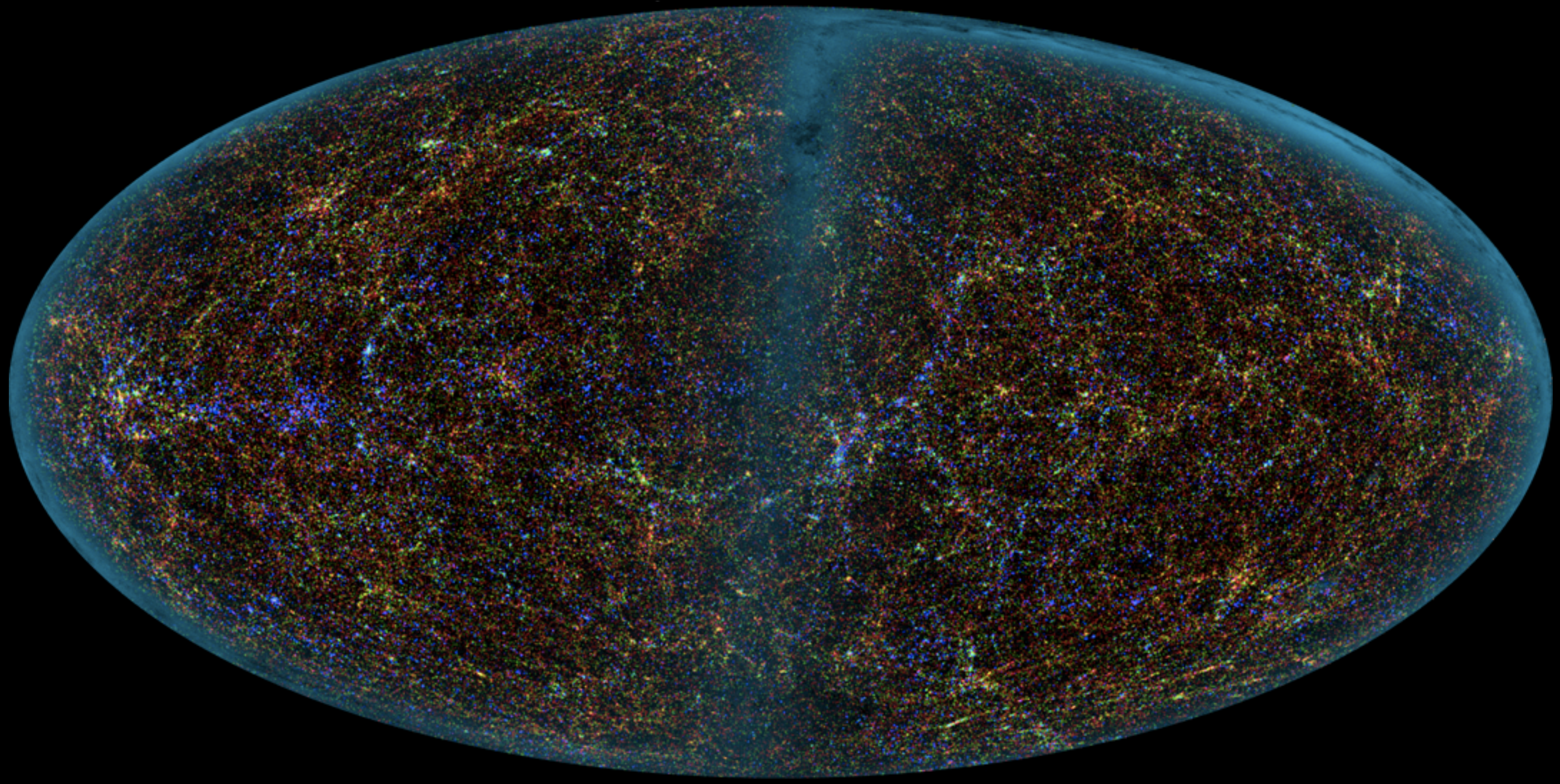
Cosmology

and Large Scale Structure



Today

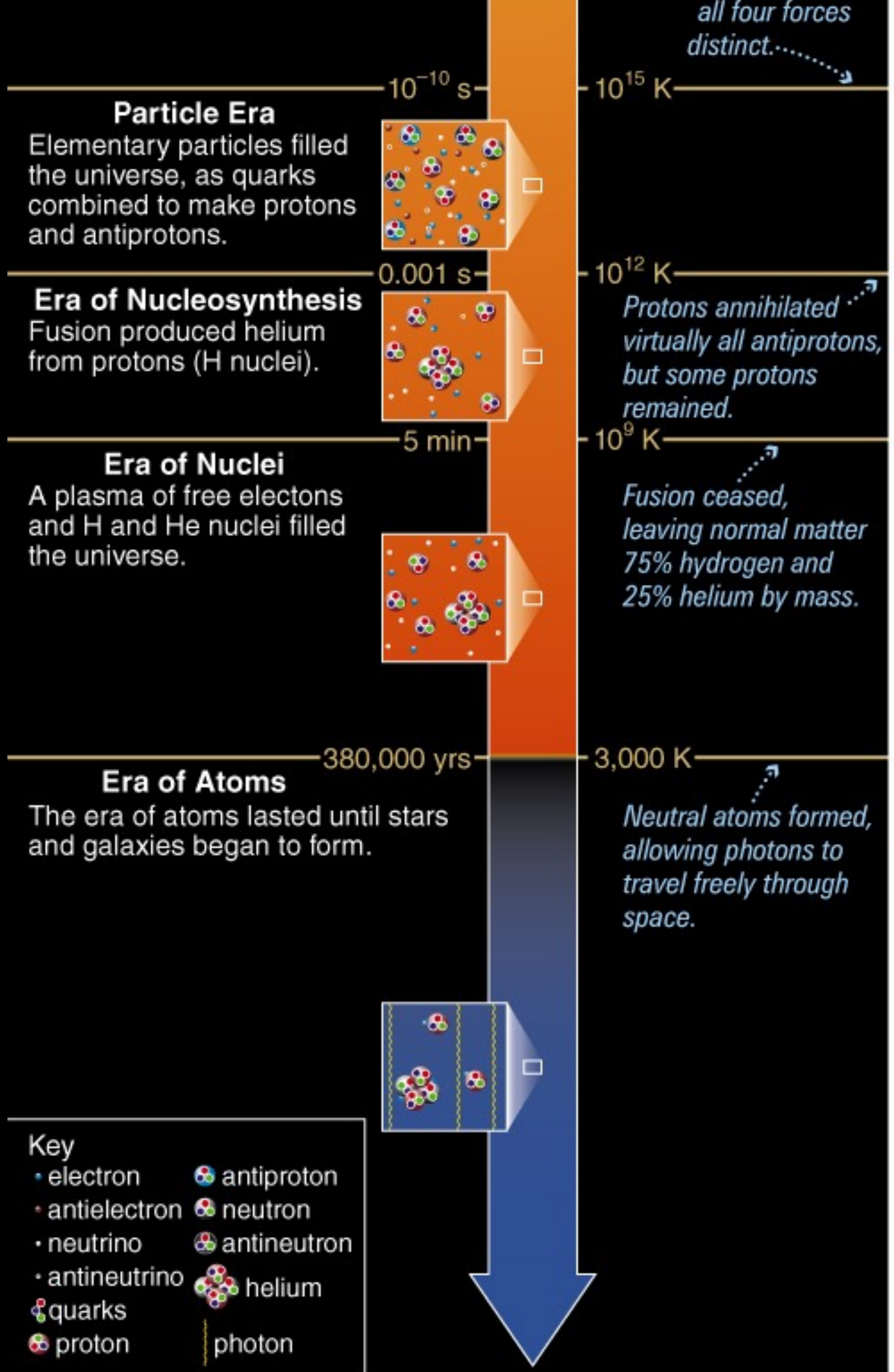
Empirical Pillars
of the Hot Big Bang

Big Bang Nucleosynthesis
Cosmic Background Radiation

1 November 2022

http://astroweb.case.edu/ssm/ASTR328/

Time	Event
$t \sim 10^{-43}$ s	Planck scale (*speculative*)
$t \sim 10^{-38}$ s	GUT scale (*speculative*)
$t \sim 10^{-35}$ s	Inflation (*speculative*)
$t \sim 10^{-12}$ s	Standard Model forces emerge
$t \sim 10^{-8}$ s	WIMPs decouple (*speculative*)
$t \sim 10^{-5}$ s	quarks condense into baryons (*baryogenesis*)
$t \sim 10^{-4}$ s	proton-antiproton annihilation ends
$t \sim 1$ s	neutrinos decouple
$t \sim 4$ s	electron-positron annihilation ends
$t \sim 10^{2}$ s	Big Bang Nucleosynthesis
$t \sim 10^{5}$ yr	Matter-radiation equality
$t \sim 4 \times 10^{5}$ yr	Atoms form, CMB emerges
$t \sim 5 \times 10^{6}$ yr	Gas temperature decouples from radiation
$t \sim 10^{7}$ yr	Dark Ages
$t \sim 5 \times 10^{8}$ yr	Cosmic dawn (first stars)
$t \sim 10^{9}$ yr	Galaxies form
$t \sim 4 \times 10^{9}$ yr	Peak star formation
$t \sim 9 \times 10^{9}$ yr	Sun forms
$t \sim 13 \times 10^{9}$ yr	Life on earth



time Temp

particle soup

< millisecond

T ~ 10^{14} K

nucleosynthesis (BBN)

~ 3 minutes

T ~ 10^{10} K

Early Universe

recombination

~380,000 year

T ~ 3000 K

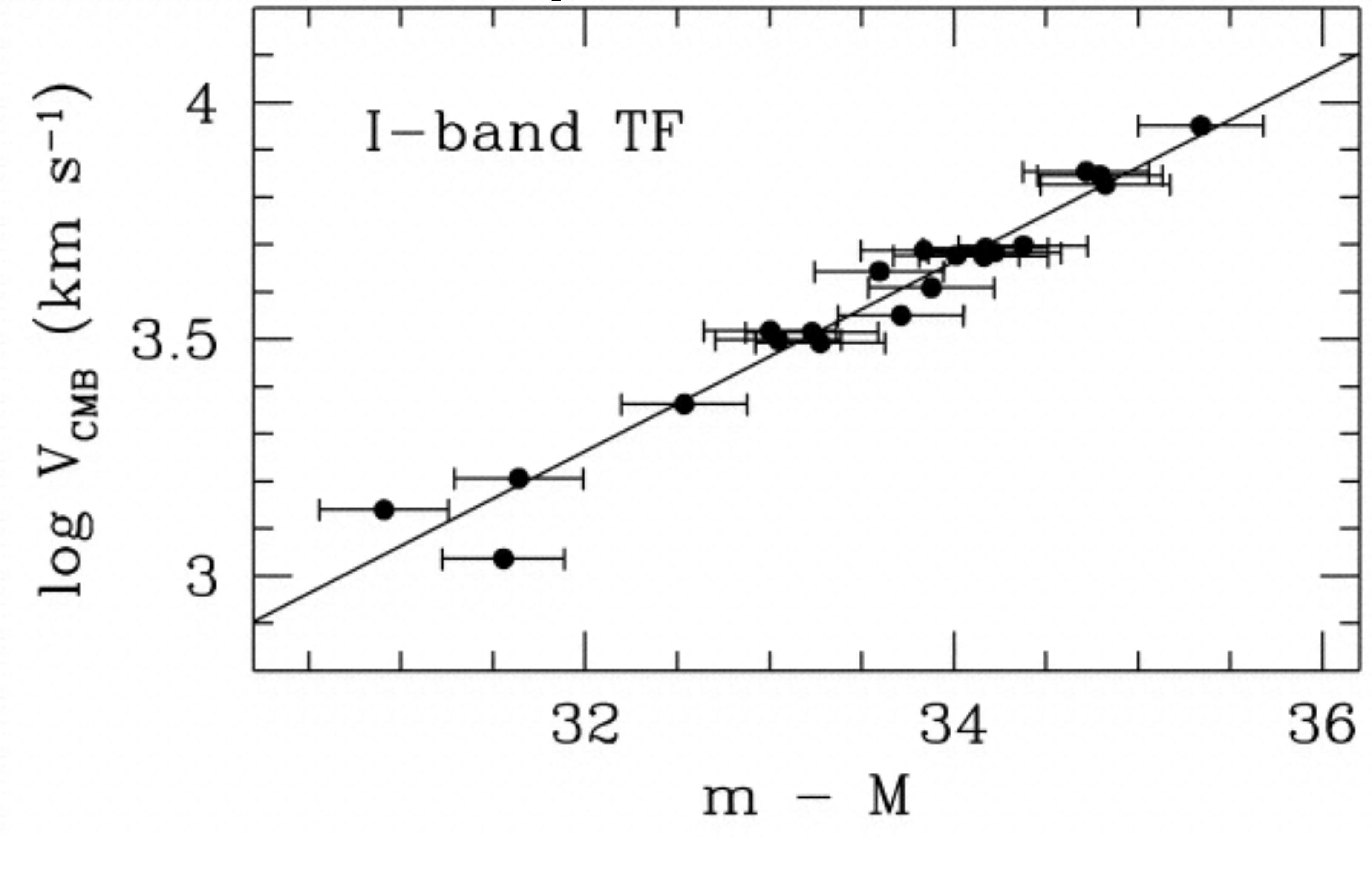
emission of CMB:

surface of last scattering - transition from opaque plasma to transparent neutral gas

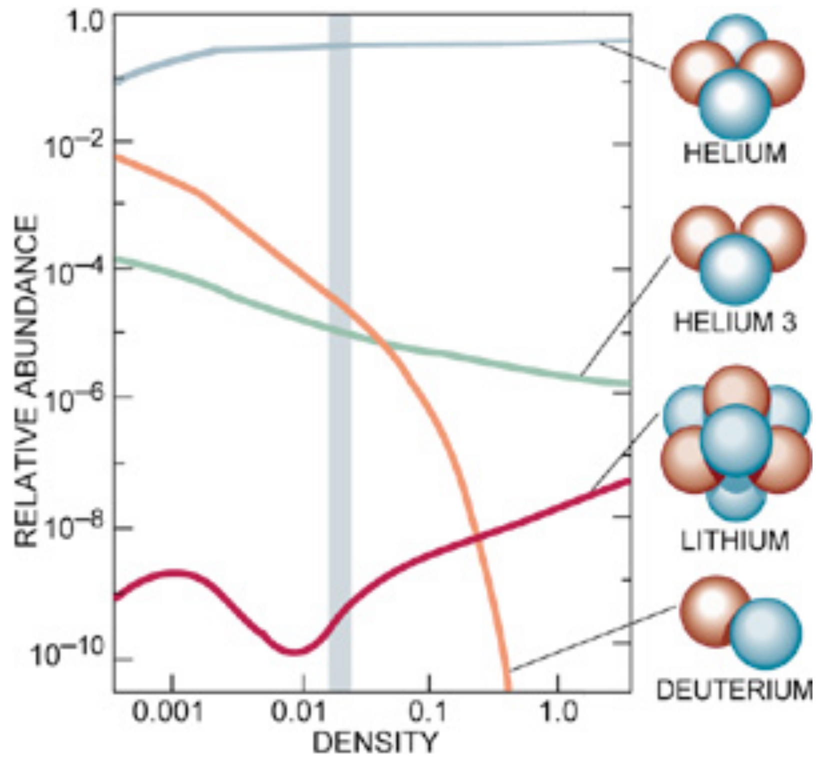
Empirical Pillars of the Hot Big Bang

1. Hubble Expansion — Hubble (1930)
2. Big Bang Nucleosynthesis (BBN) — Alpher & Gamow (1948) — $\alpha\beta\gamma$ paper
3. Cosmic Microwave Background (CMB) — Penzias & Wilson; Peebles & Dicke (1965)

