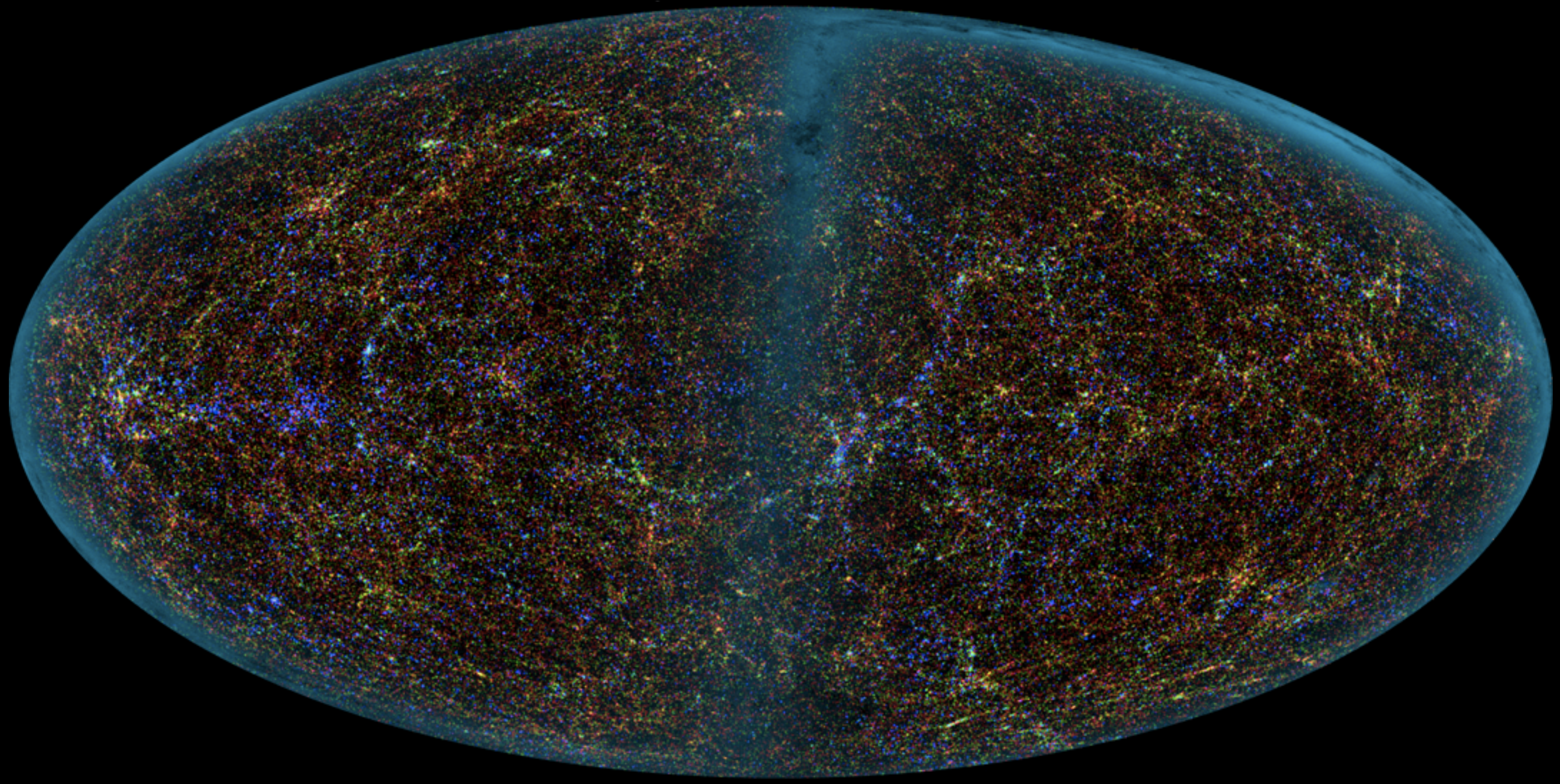
Cosmology

and Large Scale Structure



Today

Empirical Pillars
of the Hot Big Bang

A little more CMB
Nucleosynthesis

Homework 4 due one week from today

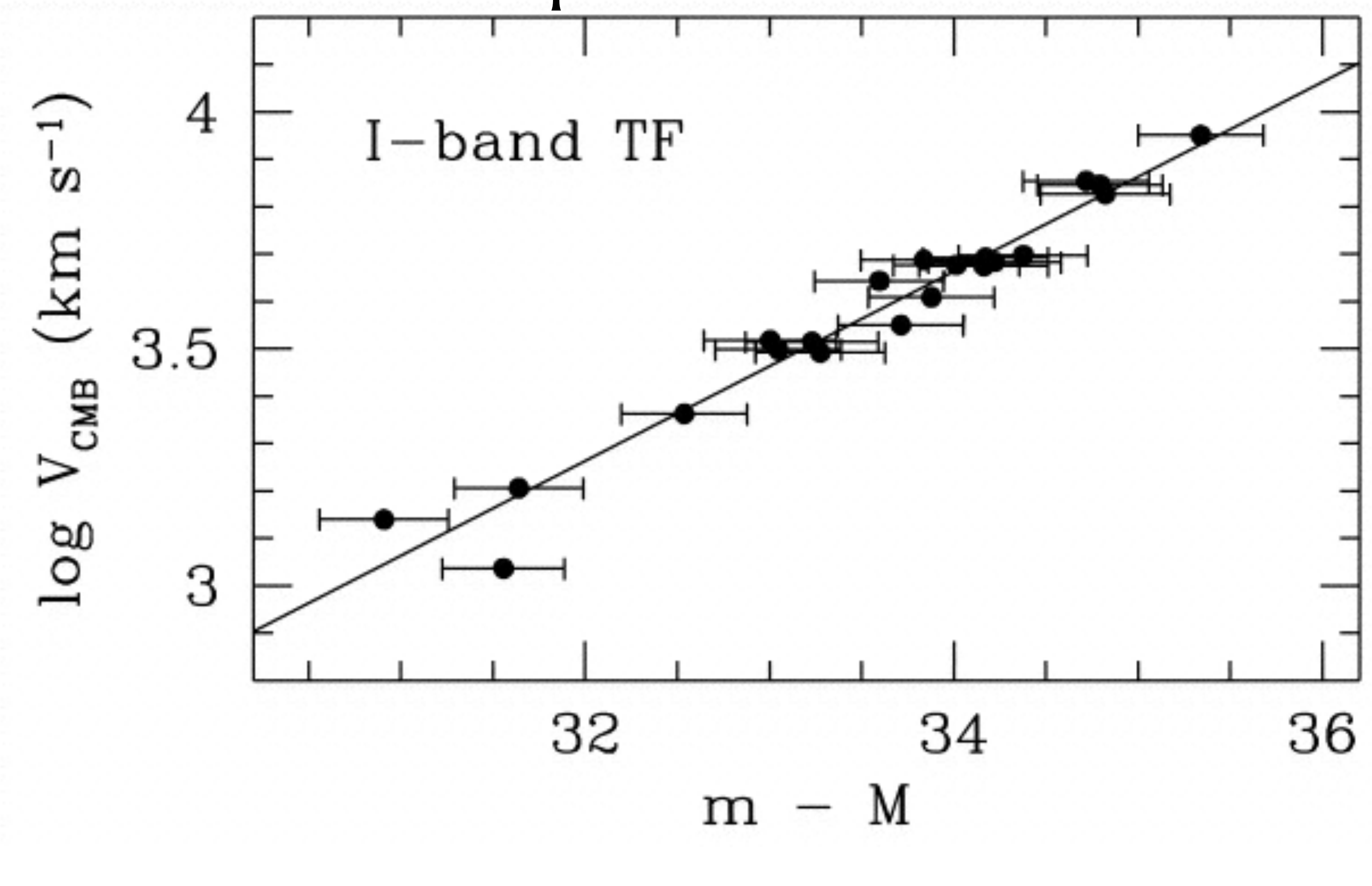
3 November 2022

http://astroweb.case.edu/ssm/ASTR328/

