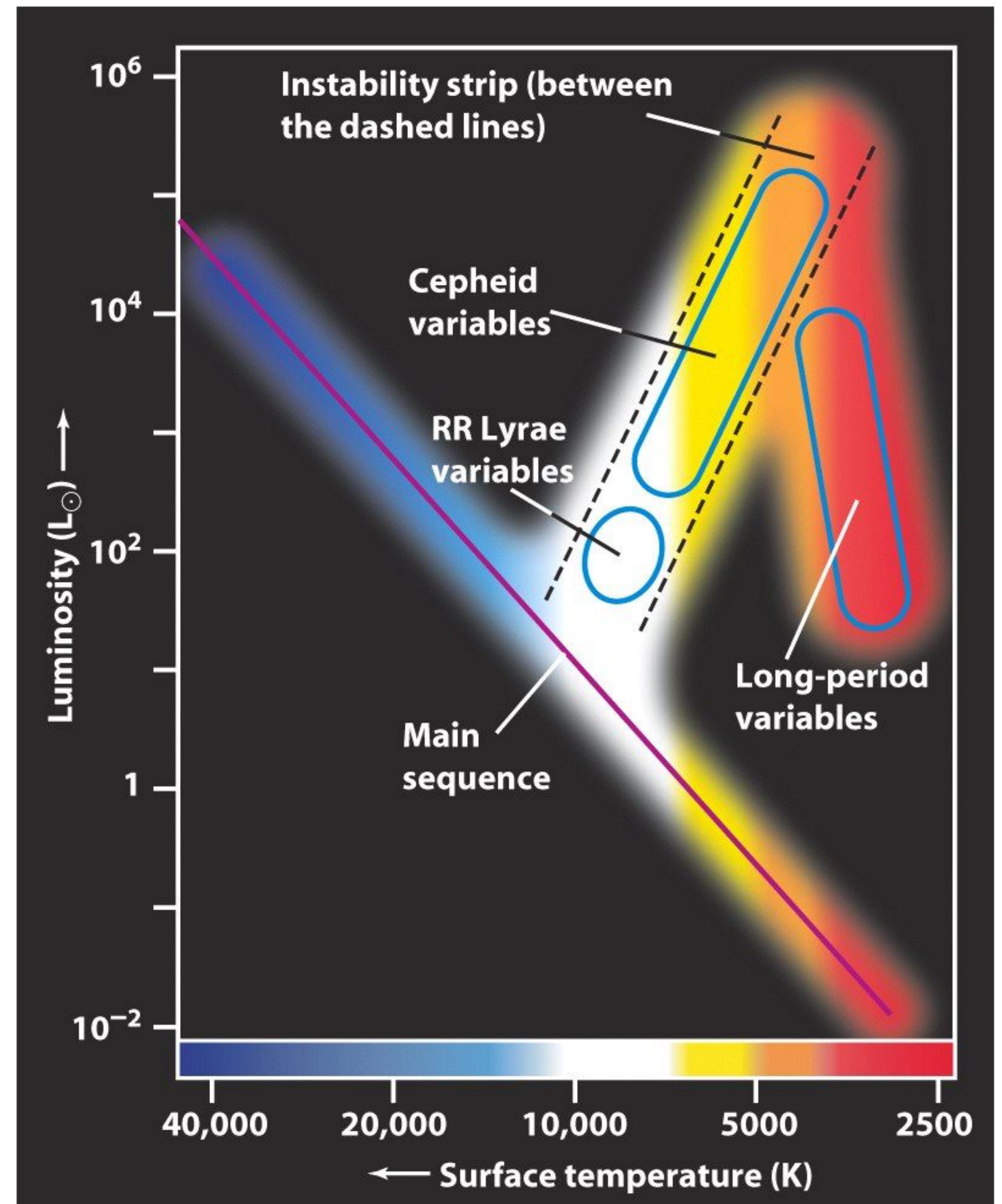
use luminosity and effective surface temperature to infer radius

Baade-Wesselink method

$$\int_{R_1}^{R_2} dR = -p \int_{t_1}^{t_2} V_{\text{los}} dt$$

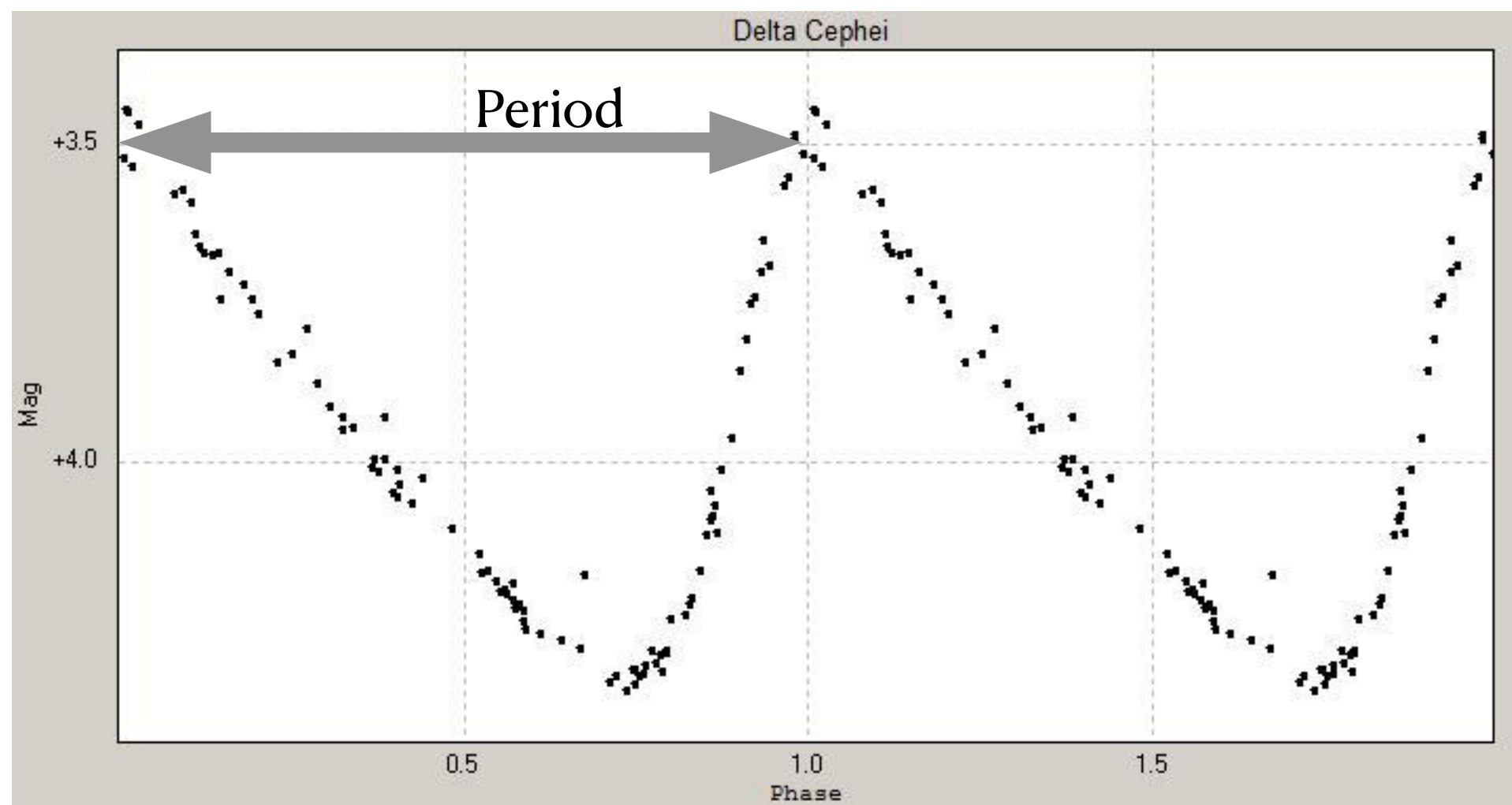
$p \approx 1.4$  corrects line of sight velocity to radial velocity, accounting for limb darkening

Instability strip in the HR diagram (not the same as the giant branch)