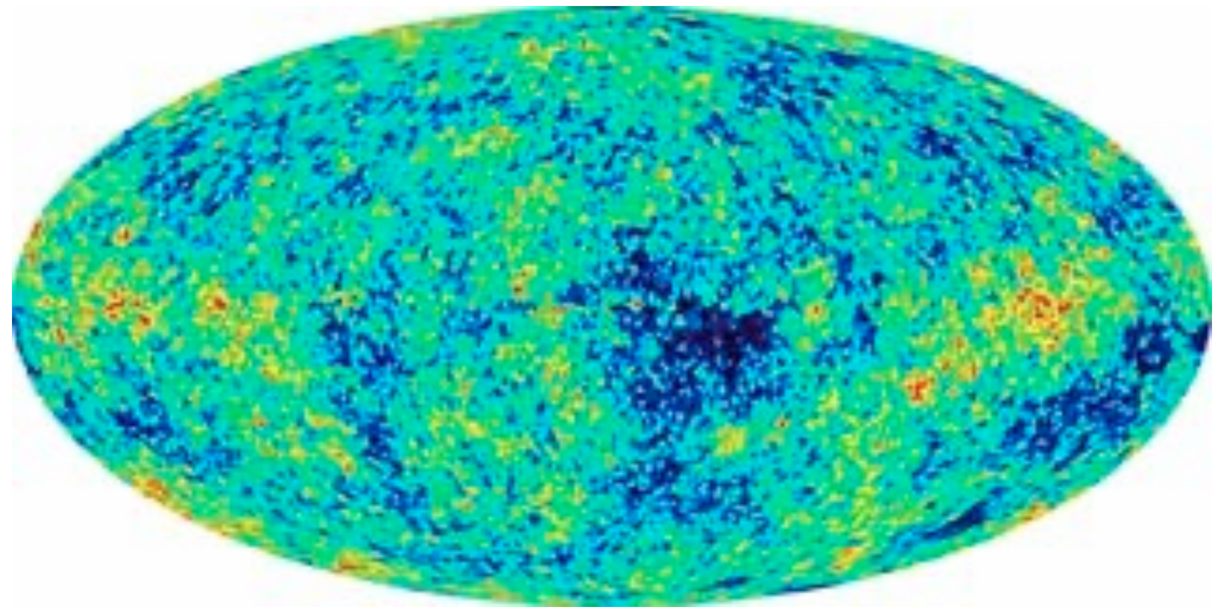
Hubble expansion



BBN
t ~ 3 minutes

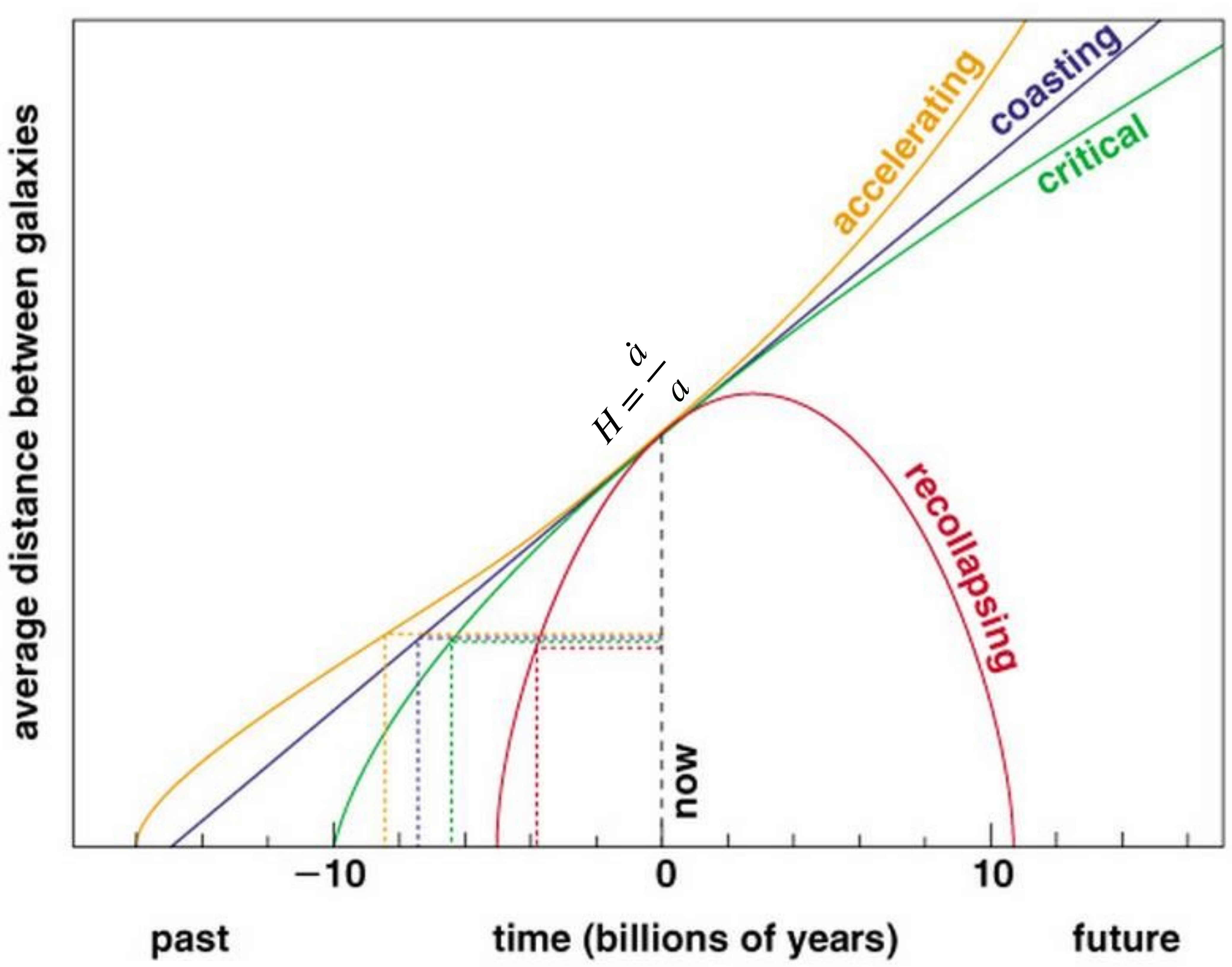


CMB
z = 1090
t = 380,000 yr



All expansion histories point to a dense, hot beginning - the **Hot Big Bang**

$a(t)$



t

Matter (non-relativistic "dust") dilutes as the volume expands:

$$\rho_m \sim a^{-3}$$

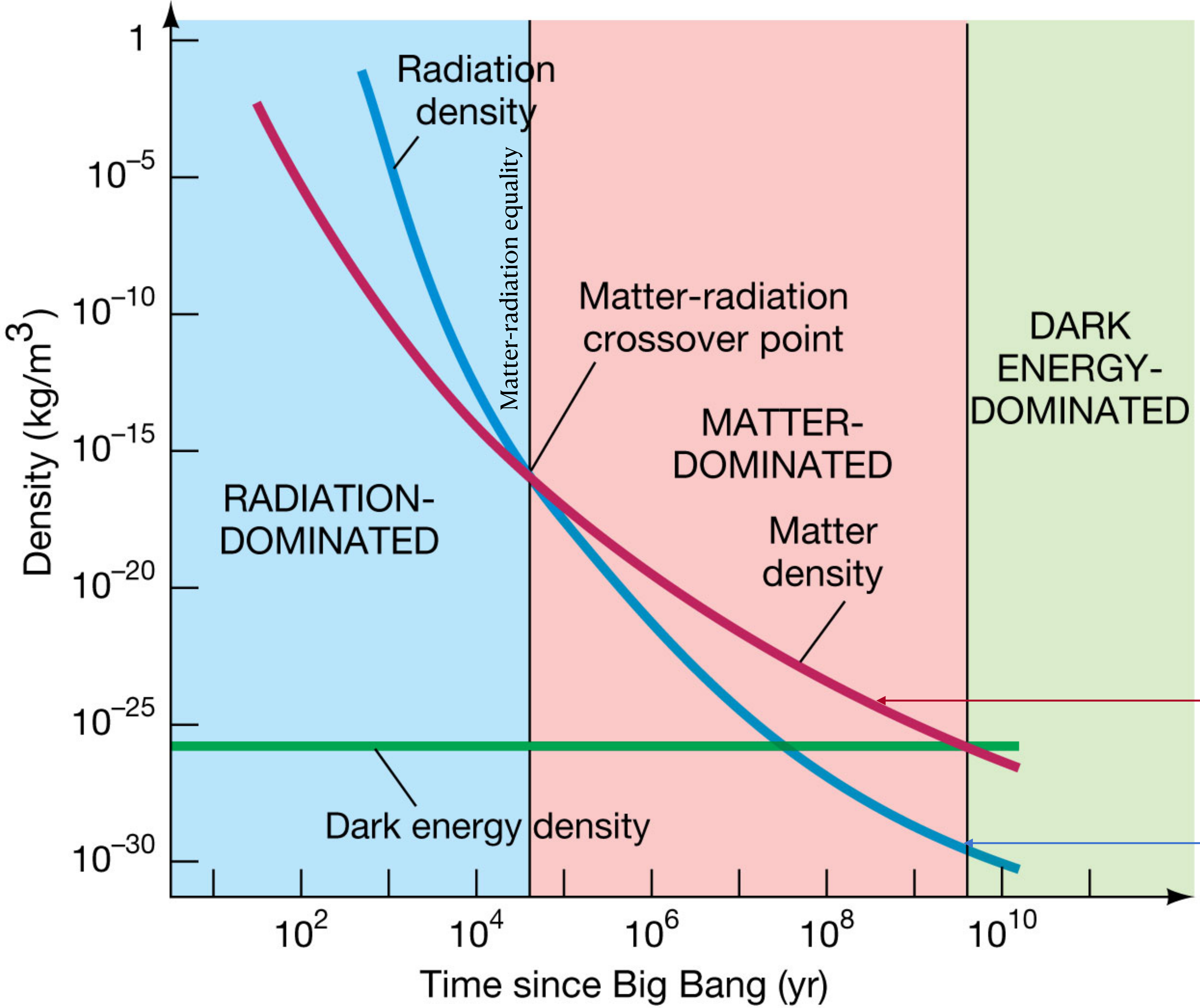
The energy density in radiation dilutes with the volume, but also stretches with space so that

$$z = \frac{\Delta\lambda}{\lambda} = \frac{1}{a} - 1$$

$$E_\lambda = \frac{hc}{\lambda} \sim a^{-1} \sim T_r$$

Consequently, $\rho_r = \frac{\varepsilon_r}{c^2} \sim a^{-4}$

and $T_r \sim a^{-1}$



Because of the different dependence on the expansion, there must have been a time in the past when the mass-energy in radiation exceeded that in matter.

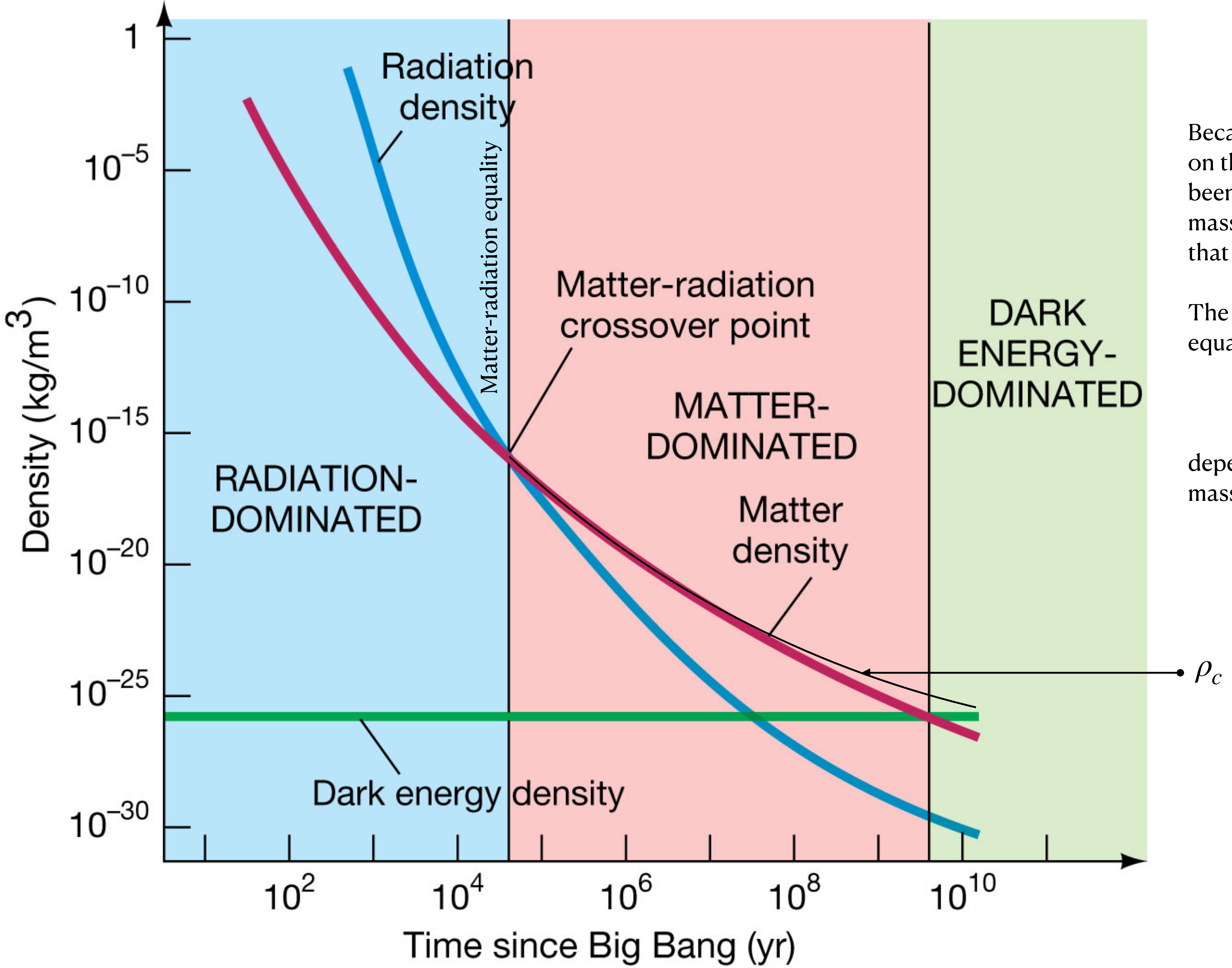
The redshift of matter-radiation equality occurred at

$$z_{\rm eq} \approx 3{,}000$$

depending in detail on the current mass and radiation density.

$\rho_m \sim a^{-3}$

$\rho_r \sim a^{-4}$



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The redshift of matter-radiation equality occurred at

$$z_{\rm eq} \approx 3{,}000$$

depending in detail on the current mass and radiation density.

The matter density has only diverged from the critical density a little bit so far.

What happens in the future?

Early universe expansion dynamics is radiation dominated

Friedmann equation

$$H^2 = H_0^2(\Omega_m + \Omega_r + \Omega_k + \Omega_\Lambda)$$

Simplifies to

$$\dot{a} = H_0 \Omega_{r_0}^{1/2} a^{-1}$$

for the early, radiation dominated universe.

We also have

$$T \sim a^{-1}$$

So we can replace *a* with *T* and solve for *T(t)* in place of *a(t)*

Radiation energy density

$$\rho_{r_0} c^2 = \varepsilon_{r_0} = \alpha T_{r_0}^4$$

$$\Omega_{r_0} = \frac{\rho_{r_0}}{\rho_c} = \frac{\varepsilon_{r_0}}{\rho_c c^2} \; ; \; \rho_c = \frac{3H_0^2}{8\pi G}$$

$$\Omega_{r_0} = \frac{8\pi G}{3H_0^2} \frac{\varepsilon_{r_0}}{c^2} = \frac{8\pi G}{3H_0^2} \frac{\alpha T_{r_0}^4}{c^2}$$

Early universe expansion dynamics is radiation dominated

Replacing *a* with *T*

$$\frac{\dot{T}}{T} = -\frac{\dot{a}}{a}$$

$$\frac{\dot{T}}{T} = -\sqrt{\frac{8\pi\alpha G}{3c^2}}T^2$$

$$T = \left(\frac{32\pi\alpha G}{3c^2}\right)^{-1/4} t^{-1/2} = Kt^{-1/2}$$

$$T^2 t = \text{constant}$$

during radiation domination. Is just the thermodynamics of adiabatic expansion, depending only on elementary physical constants.