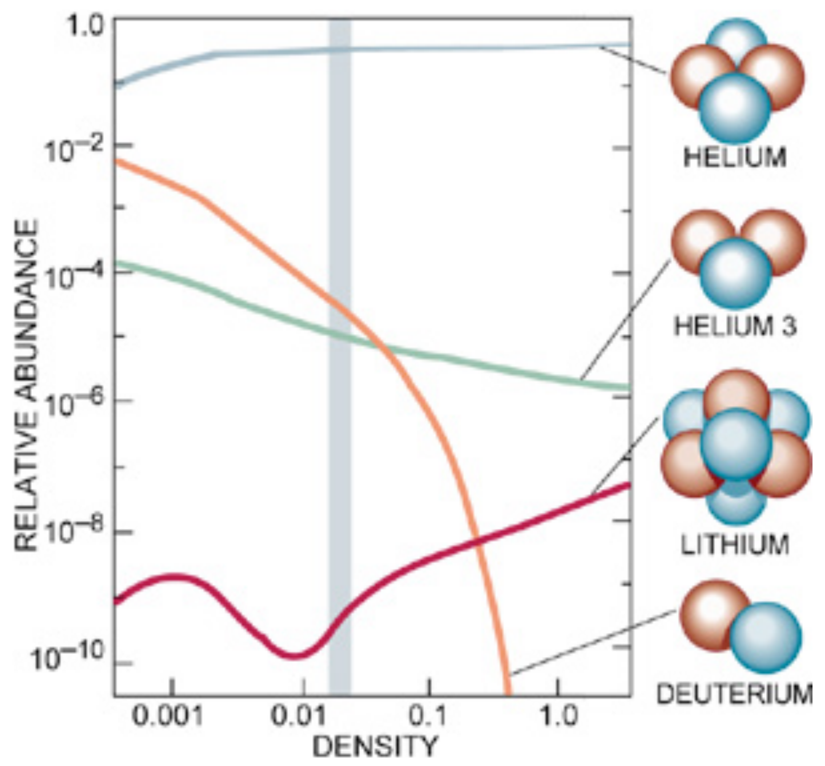
Empirical Pillars of the Hot Big Bang

1. Hubble Expansion — Hubble (1930)
2. Big Bang Nucleosynthesis (BBN) — Alpher, [Bethe], & Gamow (1948) — $\alpha\beta\gamma$ paper
3. Cosmic Microwave Background (CMB) — Penzias & Wilson; Peebles & Dicke (1965)