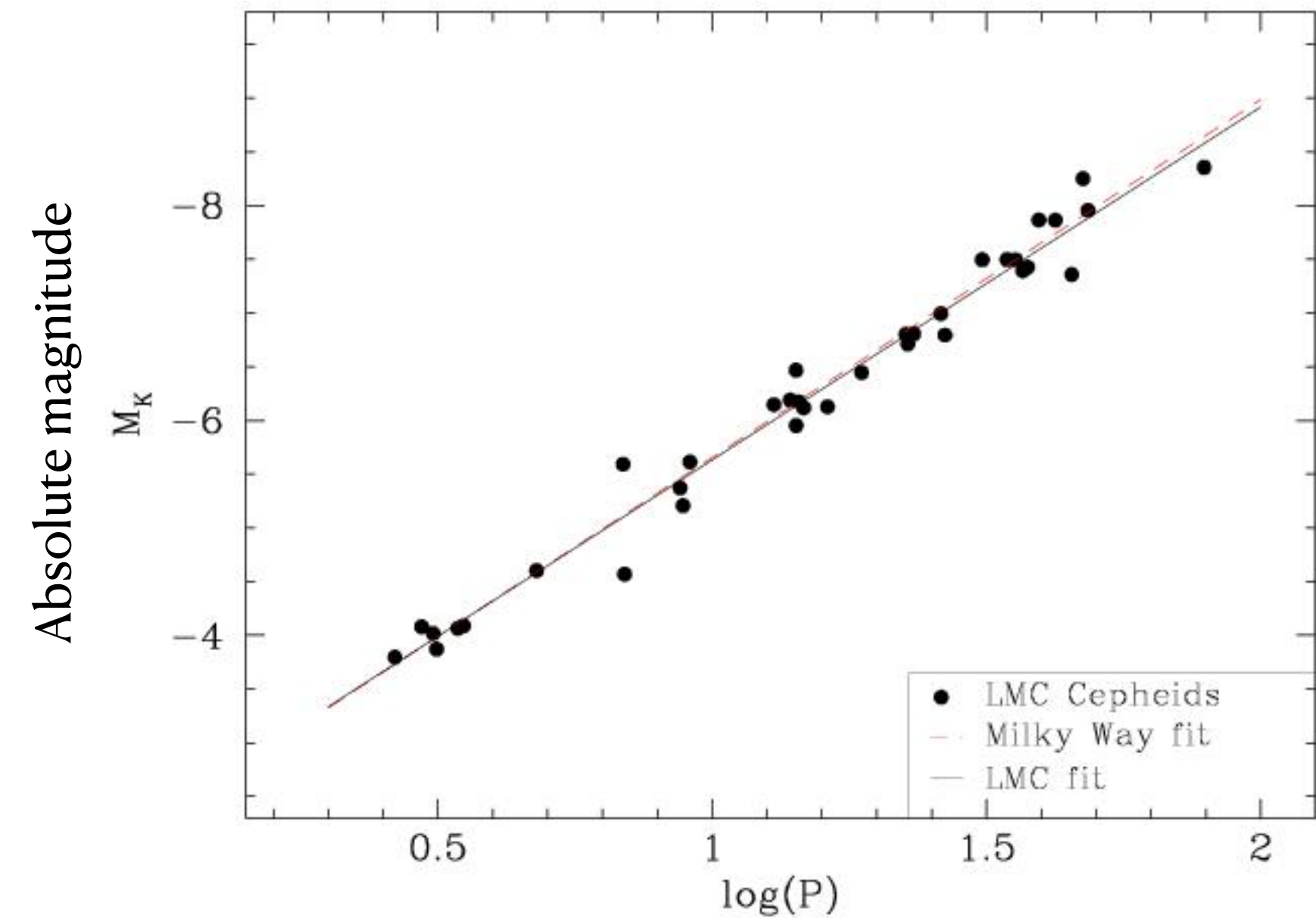
# Distance Scale

- Bright Star Standard Candles
  - Cepheids, RR Lyraes
  - calibrate by
    - parallax
    - main sequence fitting of clusters containing these stars



calibration band-pass dependent

Cepheid P-L relation



Period

Bright Cepheids have long periods;  
faint Cepheids have short periods.

Discover through repeated observation.  
Measure period, infer luminosity from P-L relation.  
Apply inverse square law, accounting for extinction  $A$ :

$$m_K - M_K = 5 \log(d) - 5 + A_K$$

metallicity dependent

Pulsations of one Cepheid in many bands

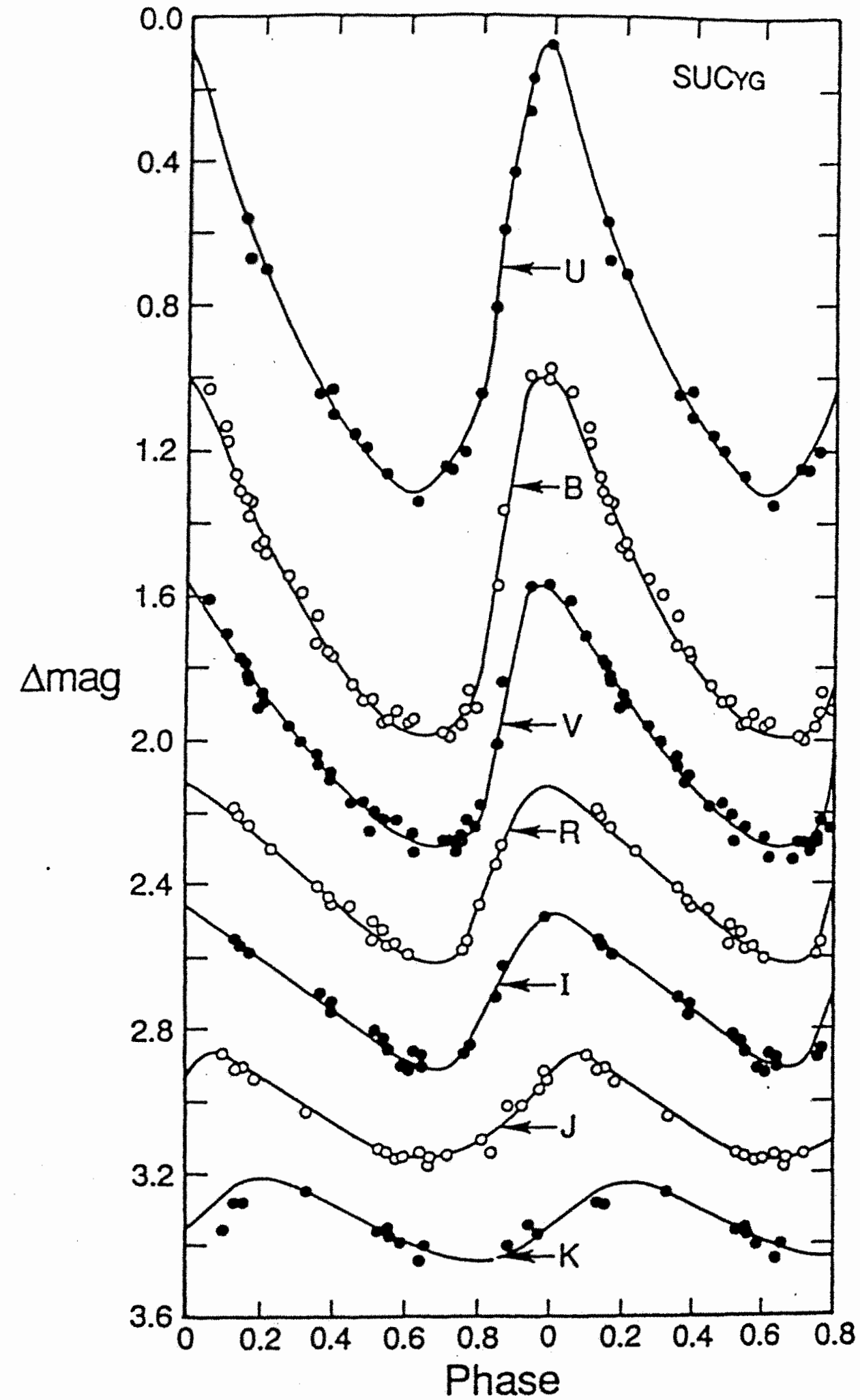


FIG. 5—Variations of amplitude and phase of maximum seen in the light curve of a typical Galactic Cepheid as a function of increasing wavelength. Note the monotonic drop in amplitude, the progression toward more symmetric light variation, and the phase shift of maximum toward later phases, all with increasing wavelength. Upper light curves are for short wavelengths (ultraviolet, blue, and visual); lower light curves are for long wavelengths (red and near-infrared out to  $K = 2.2$  microns).