Radiation energy density

$$\rho_{r_0} c^2 = \varepsilon_{r_0} = \alpha T_{r_0}^4$$

$$\Omega_{r_0} = \frac{8\pi G}{3H_0^2}\frac{\varepsilon_{r_0}}{c^2} = \frac{8\pi G}{3H_0^2}\frac{\alpha T_{r_0}^4}{c^2}$$

$$K \approx 10^{10}\ \mathrm{K\,s}^{1/2}$$

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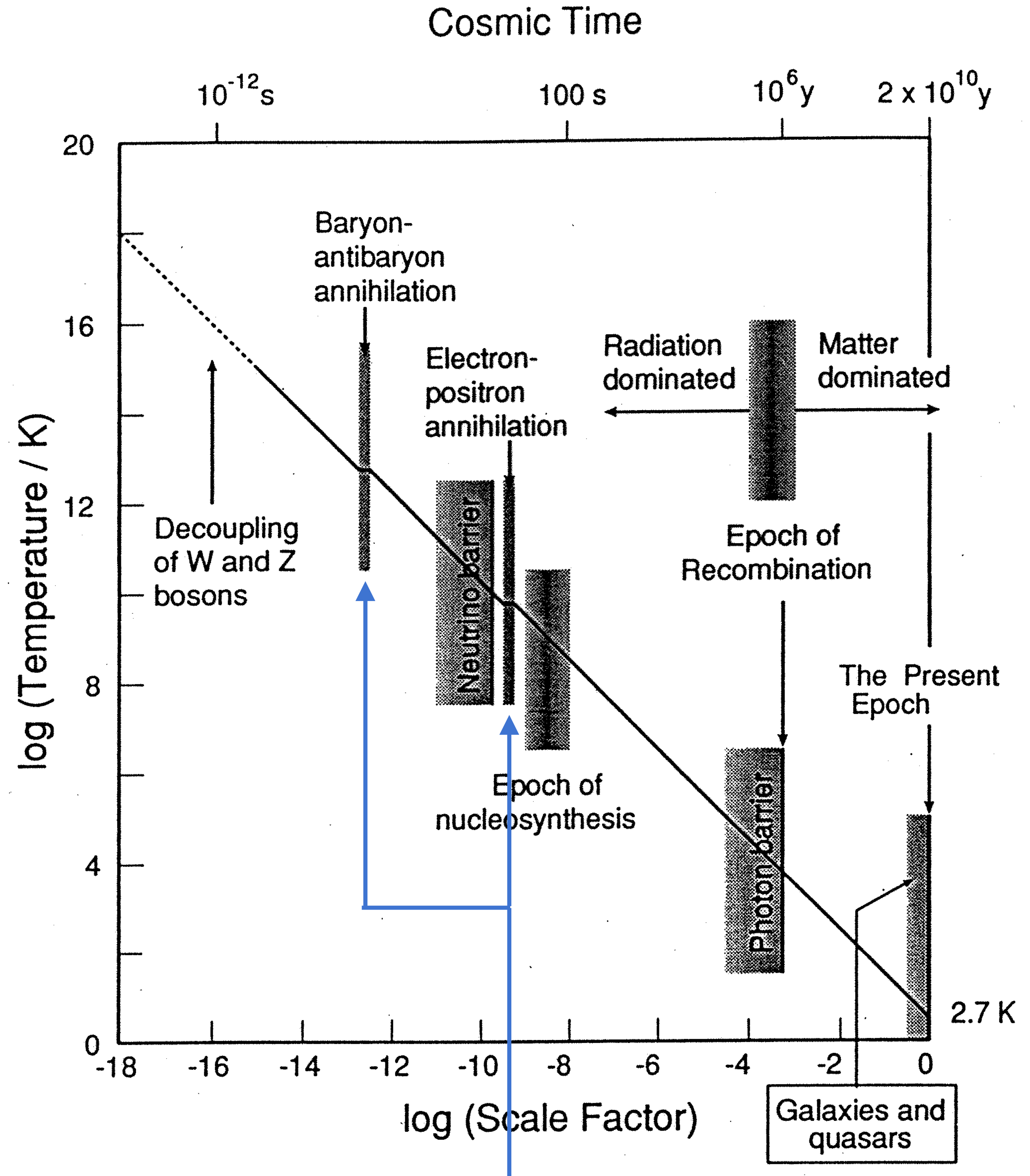


Fig. 6.1 The thermal history of the standard Hot Big Bang. The radiation temperature decreases as $T_r \propto R^{-1}$ except for abrupt jumps as different particle-antiparticle pairs annihilate at $kT \approx mc^2$. Various important epochs in the standard model are indicated. An approximate time scale is indicated along the top of the diagram. The neutrino and photon barriers are indicated. In the standard model, the Universe is optically thick to neutrinos and photons prior to these epochs.

Time	Event
$t \sim 1$ s	neutrinos decouple

neutrinos drop out of equilibrium as photons will later do. They lose energy with expansion the same as the radiation field, $T_\nu \sim a^{-1}$, with an initial energy density $\varepsilon_\nu \sim T_\nu^4$ fixed at this point.

$t \sim 4$ s	electron-positron annihilation ends

Being less massive than protons, electrons freeze out from positrons at this later time. The excess energy feeds the radiation background but not the neutrino background.

$$K = 0.95 \times 10^{10}\ \mathrm{K\,s^{1/2}}$$ before electron-positron freeze-out

$$K = 1.3 \times 10^{10}\ \mathrm{K\,s^{1/2}}$$ after electron-positron freeze-out

- Note the brief pause in the decline of the temperature of the radiation field as first proton-antiproton annihilation, then later electron-positron annihilation dump energy into radiation.

$$T = Kt^{-1/2}$$

$$K = 0.95 \times 10^{10}\ \mathrm{K\,s}^{1/2}$$ before electron-positron freeze-out

$$K = 1.3 \times 10^{10}\ \mathrm{K\,s}^{1/2}$$ after electron-positron freeze-out

Electron-positron freeze-out dumps a bit of extra entropy into the radiation field. Since neutrinos have already decoupled, they compose a radiation field that does not receive this extra bit of heating. Therefore one expects both a cosmic [microwave] background and a cosmic neutrino background of slightly different temperatures:

$$\frac{T_\nu}{T_\gamma} = \left(\frac{g_1}{g_2}\right)^{1/3} = \left(\frac{4}{11}\right)^{1/3}$$

$$T_{\nu_0} = 1.95\ \mathrm{K}\ \text{when}\ T_{\gamma_0} = 2.75\ \mathrm{K}$$

Time	Event
$t \sim 1$ s	neutrinos decouple

neutrinos drop out of equilibrium as photons will later do. They lose energy with expansion the same as the radiation field, $T_\nu \sim a^{-1}$, with an initial energy density $\varepsilon_\nu \sim T_\nu^4$ fixed at this point.

$t \sim 4$ s	electron-positron annihilation ends

Being less massive than protons, electrons freeze out from positrons at this later time. The excess energy feeds the radiation background but not the neutrino background.

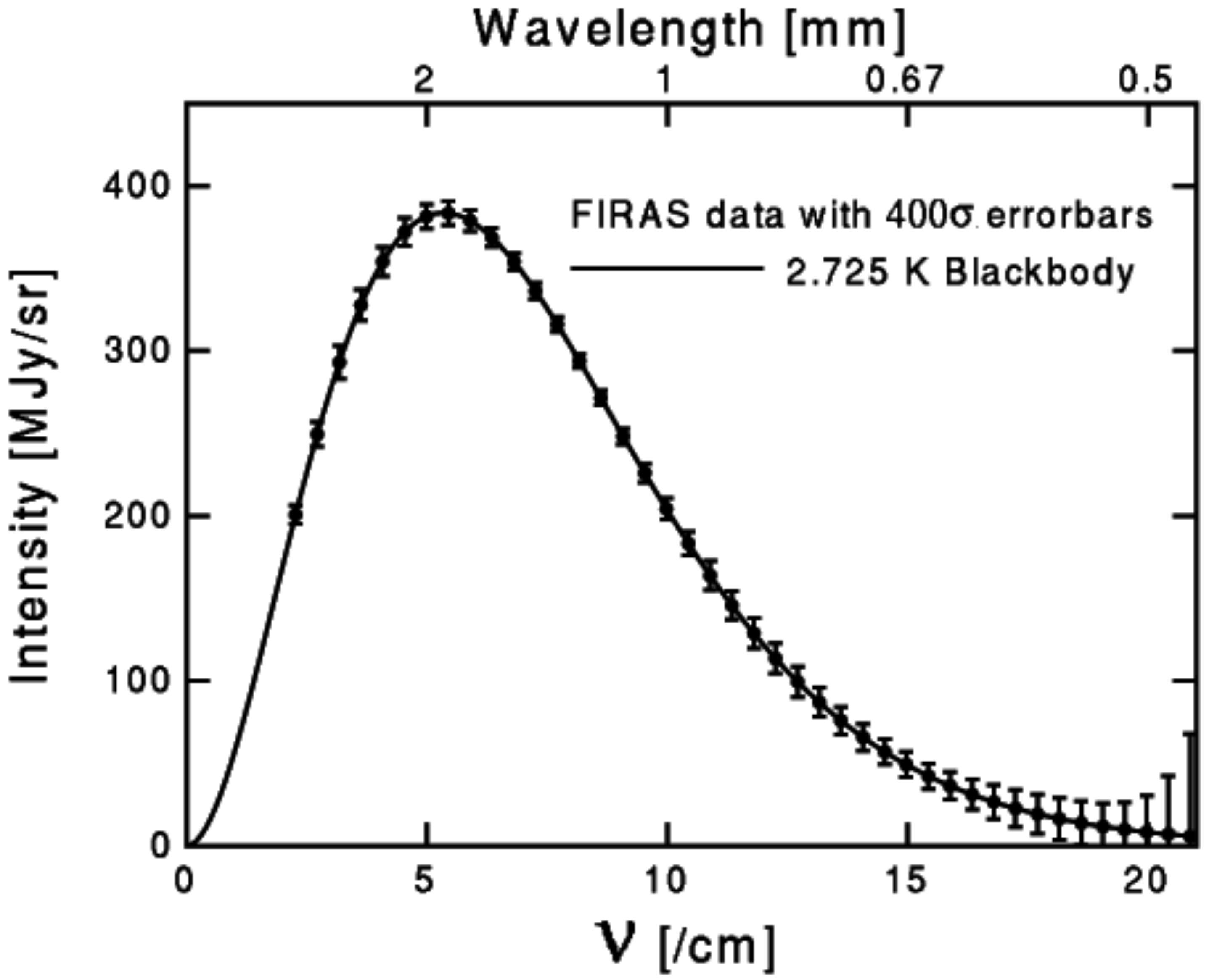
photons only — photons plus neutrinos

$$g_1 = 2\ ;\ g_2 = 2 + 2\frac{7}{8} + 2\frac{7}{8}$$

bosons — bosons + fermions

The existence of a cosmic neutrino background of this specific temperature is a major outstanding prediction of the Hot Big Bang.

Cosmic Microwave Background



The existence of the CMB had been anticipated by Gamow in the '50s, but this was largely forgotten by the '60s. It's predicted existence we rediscovered by Dicke & Peebles, who were actively searching for it when it was accidentally found by Penzias & Wilson.

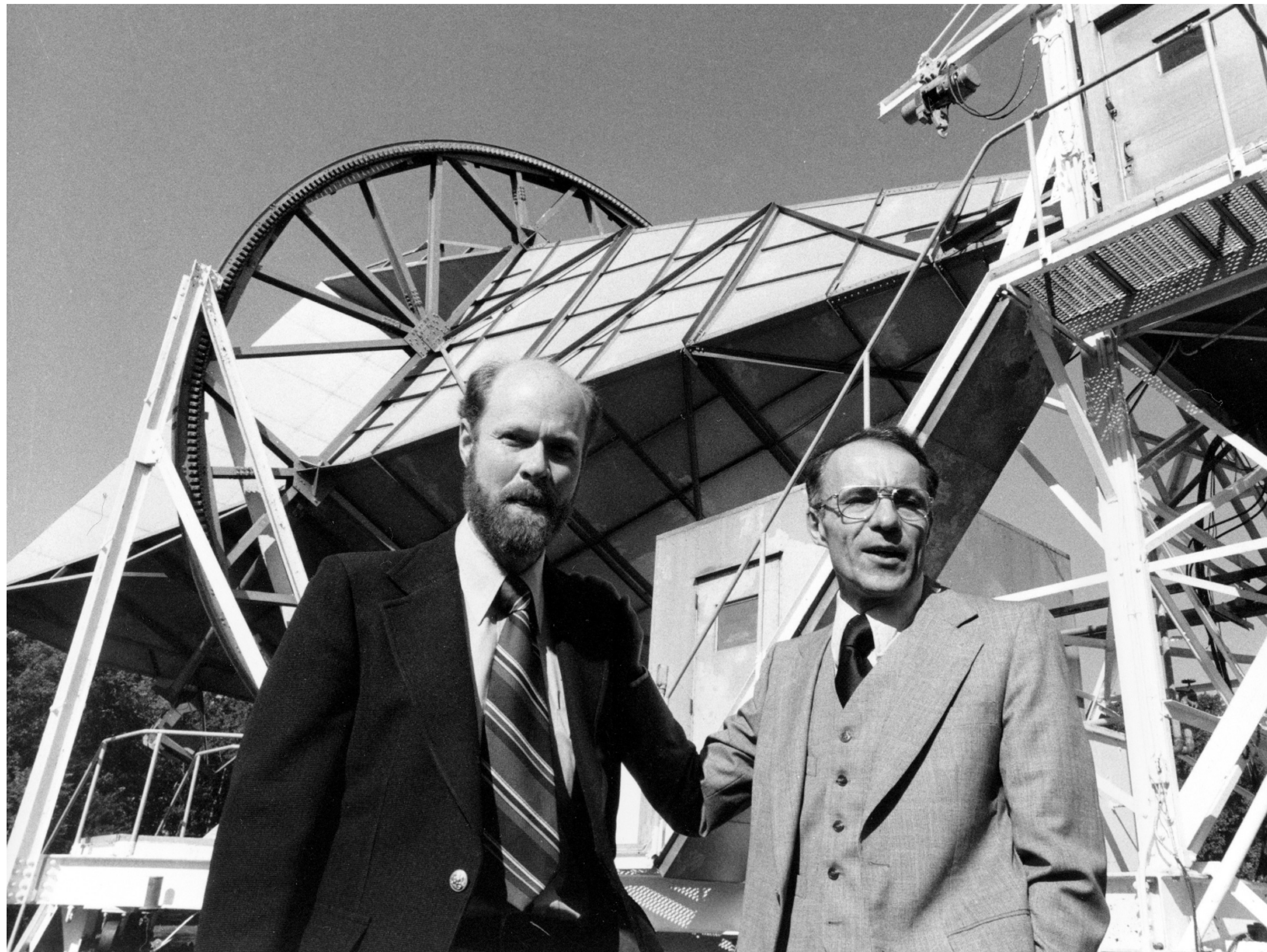
The Cosmic Microwave Background (CMB) is the relic radiation field of the Hot Big Bang. It has an essentially a perfect thermal spectrum with a current temperature of 2.725 K.

This was higher in the past by (1+z), so
T ~ 3,000 K at the epoch of recombination. This is the point at which the high-energy tail of the blackbody distribution no longer had enough photons in excess of 1 Rydberg to keep the universe ionized.