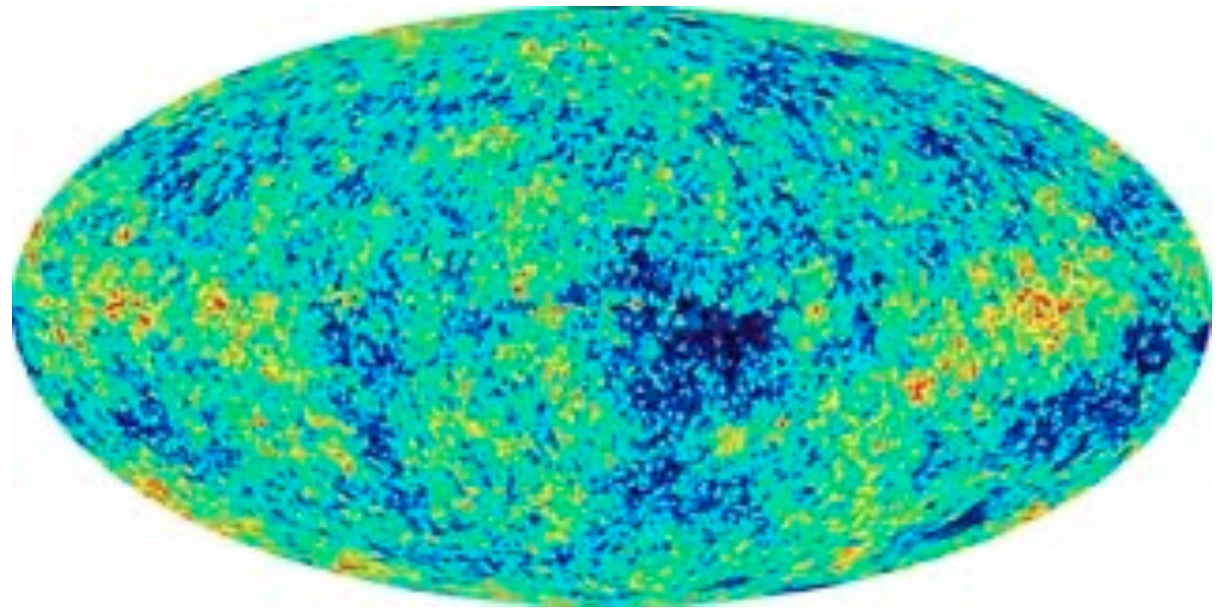
Hubble expansion



BBN
t ~ 3 minutes



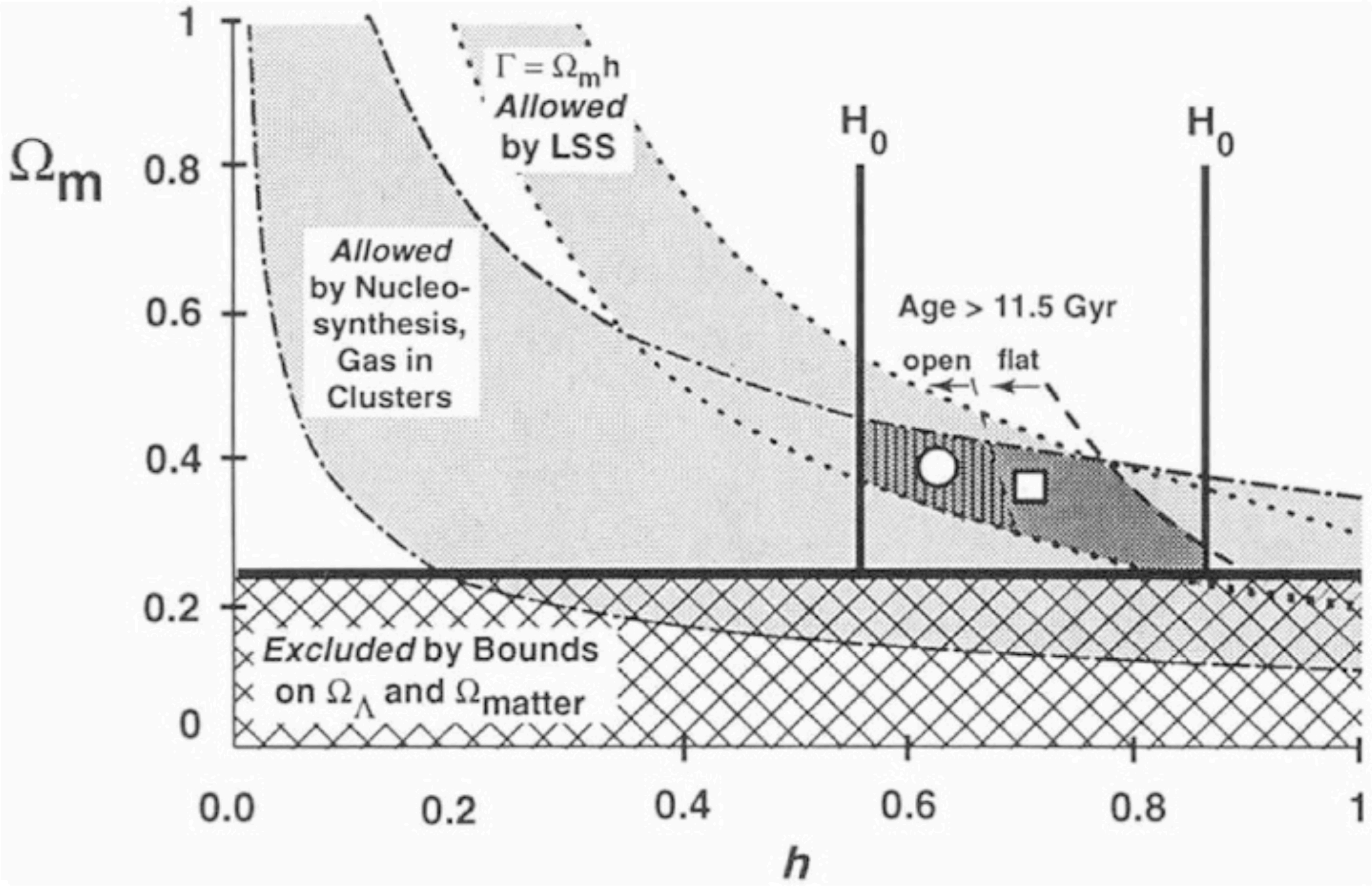
CMB
z = 1090
t = 380,000 yr



How we got LCDM

as it applies to the first problem of Homework 4

Fig. 1 of Ostriker & Steinhardt (1995, Nature, 377, 600)



In addition to the 3 empirical pillars, we now have 2 auxiliary hypotheses

- Dark matter
- Dark Energy

Each of the given constraints

- $t_0 = 13.5 \pm 0.15$ (stat) ± 0.5 (sys) Gyr
- $H_0 = 75.1 \pm 0.2$ (stat) ± 3 (sys) km/s/Mpc

trace out a locus of allowed values in this diagram.
Find where they intersect.