P-L relations in many bands

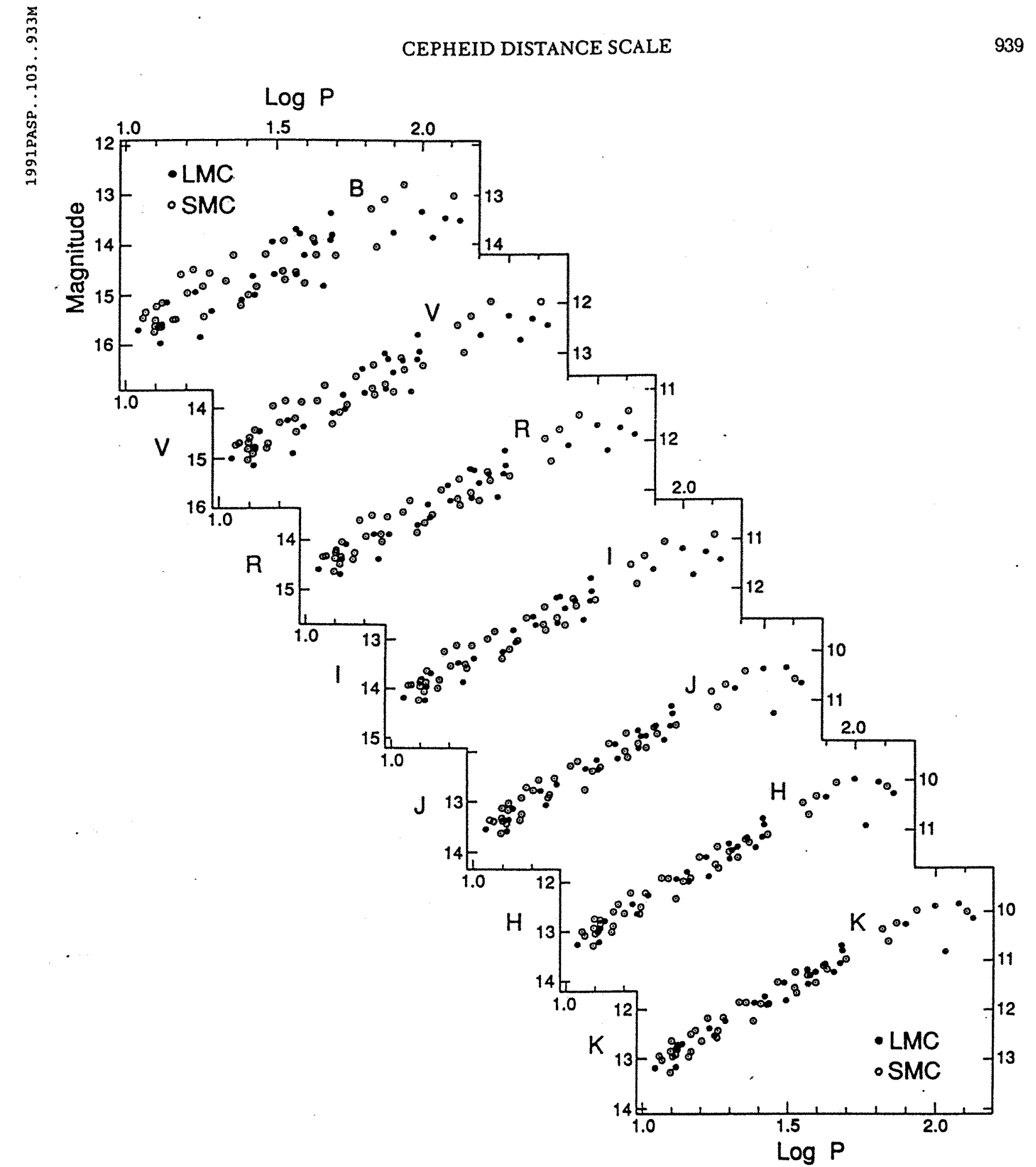