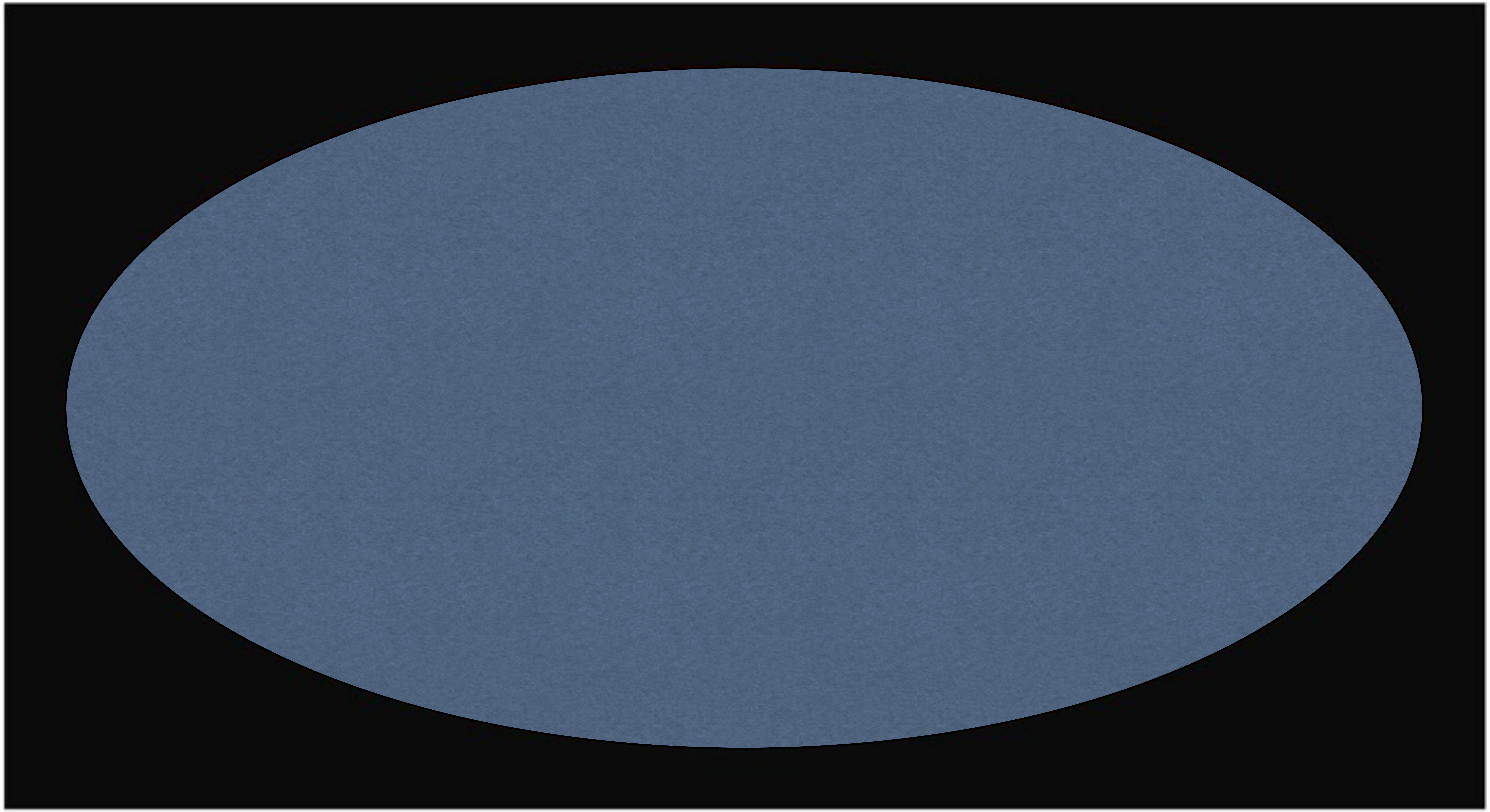
$$1 + z_{re} = \frac{T_{re}}{T_0} = \frac{3000\ \mathrm{K}}{2.75\ \mathrm{K}} = 1091$$

$$z_{re} = 1090$$

Penzias & Wilson with the horn they discovered the CMB with at AT&T Bell Labs

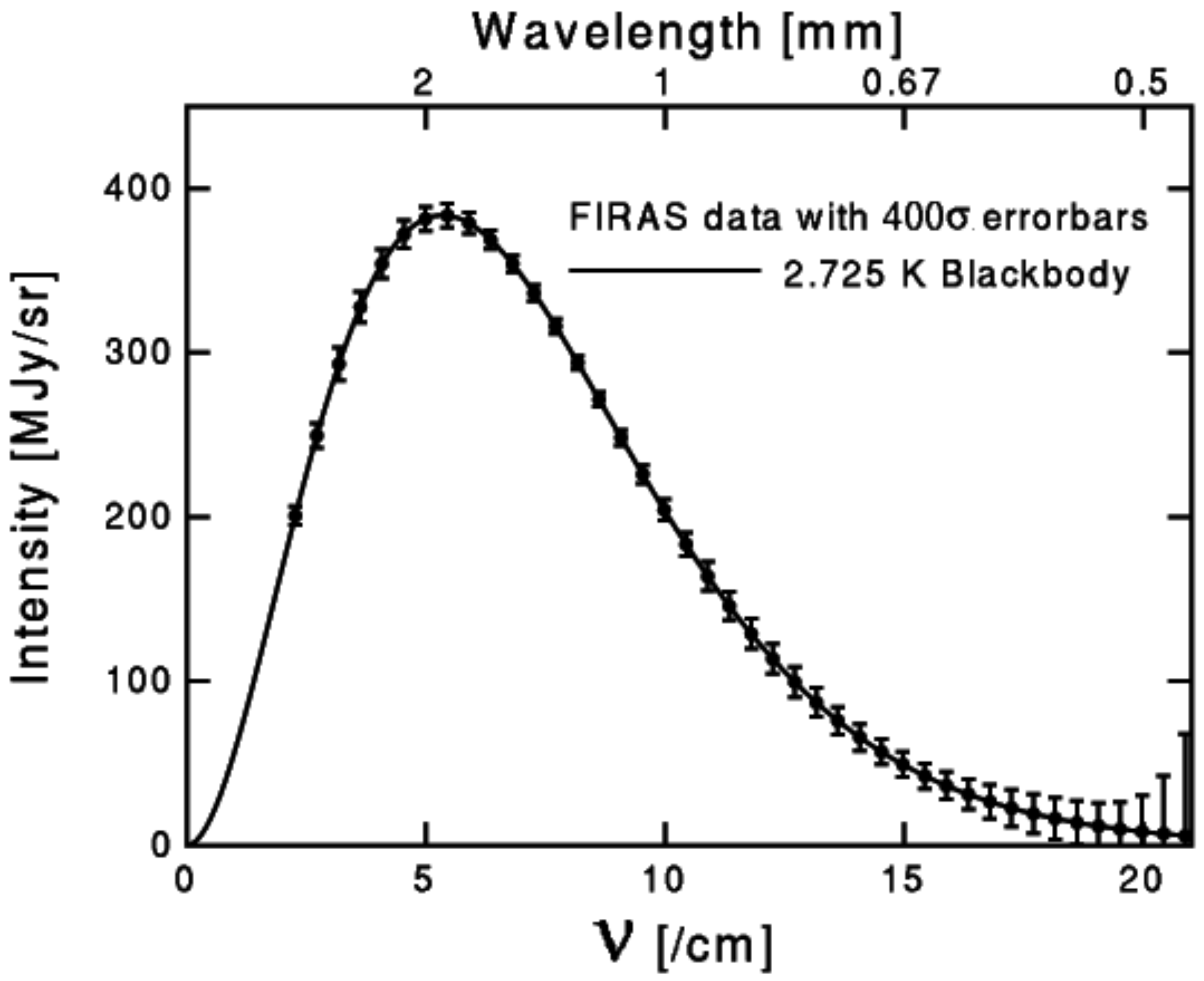


CMB: Baby picture of the universe (380,000 years old)



Same temperature everywhere on the - “monopole”

Cosmic Microwave Background



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Current temperature $T_{\gamma_0} = 2.725\ \mathrm{K}$

Energy density $\varepsilon_{r_0} = \alpha T_{\gamma_0}^4 = 0.261\ \mathrm{MeV\,m^{-3}}$

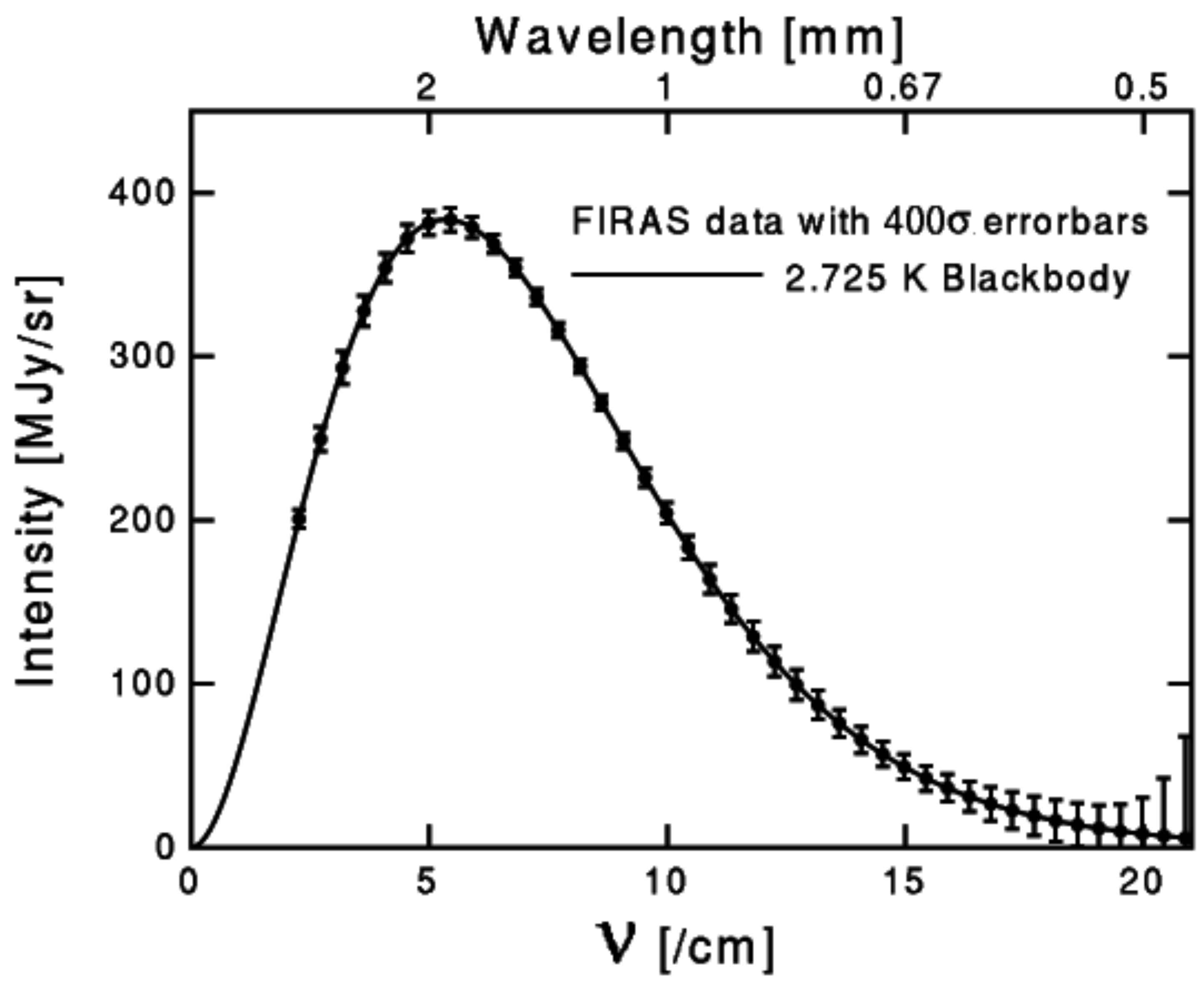
α is the radiation density constant

Relative to the critical density $\Omega_{r_0} = 5 \times 10^{-5}$

In neutrinos, there's also $\Omega_{\nu_0} = 4 \times 10^{-5}$

Number density of photons $n_{\gamma,0} = 411\ \mathrm{cm}^{-3}$

This greatly exceeds all the photons produced by all the stars throughout the universe over all time.



We observe the CMB as the "surface of last scattering" that occurs at the epoch of recombination. This is the point at which the early universe transitions from an opaque plasma of protons and electrons to a mostly transparent neutral gas of hydrogen and helium.