For a brief history, see https://tritonstation.com/2019/01/28/a-personal-recollection-of-how-we-learned-to-stop-worrying-and-love-the-lambda/

How we got LCDM

as it applies to the first problem of Homework 4

Fig. 2 of Ostriker & Steinhardt (1995, Nature, 377, 600)

In addition to the 3 empirical pillars, we now have 2 auxiliary hypotheses

- Dark matter
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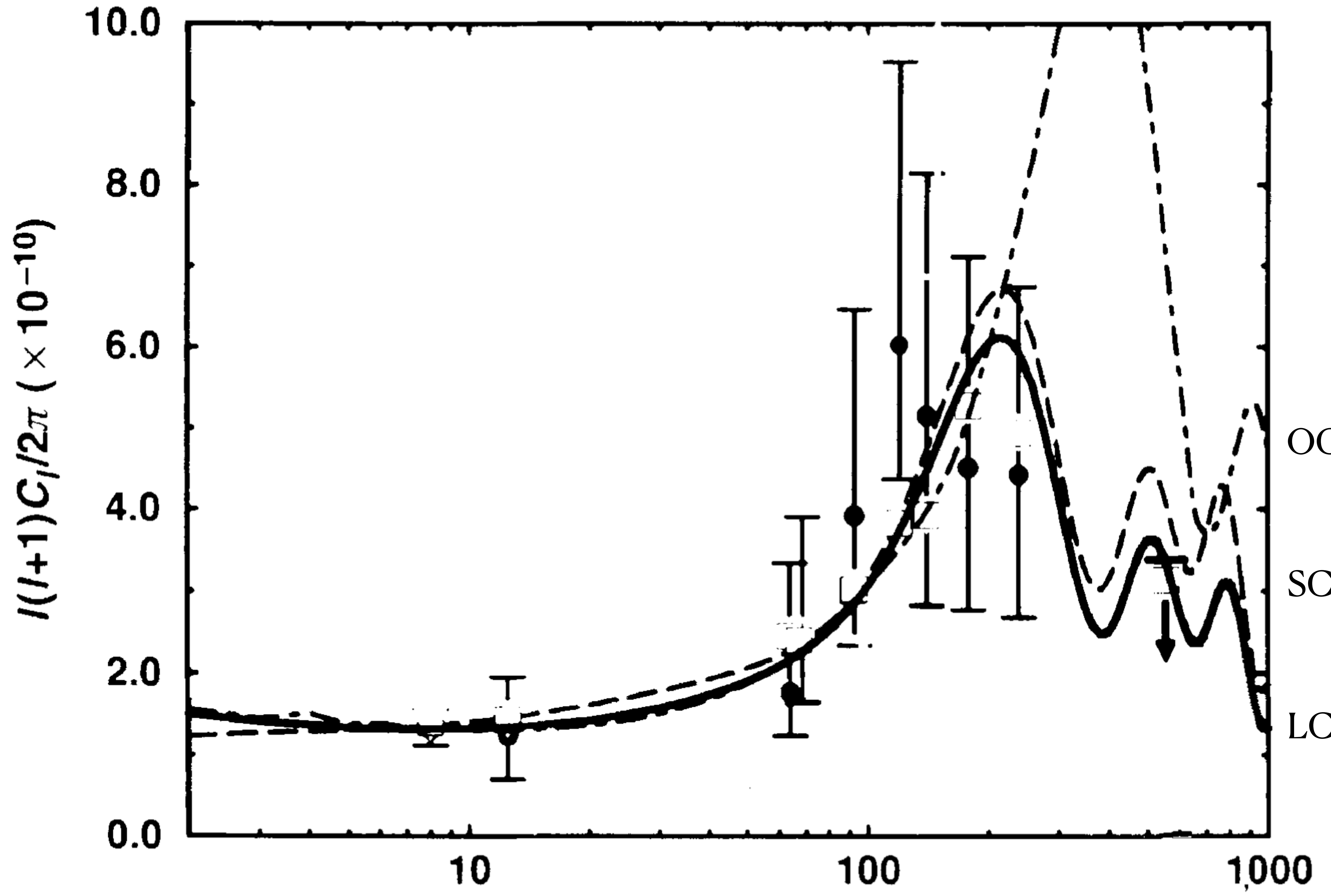


Early prediction for the acoustic power spectrum of the cosmic microwave background.