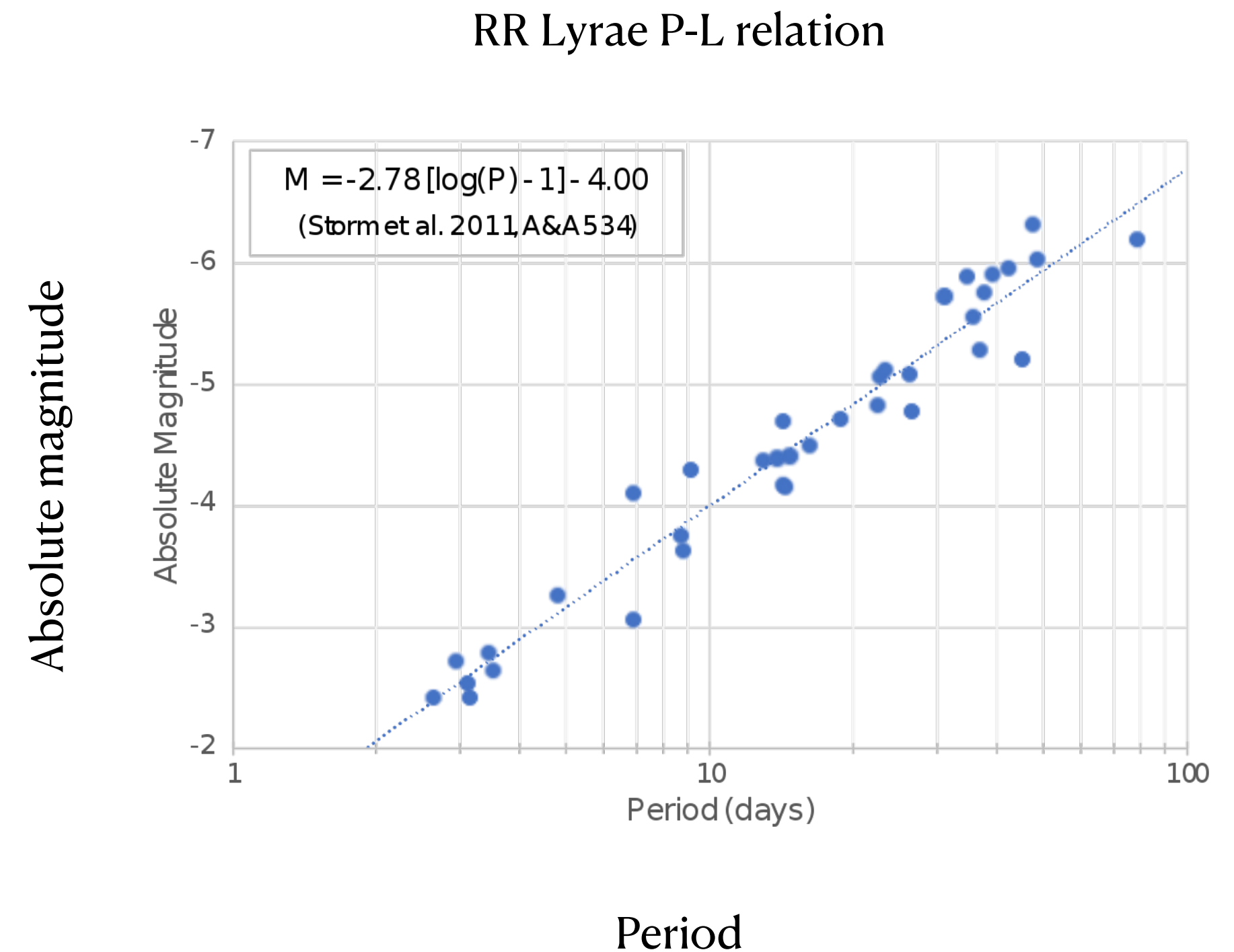
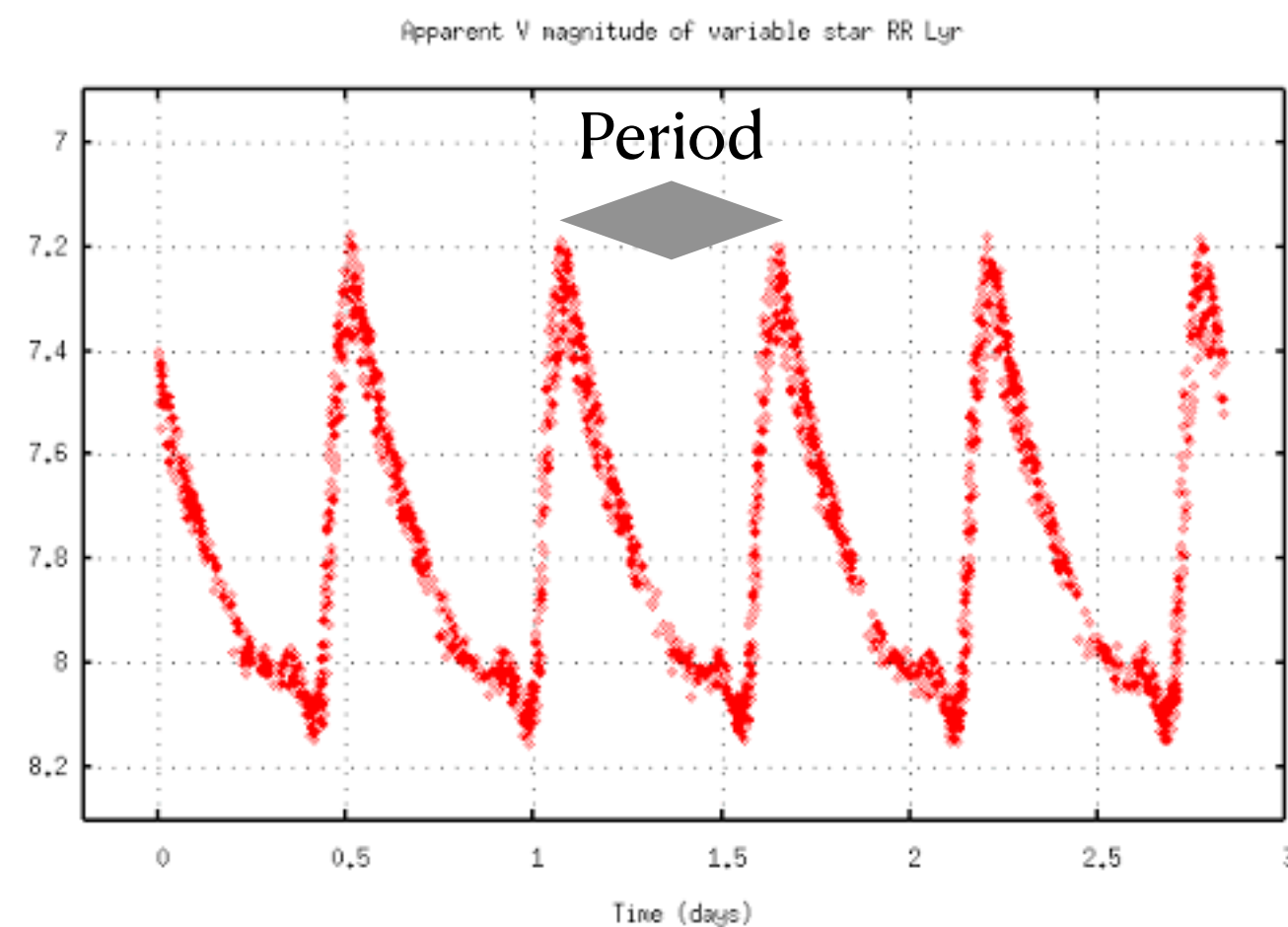