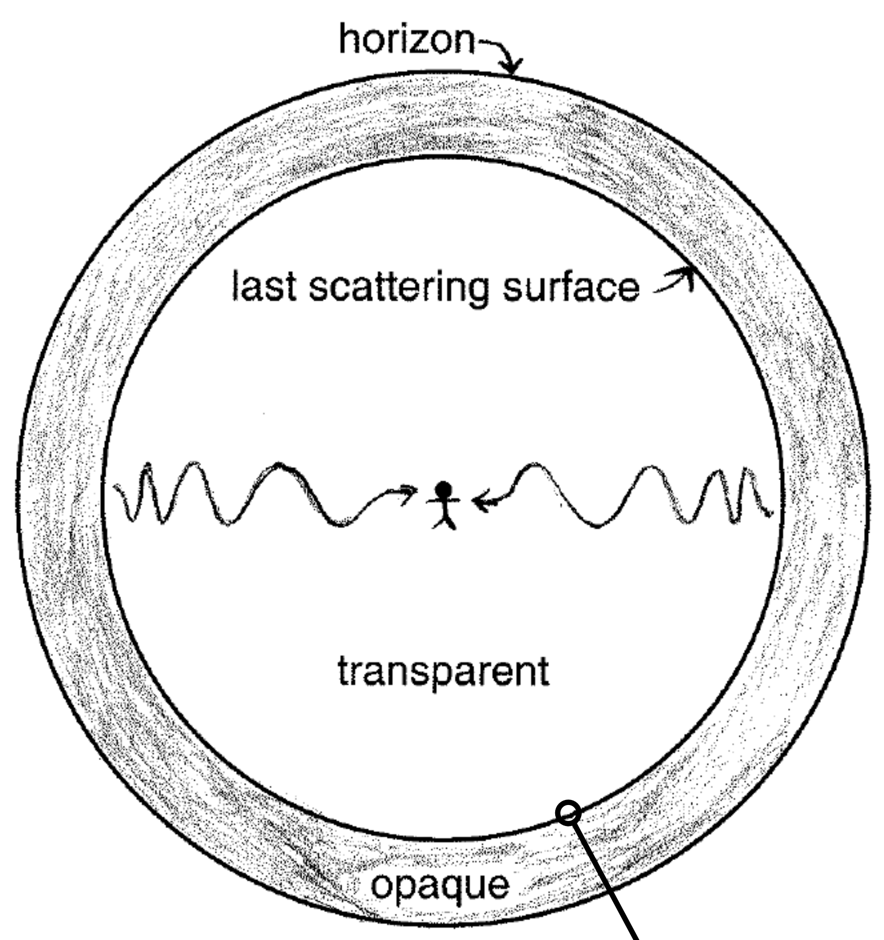
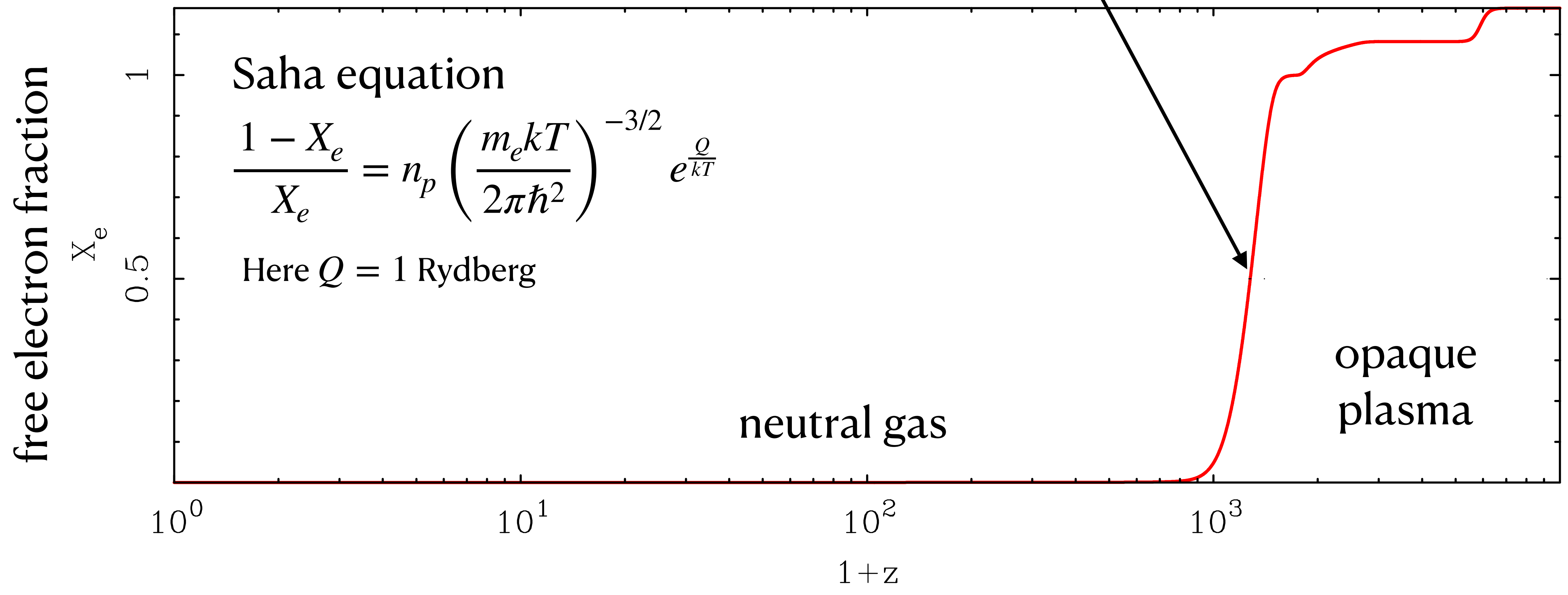
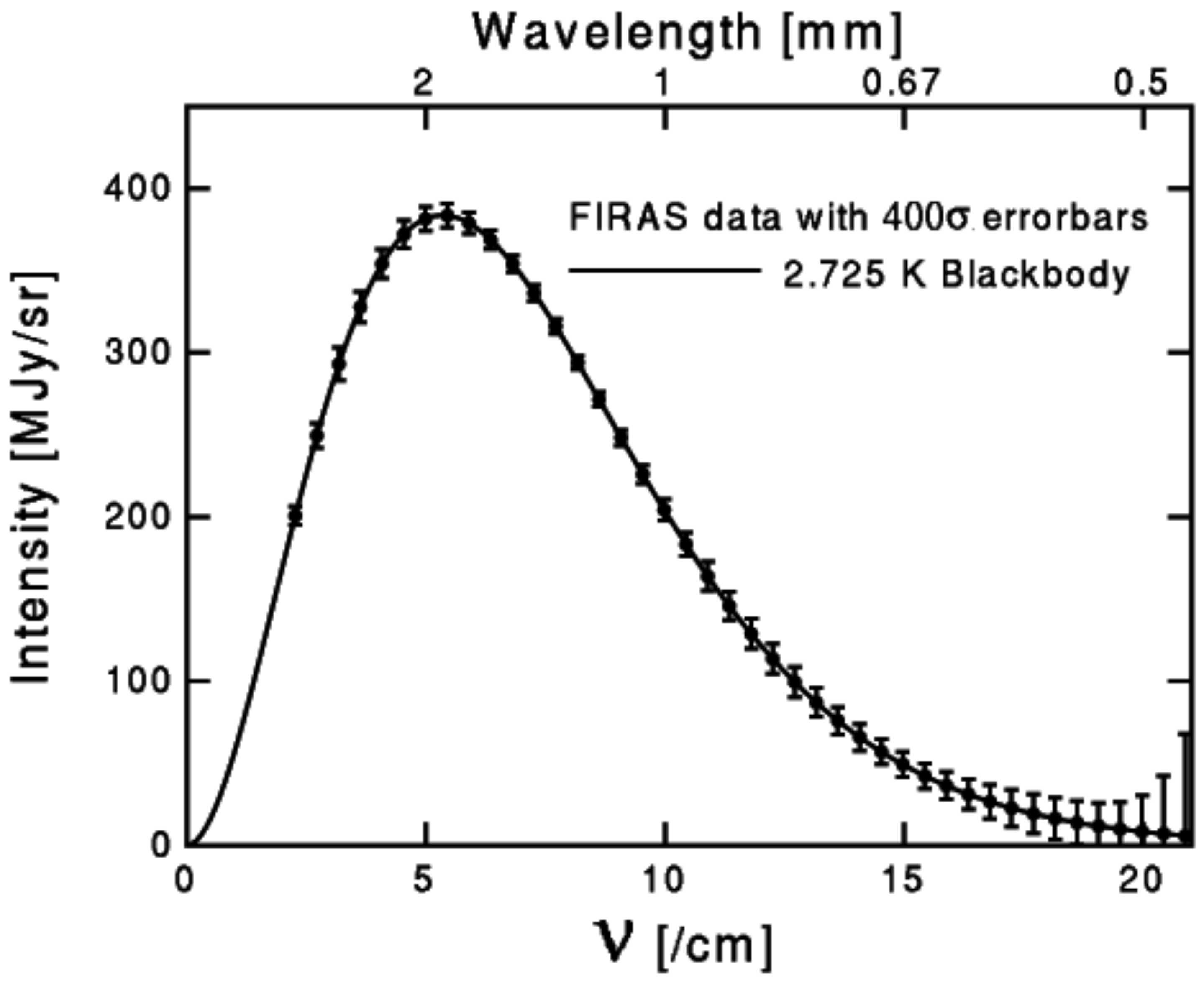


Figure 9.3: An observer is surrounded by a spherical last scattering surface. The photons of the CMB travel straight to us from the last scattering surface, being continuously redshifted.





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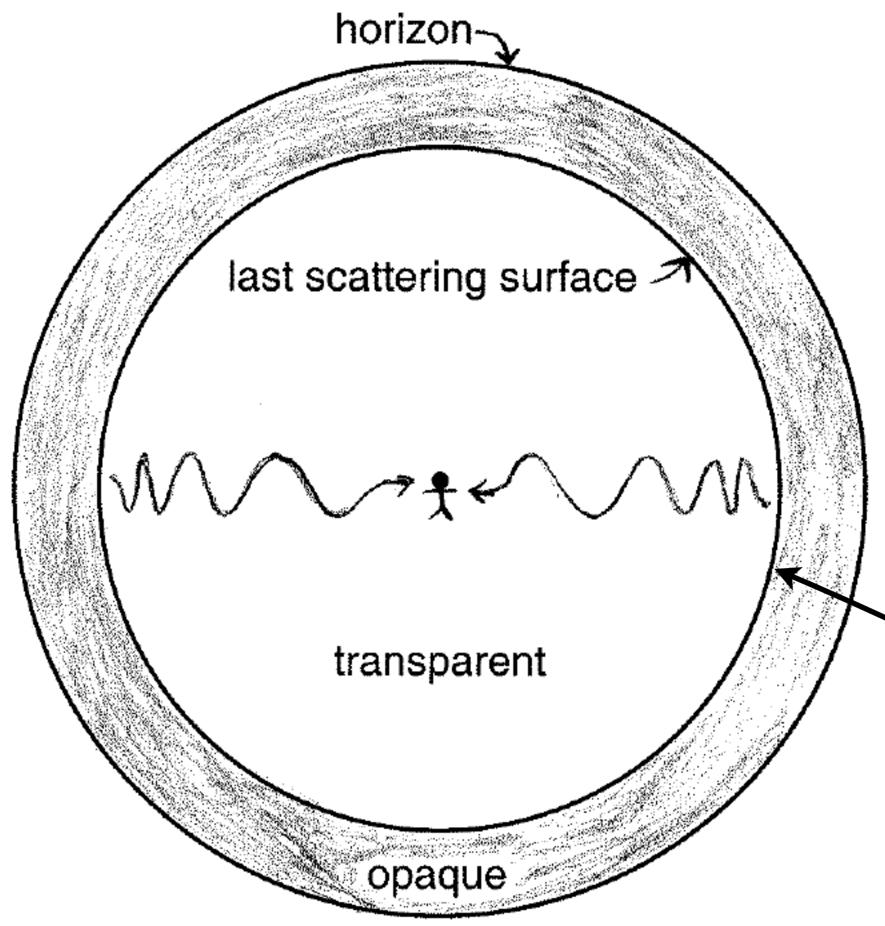

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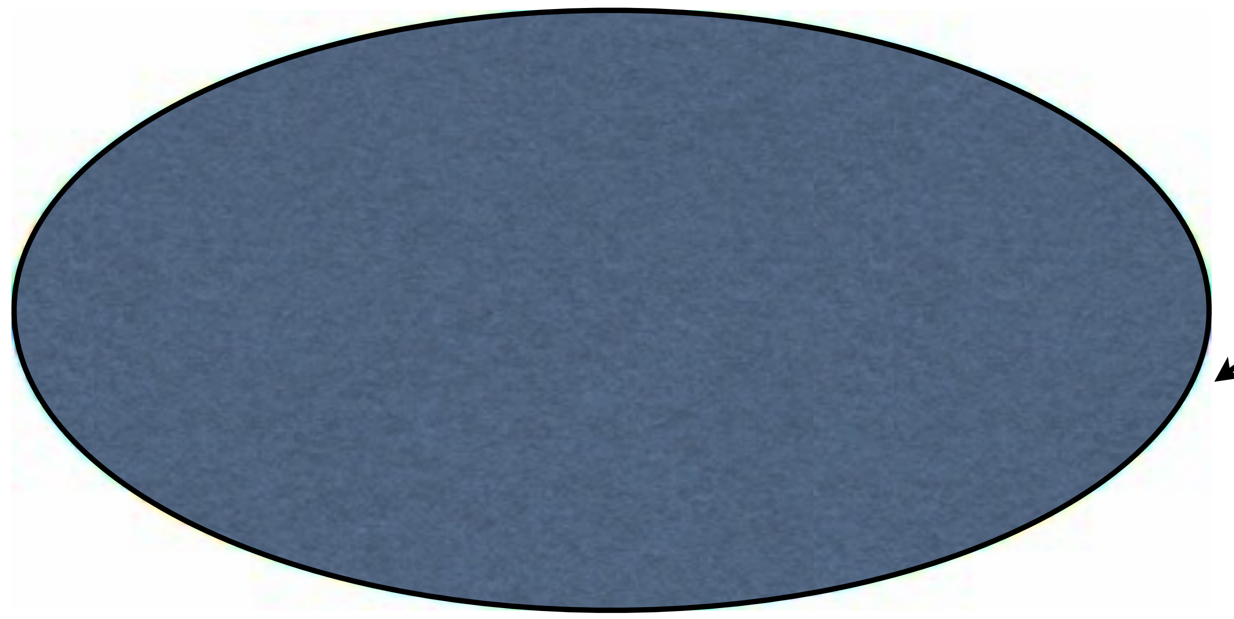
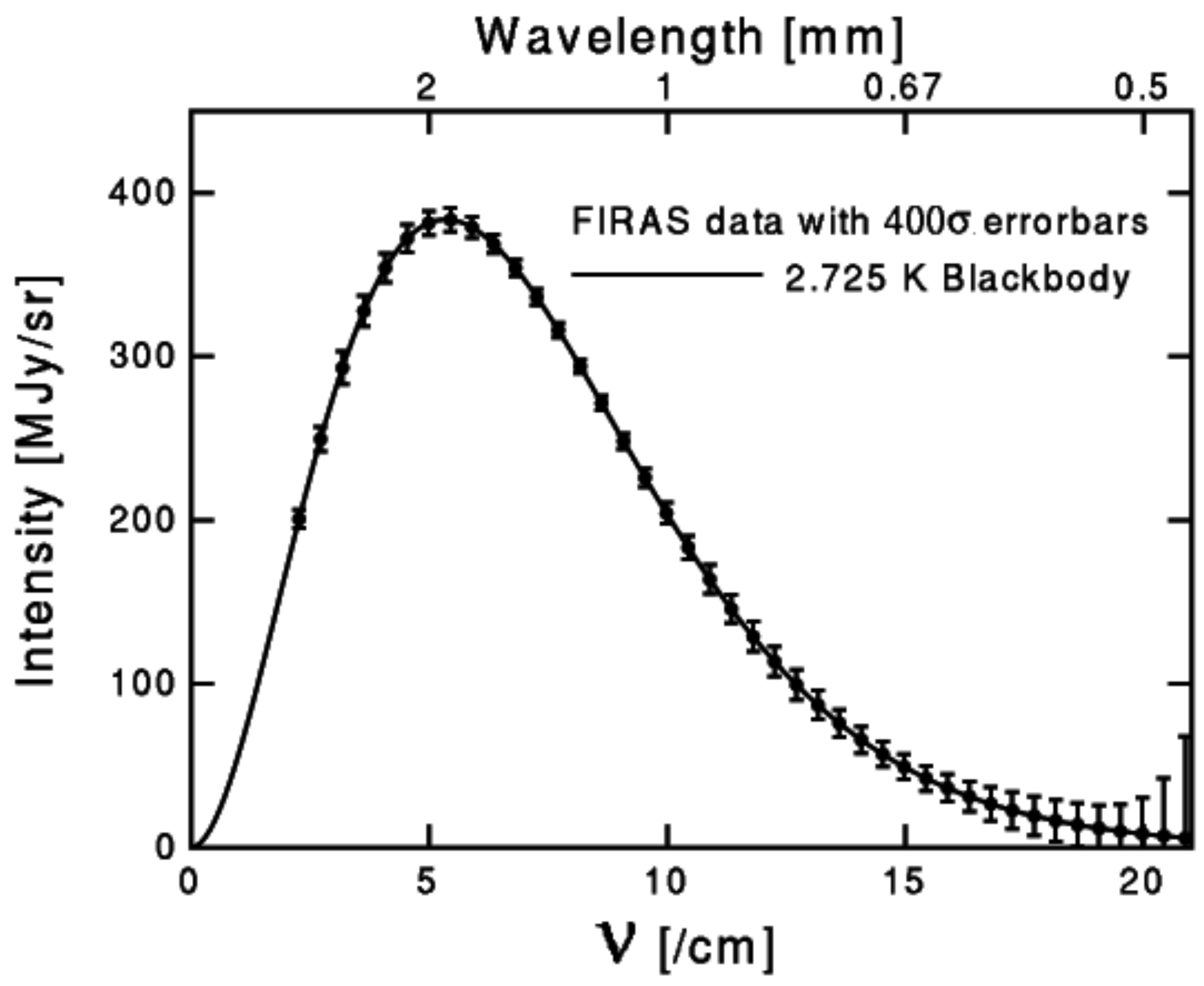
monopole



Same temperature all over the sky

($T^2 t$ = constant)



Soon after the discovery of the CMB in the 1960s, it was realized that its temperature couldn't be *perfectly* uniform. A decades-long hunt for fluctuations was launched.

First, there should be some dipole term owing to our peculiar motion with respect the the CMB frame.