FIG. 4—Magellanic Cloud Cepheid period-luminosity relations at seven wavelengths, from the blue to the near-infrared, constructed from a self-consistent data set (Freedman & Madore 1992). LMC Cepheids are shown as filled circles; SMC data, shifted to the LMC modulus, are shown as open circles. Note the decreased width and the increased slope of the relations as longer and longer wavelengths are considered.

# Distance Scale

- Bright Star Standard Candles
  - Cepheids, RR Lyraes
  - calibrate by
    - parallax
    - main sequence fitting of clusters containing these stars



Bright RR Lyraes have long periods;  
faint RR Lyraes have short periods.

Discover through repeated observation.  
Measure period, infer luminosity from P-L relation.  
Apply inverse square law, accounting for extinction  $A$ :

$$m_K - M_K = 5 \log(d) - 5 + A_K$$

calibration band-pass dependent:  $M_V = -4.12 - 2.88(\log P - 1)$

metallicity dependent

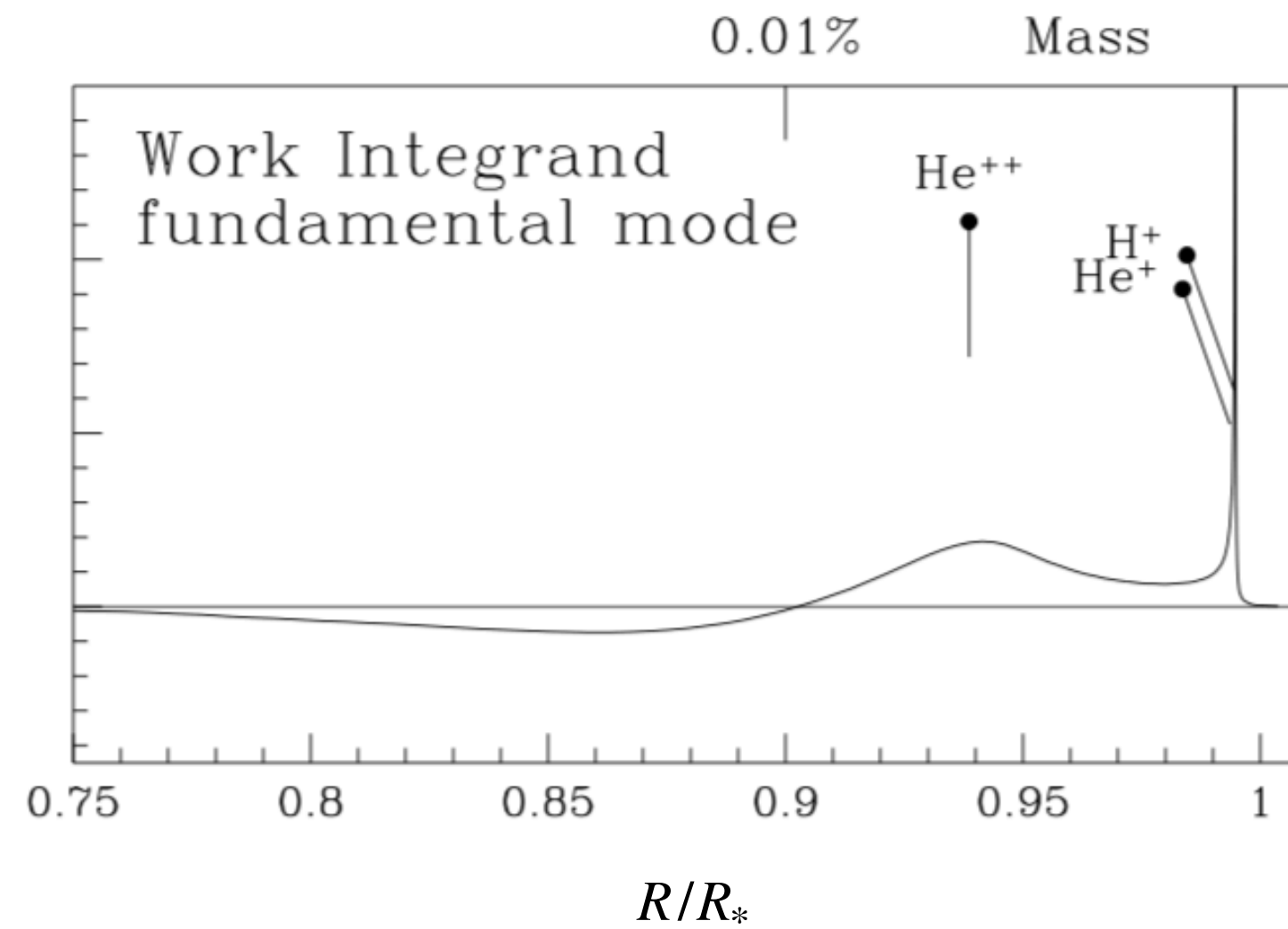
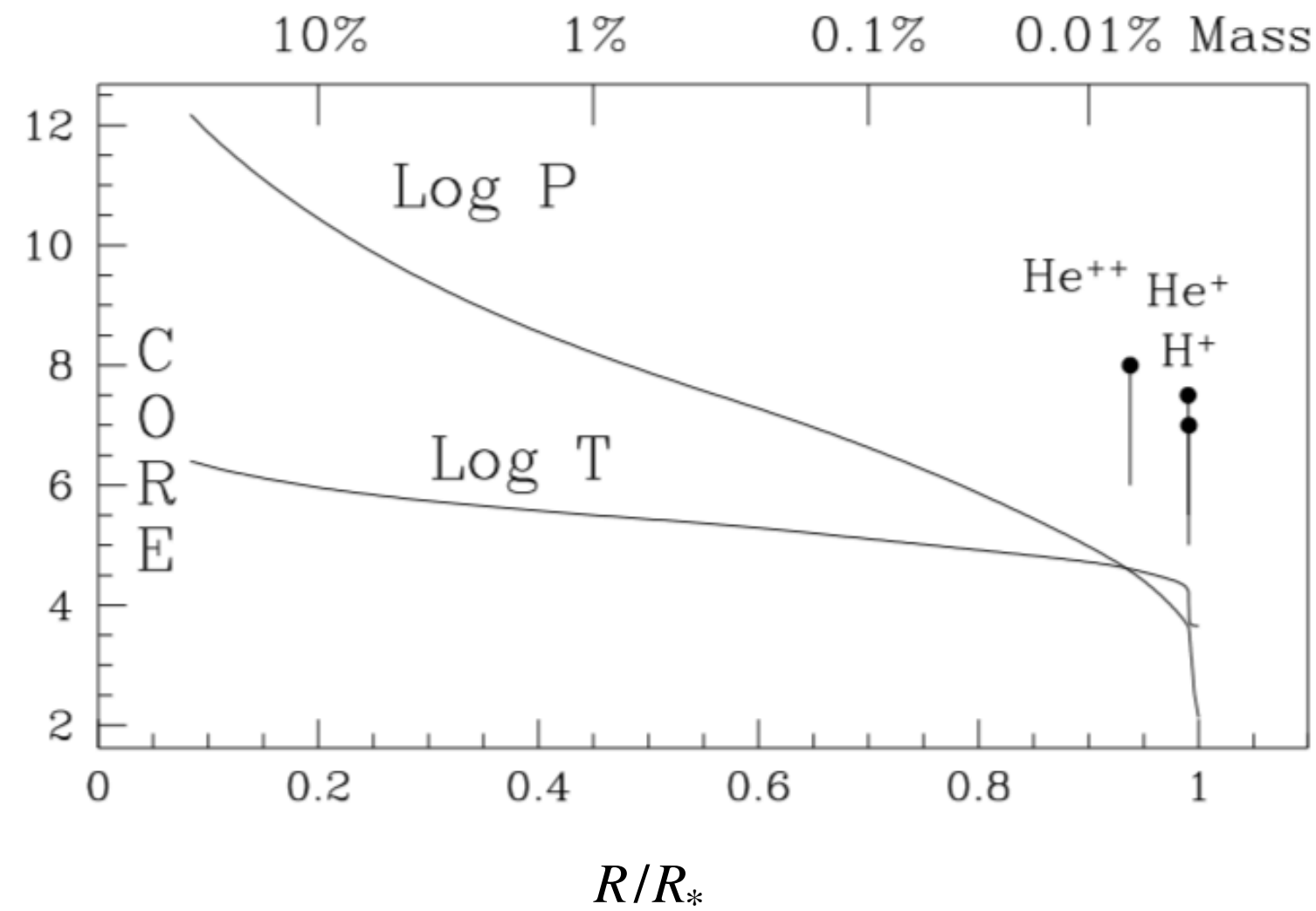
# Distance Scale

- Bright Star Standard Candles

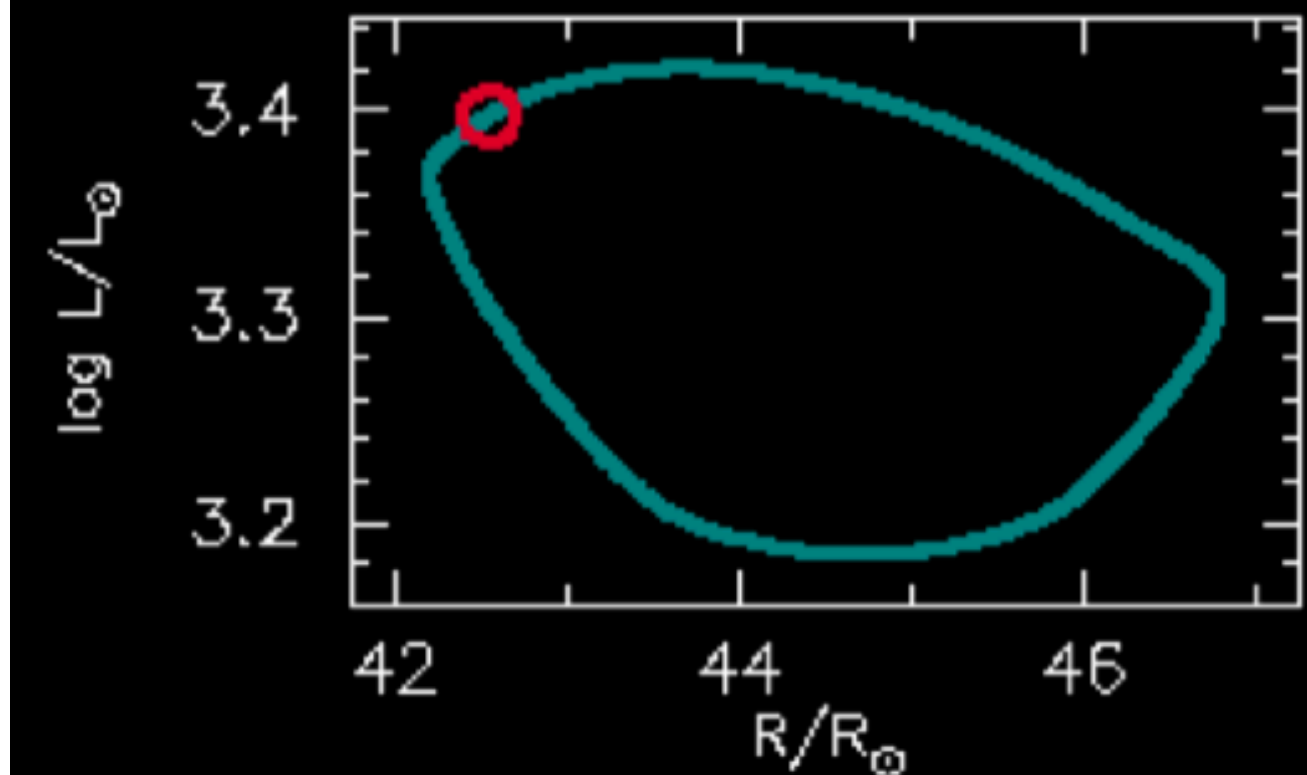
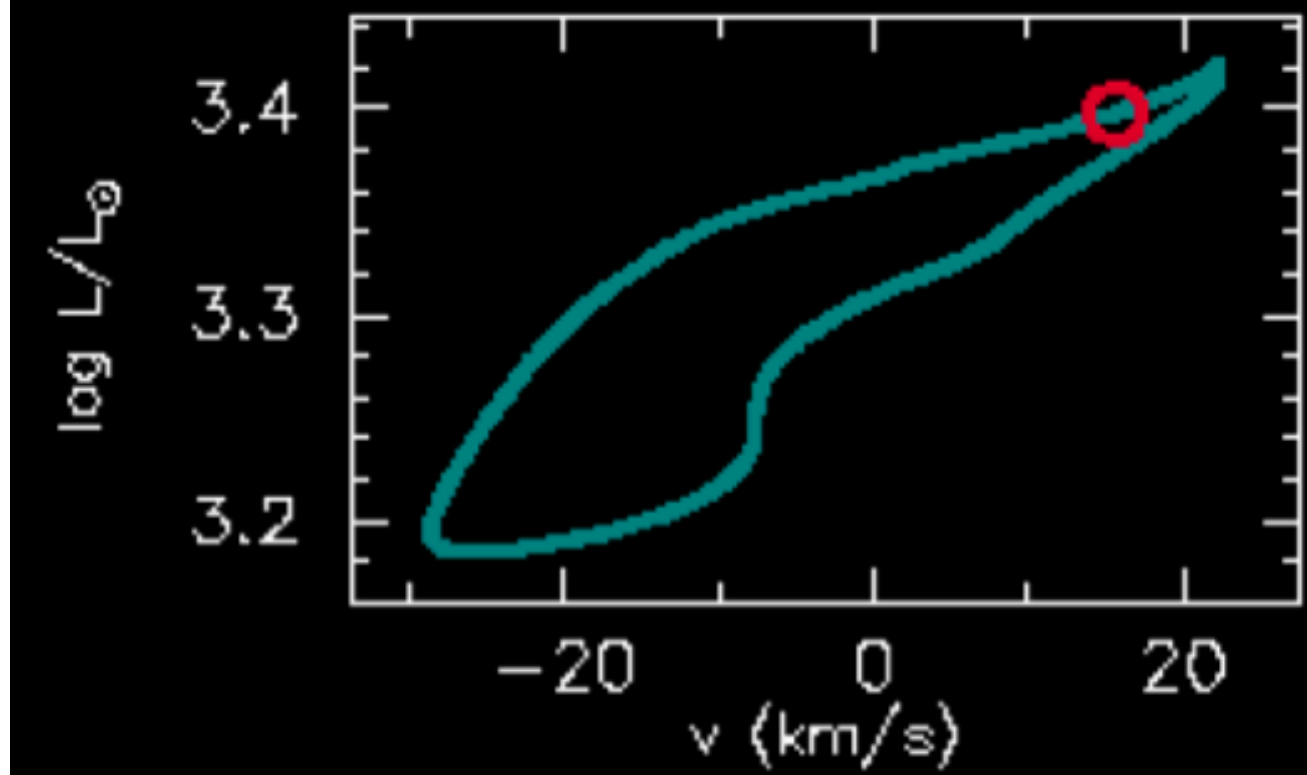
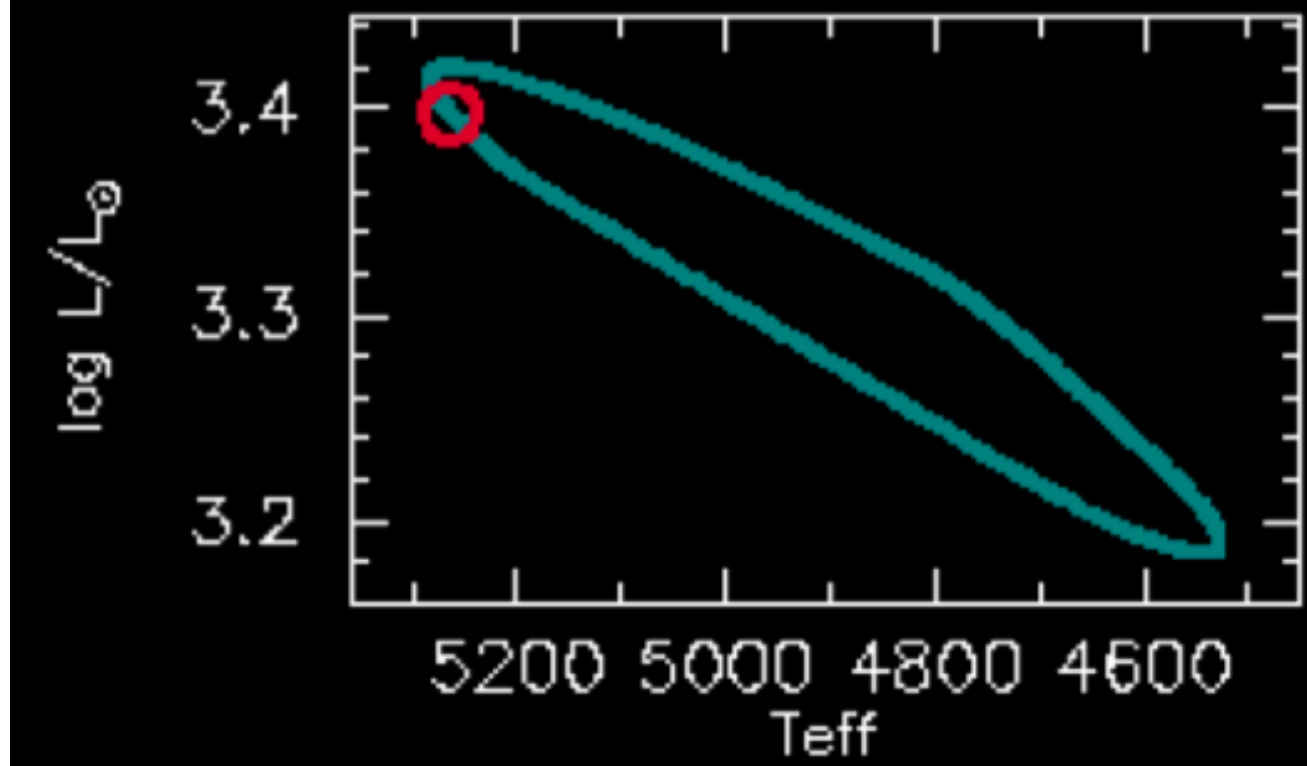
- Cepheids, RR Lyraes
- pulsating stars

Oscillations driven by opacity instability that occurs when the He<sup>+</sup> edge is near enough to the surface that there is insufficient pressure to contain it.

The opacity goes up instead of down with increasing temperature across the He<sup>+</sup> ↔ He<sup>++</sup> transition.



## delta Cep model



# Distance Scale

- Bright Star Standard Candles
  - TRGB
  - calibrate by
    - main sequence fitting of clusters or an entire galaxy like the LMC