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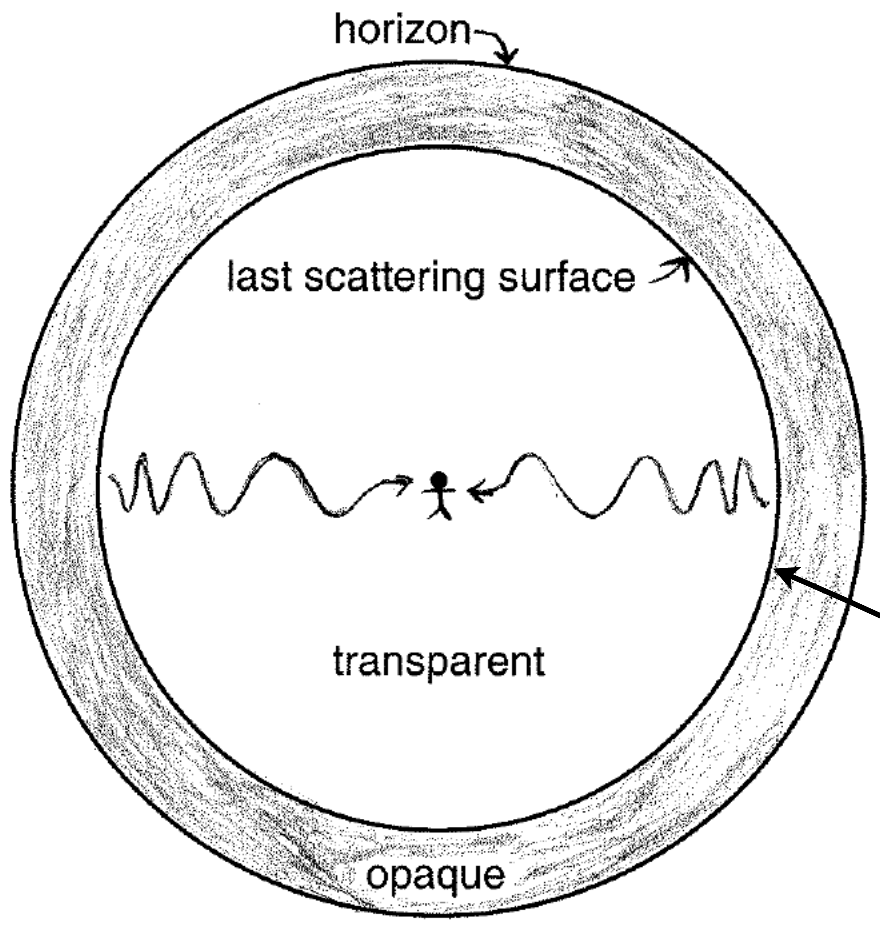

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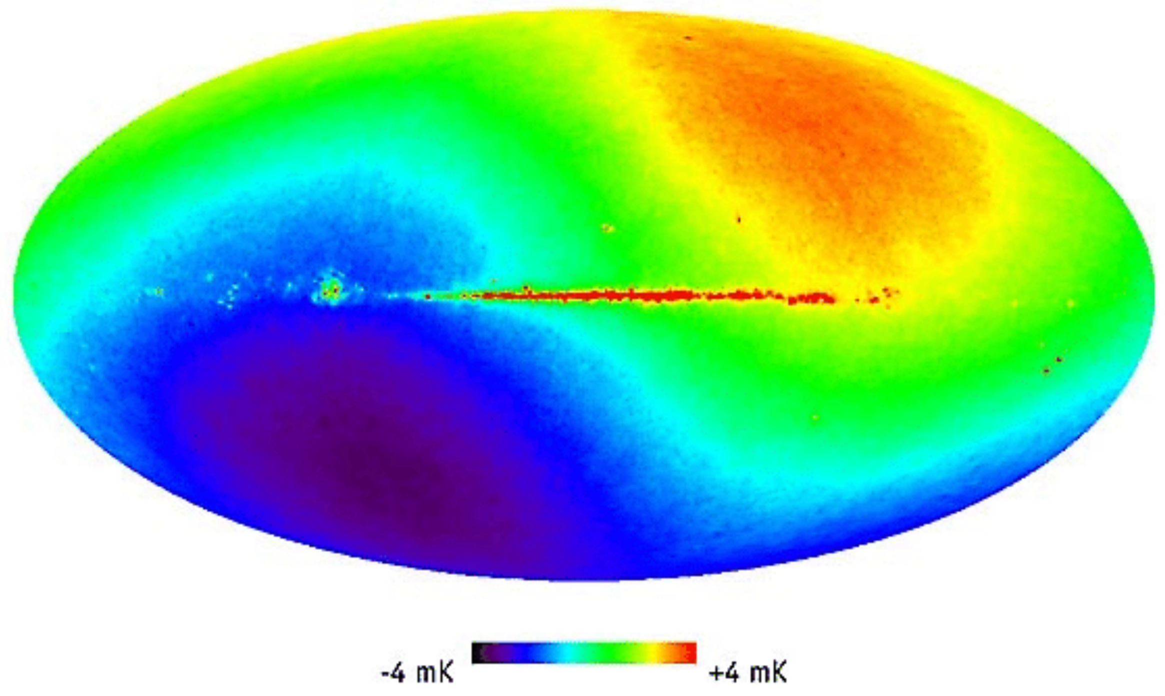
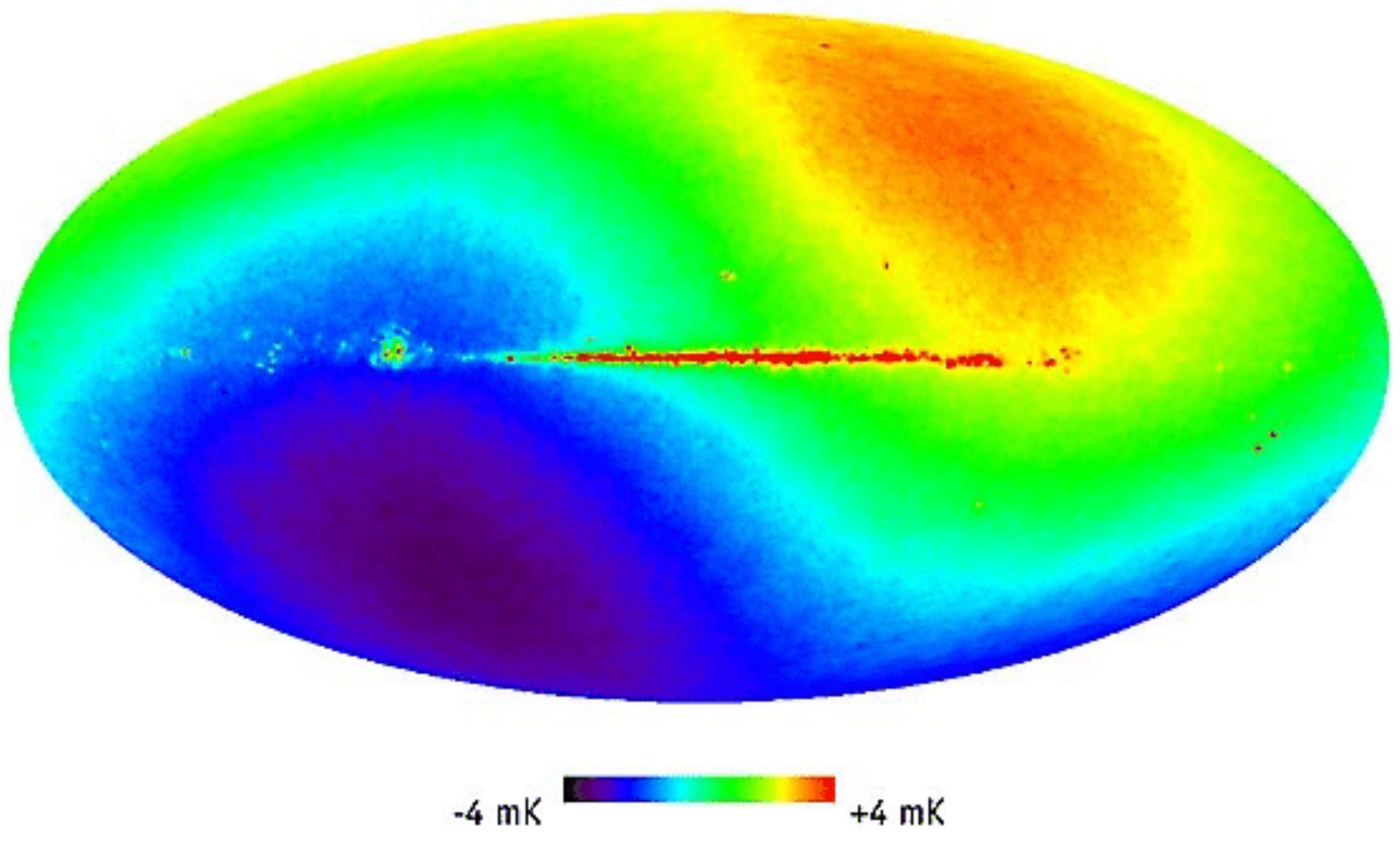
dipole



We are moving wrt the CMB frame

Dipole

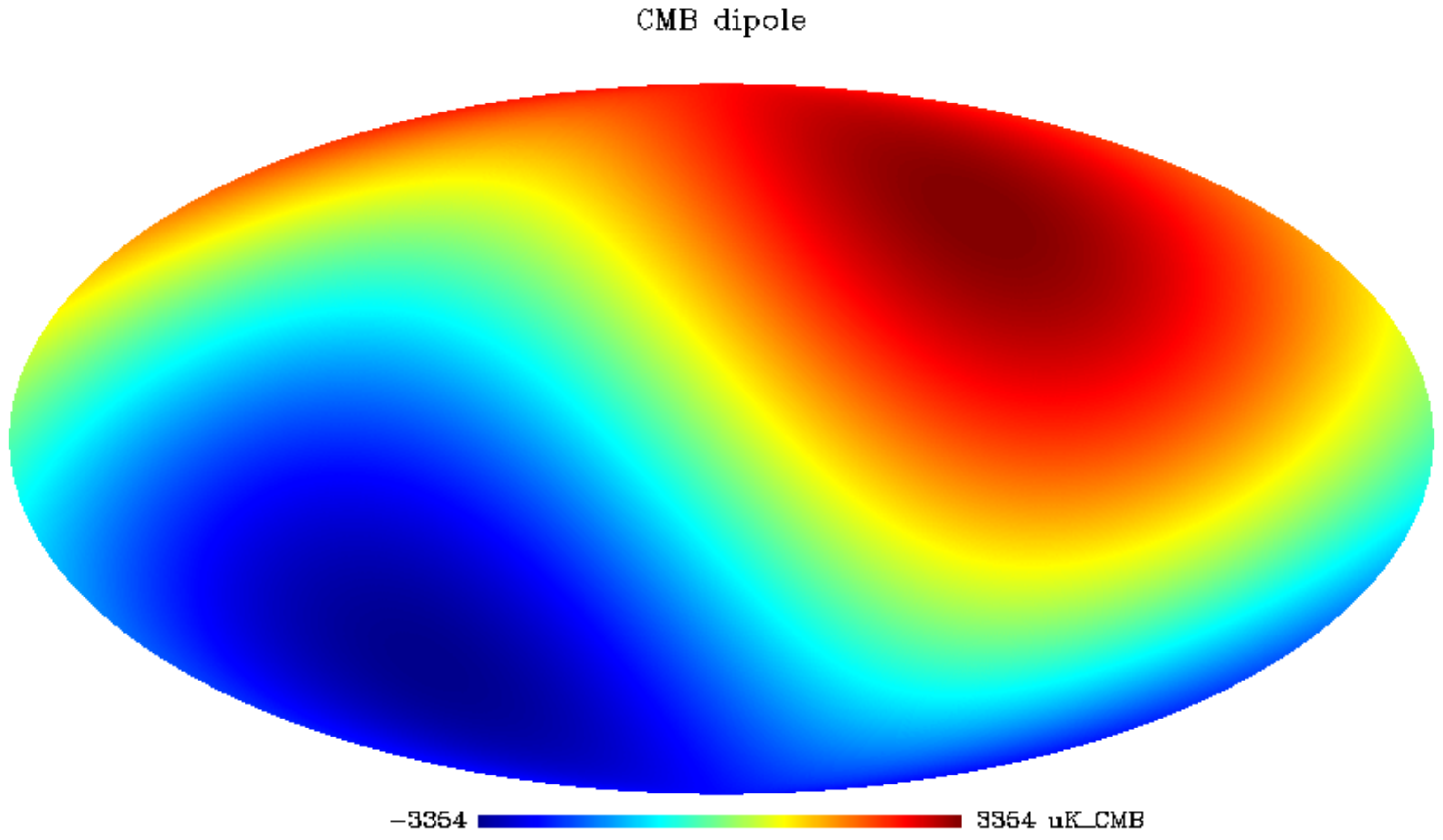


Soon after the discovery of the CMB in the 1960s, it was realized that its temperature couldn't be *perfectly* uniform. A decades-long hunt for fluctuations was launched.

Dipole discovered at the milli-Kelvin scale in the '70s.

For a brief, jaded history of the discovery of the dipole, see http://www.astro.ucla.edu/~wright/CMB-dipole-history.html

Cleaned of foregrounds and smoothed



We are moving wrt the CMB frame

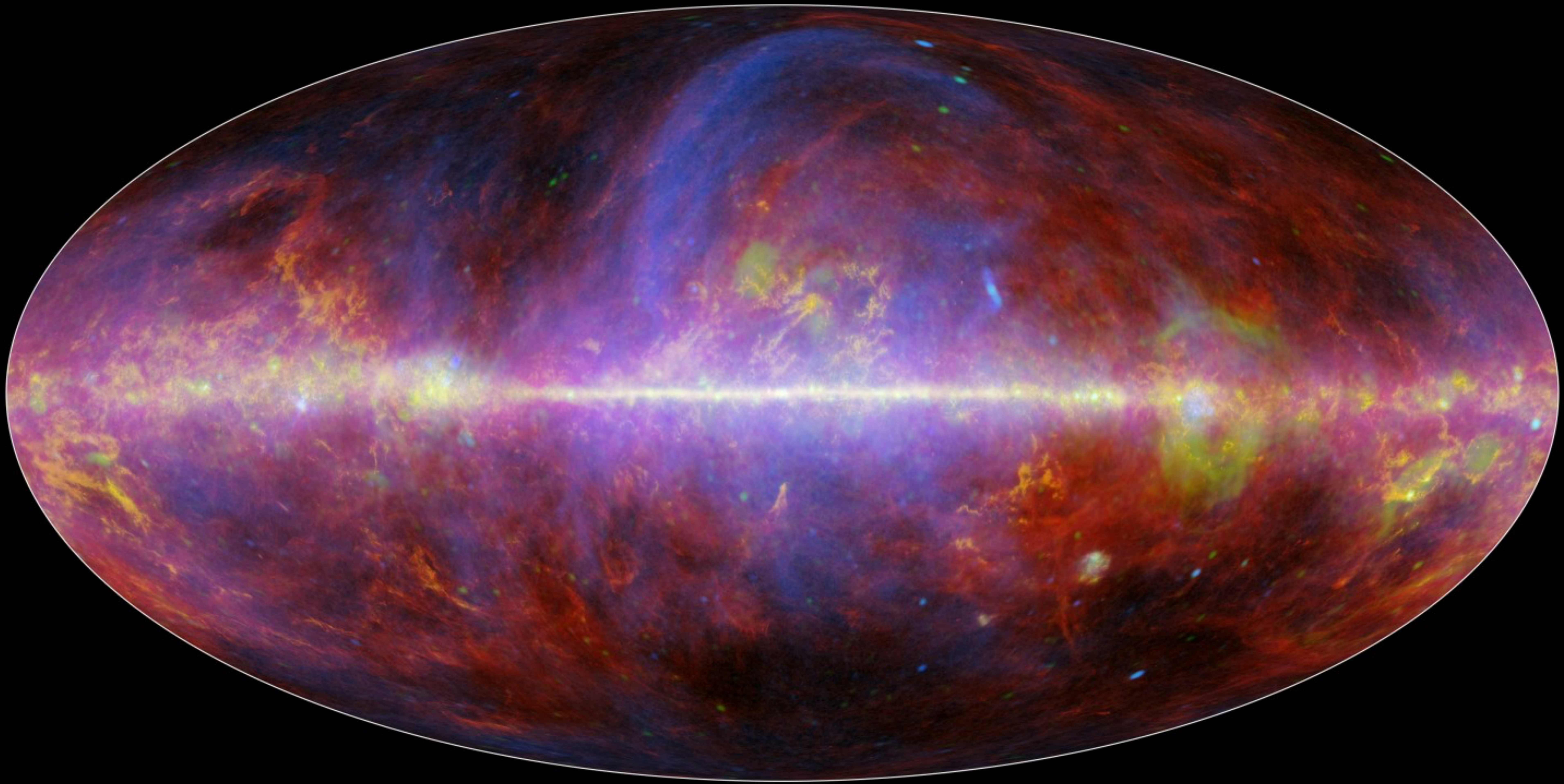
3.358 mK amplitude towards $(\ell, b) = (264^\circ, +48^\circ)$

In the frame of the Local Group,

$V_{LG} = 627 \pm 22\ \mathrm{km\,s^{-1}}$ towards $(\ell, b) = (276^\circ, +30^\circ)$

That is a disturbingly large peculiar velocity for the Local Group.