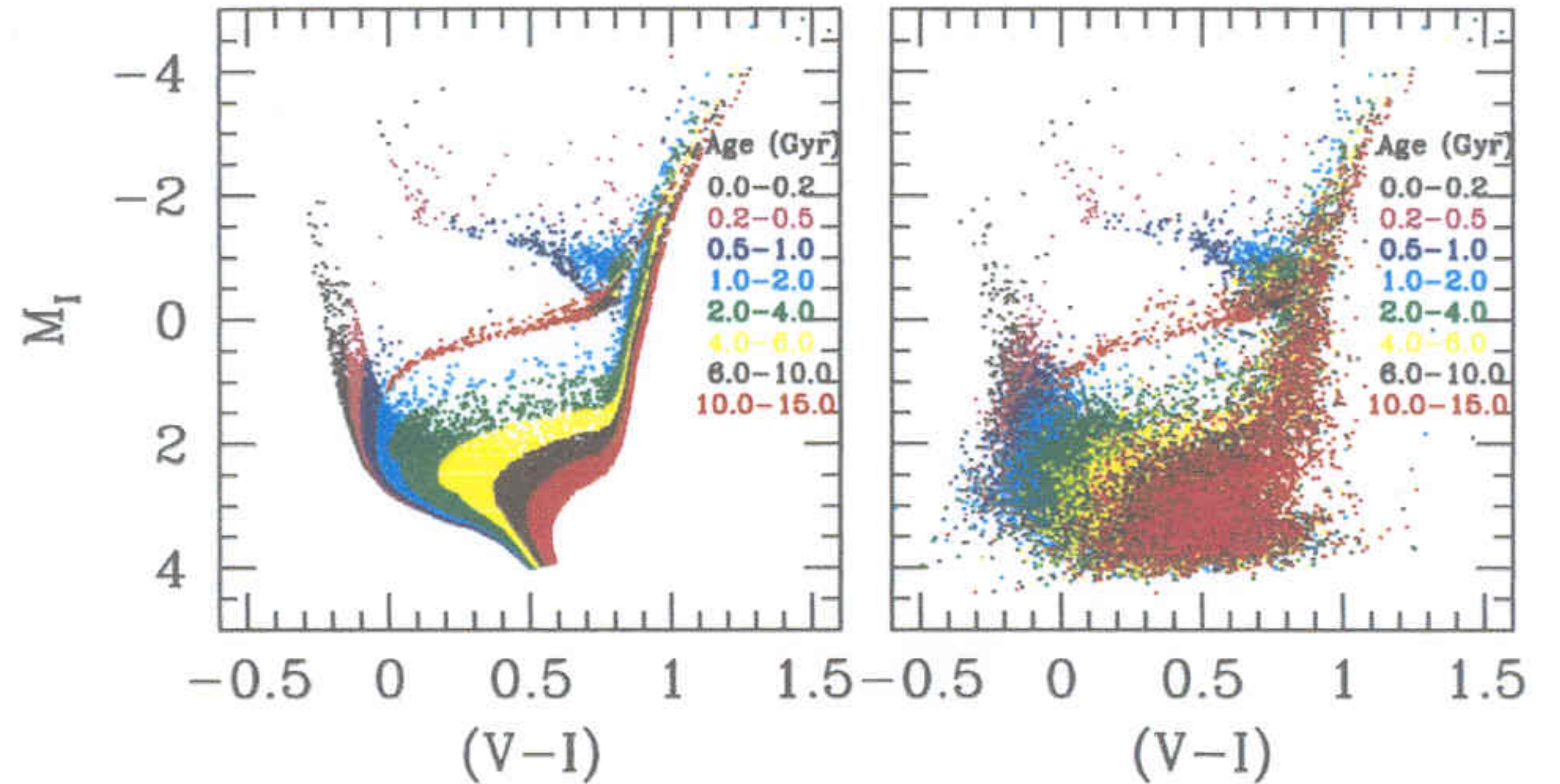
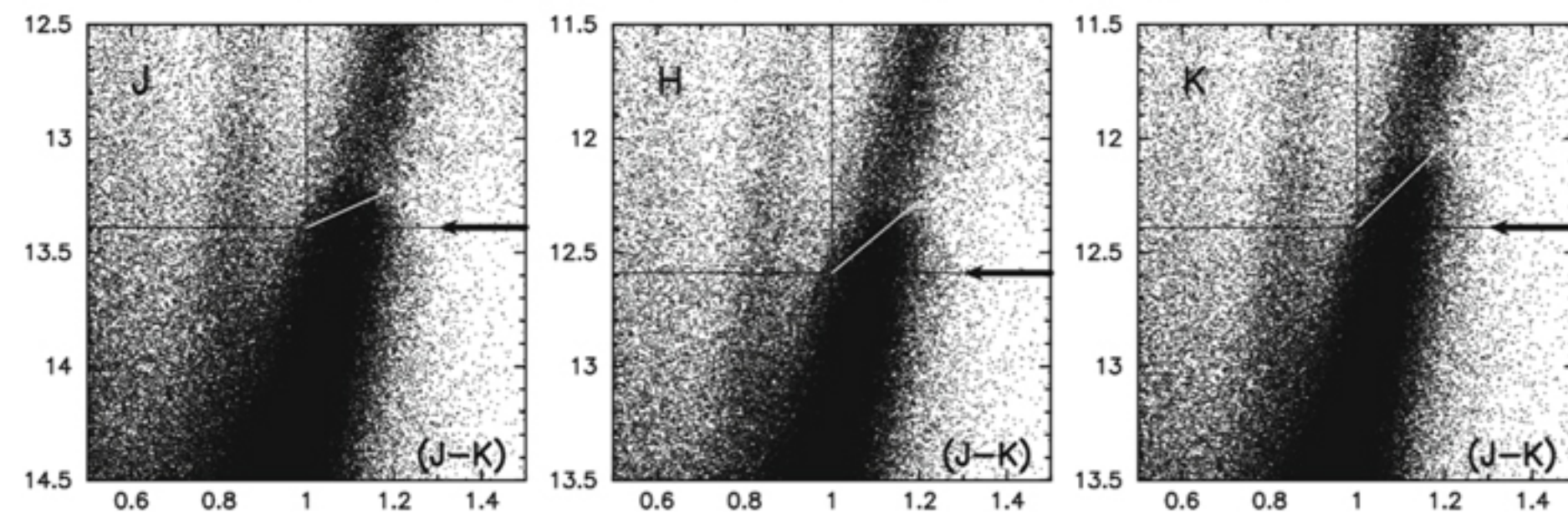
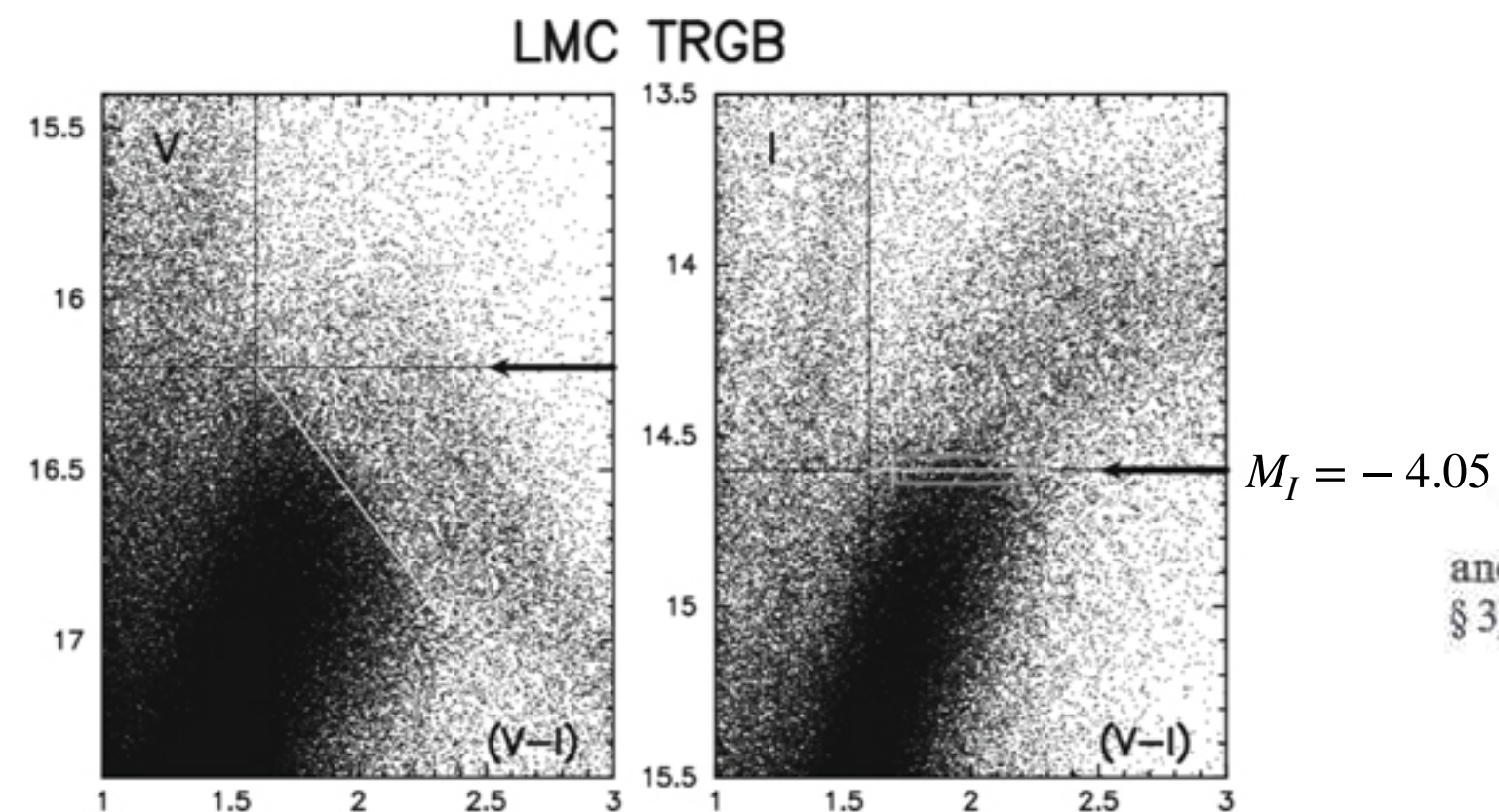


FIG. 2.—Set of partial models for a synthetic CMD computed with constant  $SFR(t)$  from 15 Gyr ago to the present,  $Z = 0.0004$ , Kroupa et al. (1993) IMF, and no binary stars. The left panel shows the theoretical synthetic CMD, while in the CMD of the right panel observational errors have been simulated (see § 3). Note the sequence of ages in both the MS and the subgiant branch and, although less definite, also in the RC and HB.