The radio sky has many foreground sources, mostly from our Galaxy, obscuring the cosmic background radiation



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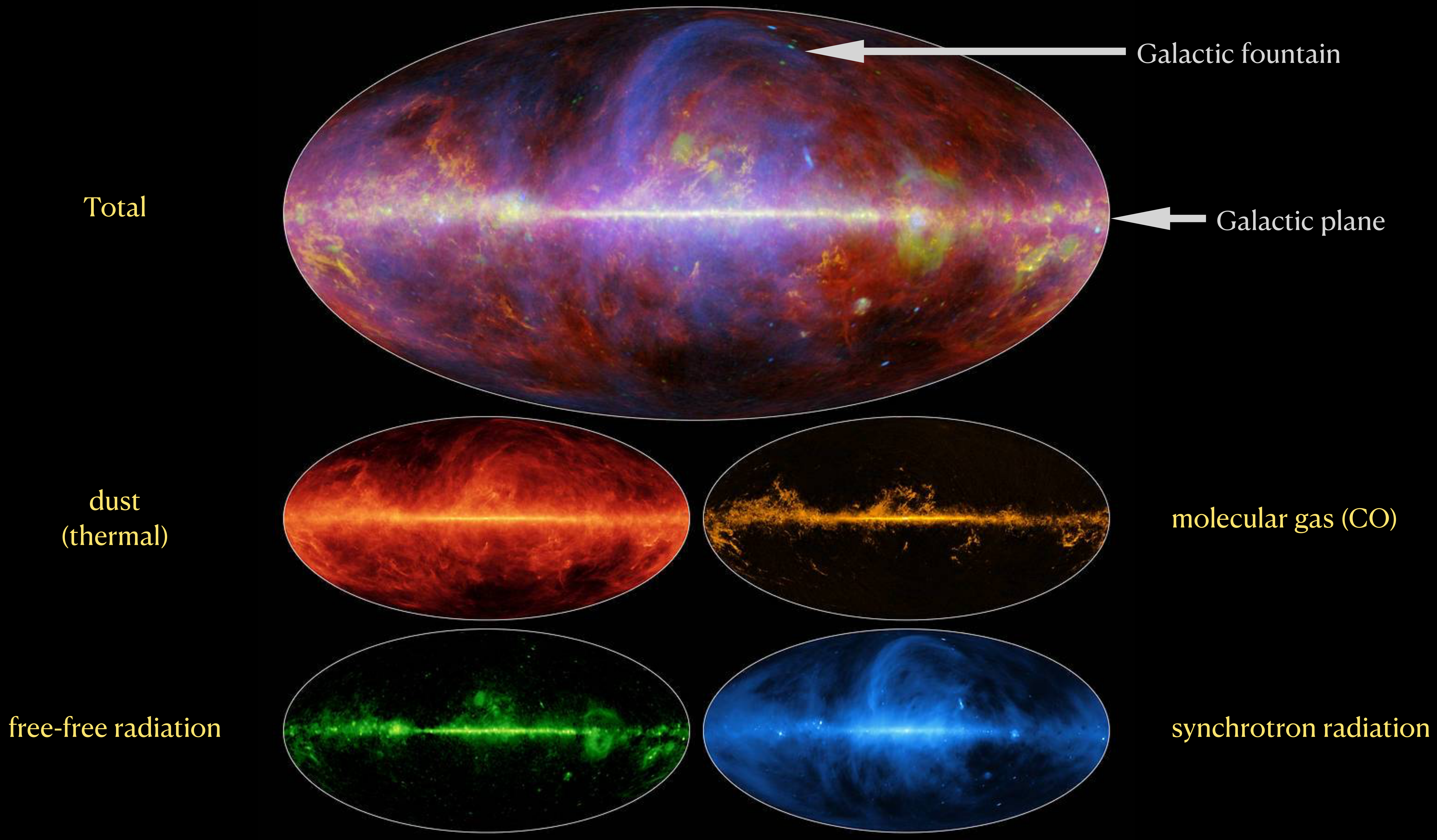
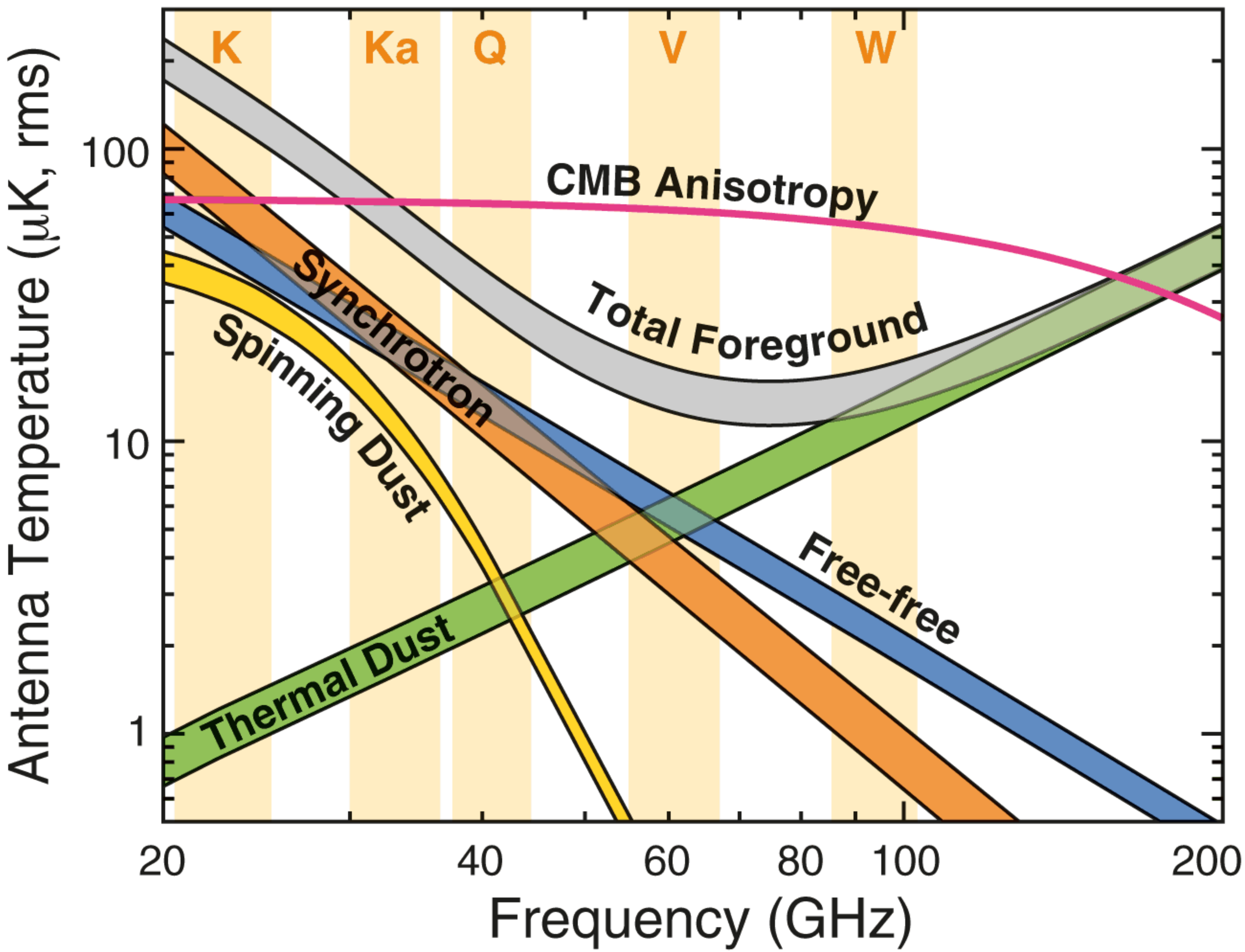
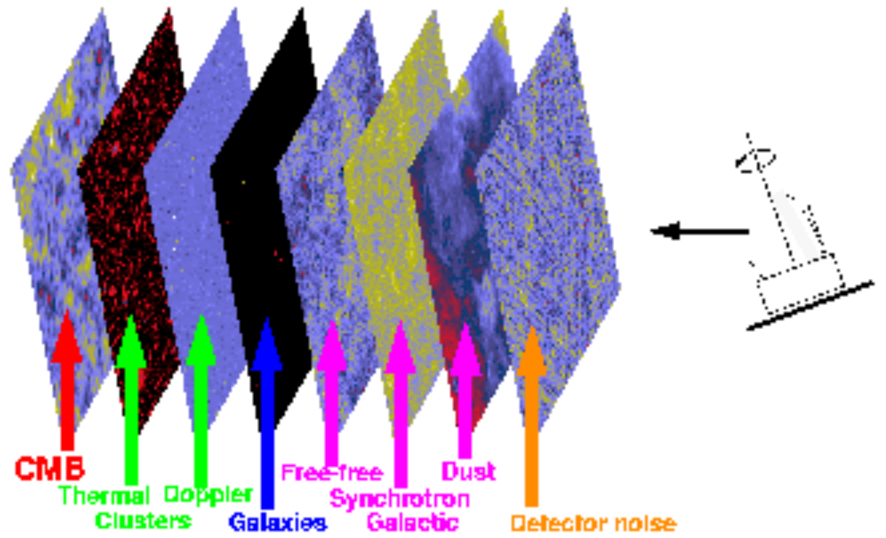


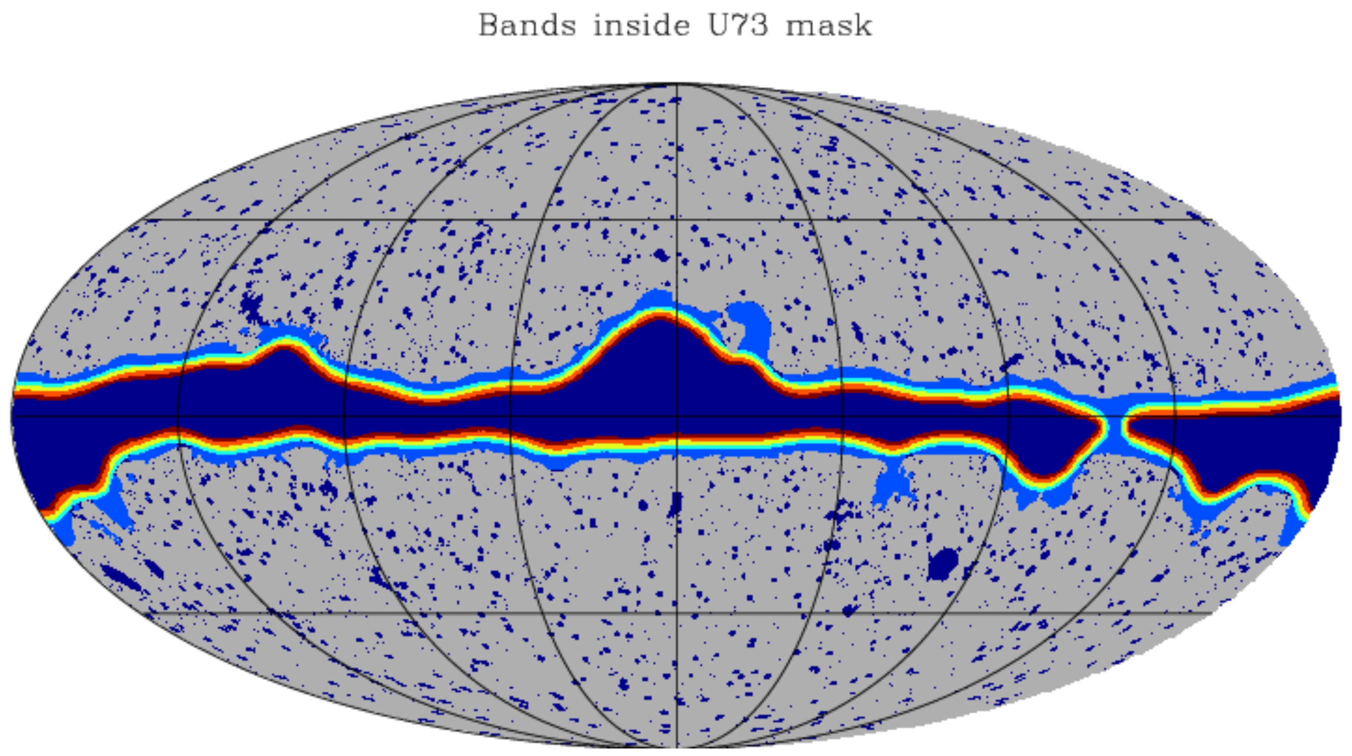


Foregrounds have different frequency dependence, so can be subtracted with multi-band data...



maps at various frequencies

Or, if too strong, masked & ignored.



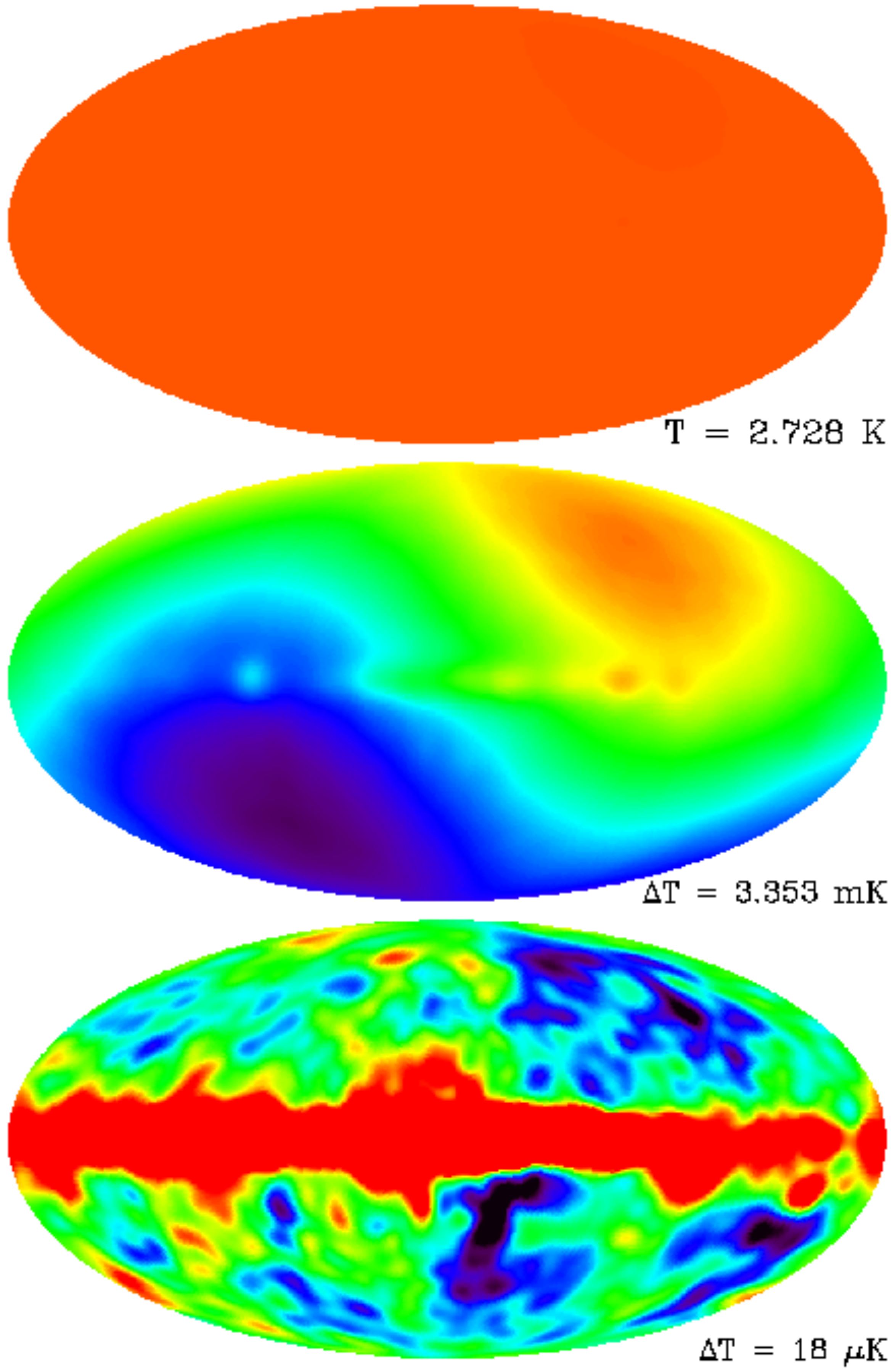
point sources are AGN & SZ clusters

BICEP *cautionary tale.*

Foreground subtracted Planck map

Gain cranked up to show minute fluctuations

Quadrupole



Monopole

Dipole

Quadrupole

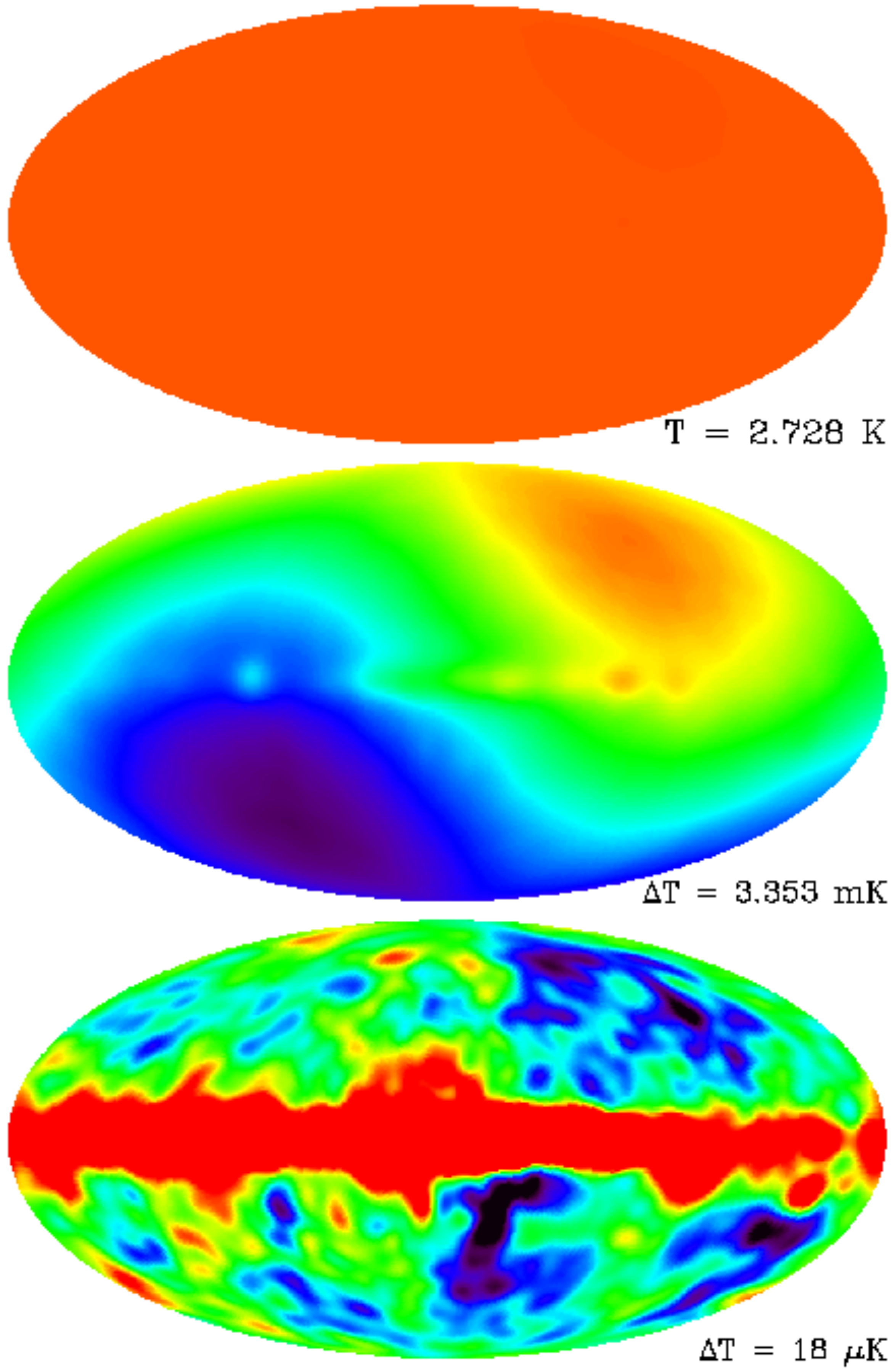
Soon after the discovery of the CMB in the 1960s, it was realized that its temperature couldn't be *perfectly* uniform. A decades-long hunt for fluctuations was launched.

Finding anything beyond the dipole was harder than expected.

Quadrupole finally discovered at the ~10 micro-Kelvin scale by COBE/FIRAS satellite in 1992. (Earned 2006 Physics Nobel prize for John Mather & George Smoot).

$$\frac{\Delta T}{T} = \frac{1.8 \times 10^{-5}\ \mathrm{K}}{2.725\ \mathrm{K}} \approx 10^{-5}$$ observed

Quadrupole



Monopole

Dipole

Quadrupole

Structure formation basics:

Expect density perturbations $\delta = \dfrac{\rho - \langle \rho \rangle}{\langle \rho \rangle}$

to grow as $\delta(t) \sim a(t)$.

At $z = 0$, we observe $\delta \approx 1$ on scales of 8 Mpc.
So at $z = 1000$ we expect $\delta \approx 10^{-3}$
Instead, we observe $\delta \approx 10^{-5}$ —
off by a factor of 100!

This is a prime motivation for non-baryonic **cold dark matter** — a substance for which perturbations δ can grow sufficiently large while not leaving an imprint of corresponding magnitude on the CMB.

$$\frac{\Delta T}{T} = \frac{1.8 \times 10^{-5}\ \mathrm{K}}{2.725\ \mathrm{K}} \approx 10^{-5} \qquad \text{observed}$$

Radiation and baryon plasma tightly coupled at recombination, so a fluctuation in density is reflected by one in temperature.

Etc. pole

$\ell = 1,2,3$

$\ell = 4,5,6$

$\ell = 7,8,9$

$\ell = 10,11,12$

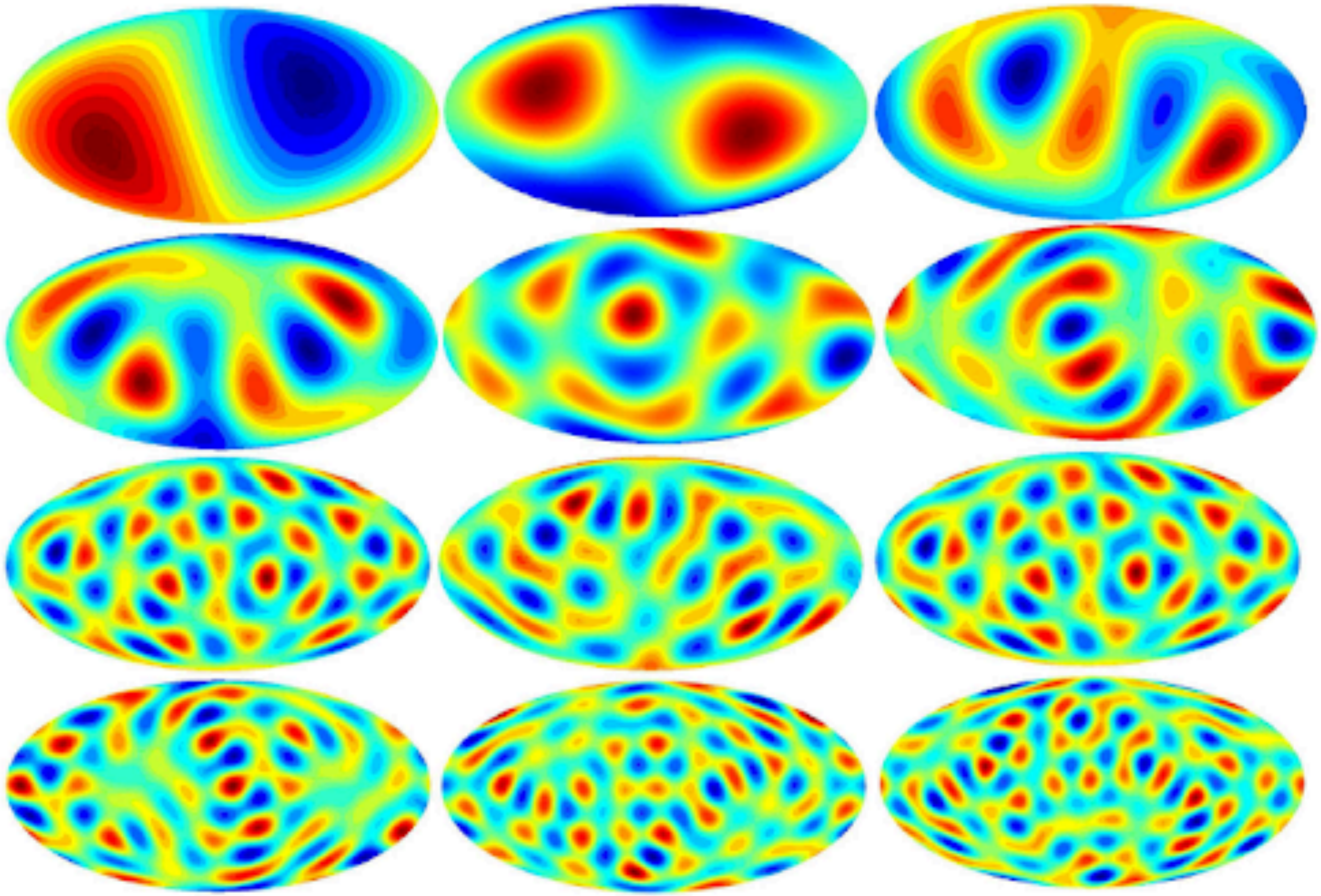
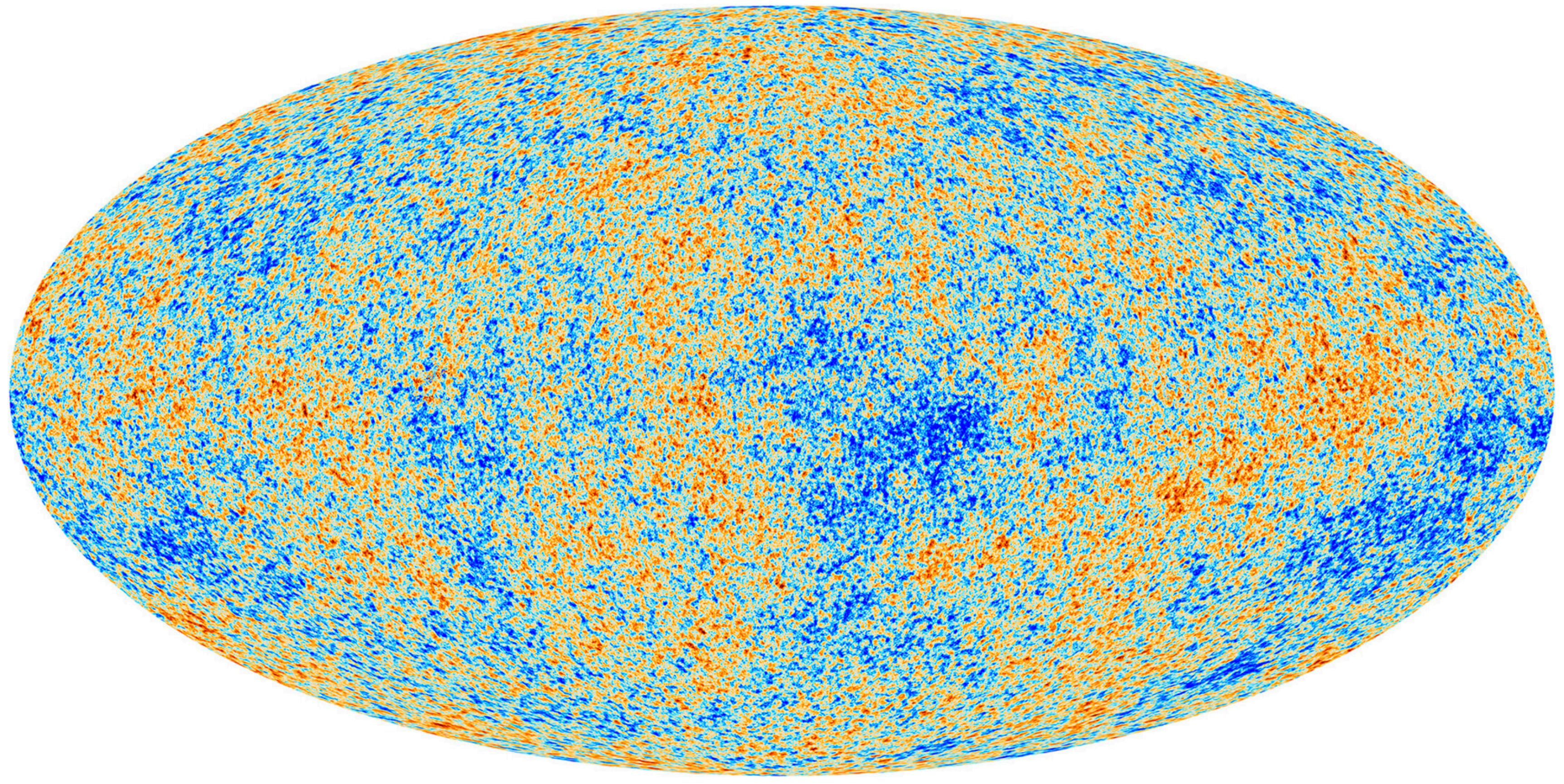


Figure 6: Randomly generated skies containing only a single multipole ℓ. Staring from top left: $\ell = 1$ (dipole only), 2 (quadrupole only), 3 (octupole only), 4, 5, 6, 7, 8, 9, 10, 11, 12. Figure by Ville Heikkilä.



Spherical harmonics provide a convenient way to decompose the fluctuations observed on the sky

$$\frac{\Delta T}{T}(\theta, \phi) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} A_{\ell m} Y_{\ell m}$$

with Fourier transform

$$A_{\ell m} = \int_{\text{sky}} \frac{\Delta T}{T}(\theta, \phi) Y^*_{\ell m} d\Omega$$

giving the power in fluctuations on an angular scale $\frac{\pi}{\ell}$

$$C_\ell = \frac{1}{2\ell + 1} \sum_m A_{\ell m} A^*_{\ell m} = \langle |A_{\ell m}|^2 \rangle$$

note: $1^\circ = 0.0175$ radians so one degree corresponds to $\frac{\pi}{\ell} = 0.0175$, hence $\ell = 180$.

Multipole ℓ varies inversely with angular scale.