Works best in the I-band for low metallicity systems, where

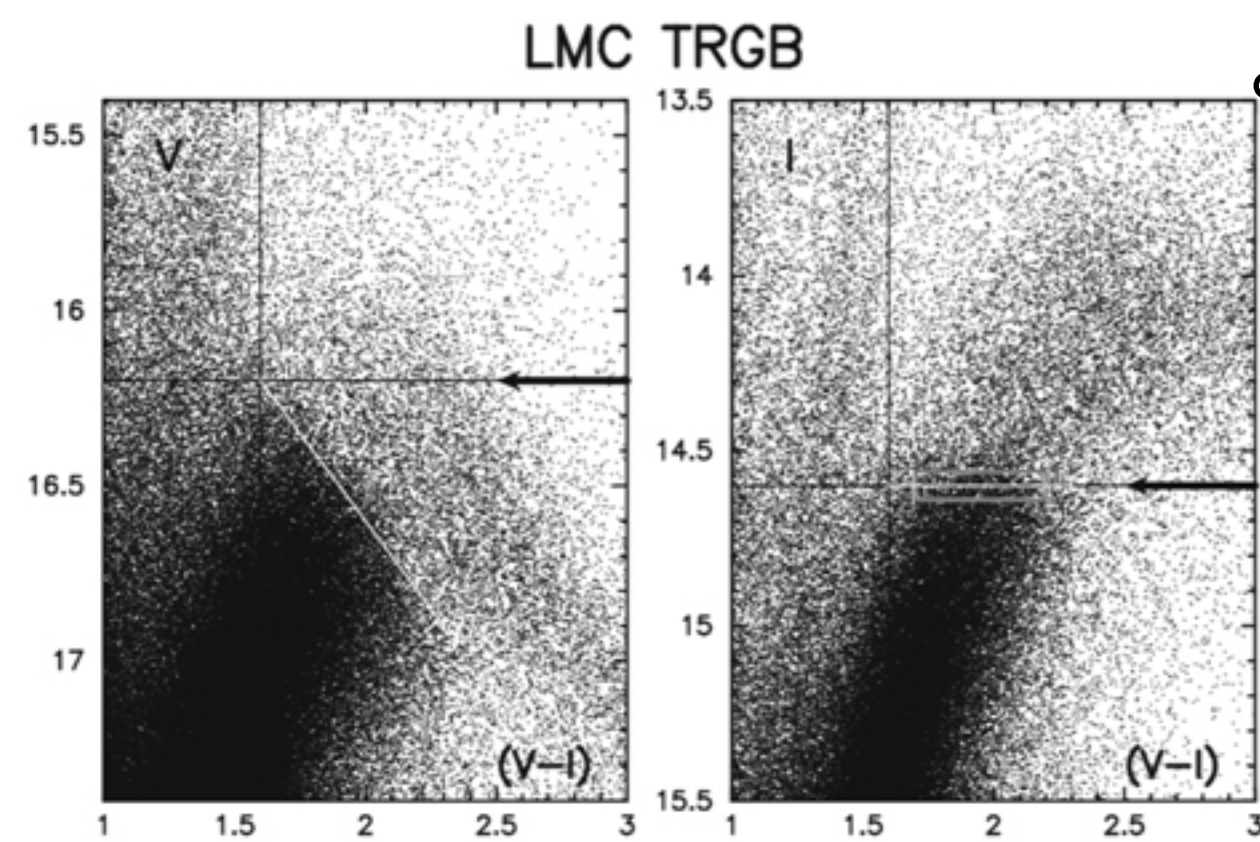
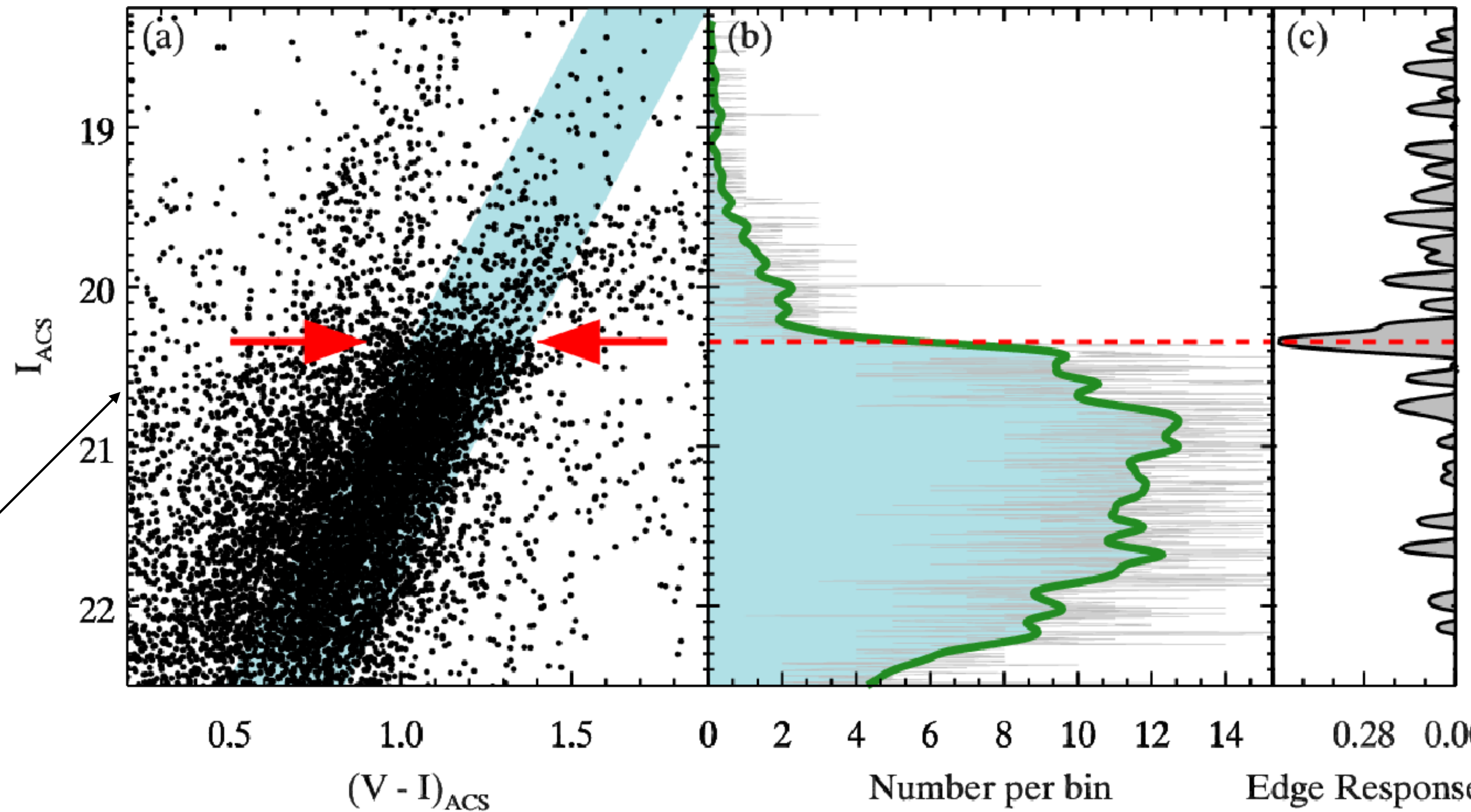
$$\text{TRGB } M_I = -4.05$$

In general, both bandpass and metallicity dependent.



# Distance Scale

- Bright Star Standard Candles
  - TRGB
  - calibrate by
    - main sequence fitting of clusters or an entire galaxy like the LMC



Works best in the I-band for low metallicity systems, where

$$\text{TRGB } M_I = -4.05$$

In general, both bandpass and metallicity dependent.