OCDM: $\Omega_{m_0} = 0.375;\ H_0 = 62.5$ (dash-dot line)

SCDM: $\Omega_{m_0} = 1.0;\ H_0 = 50$ (dashed line)

LCDM: $\Omega_{m_0} = 0.35;\ H_0 = 65;\ \Omega_{\Lambda_0} = 0.65$ (solid line)

For a short history, see https://tritonstation.com/2019/01/28/a-personal-recollection-of-how-we-learned-to-stop-worrying-and-love-the-lambda/

Etc. pole

$\ell = 1,2,3$

$\ell = 4,5,6$

$\ell = 7,8,9$

$\ell = 10,11,12$

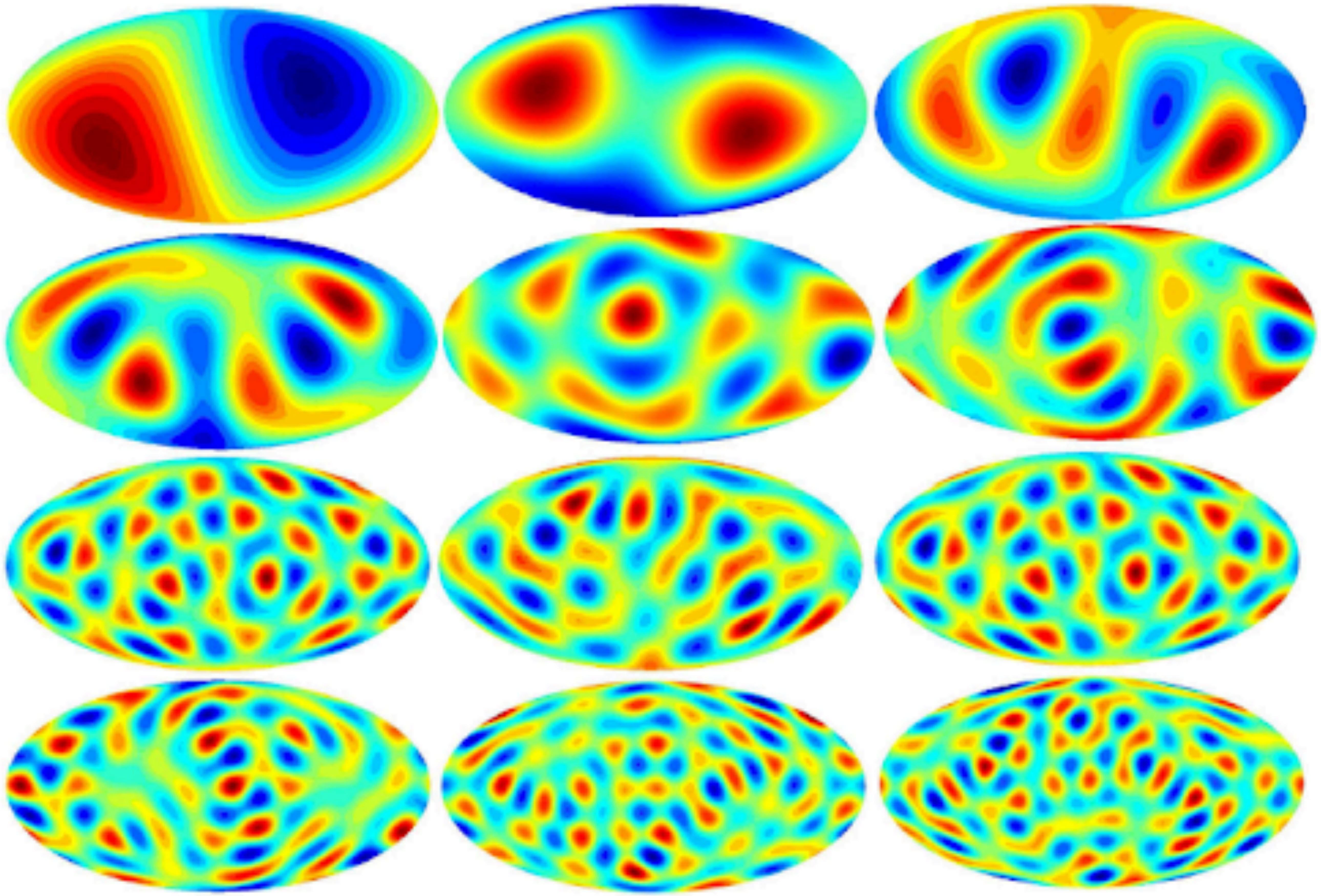
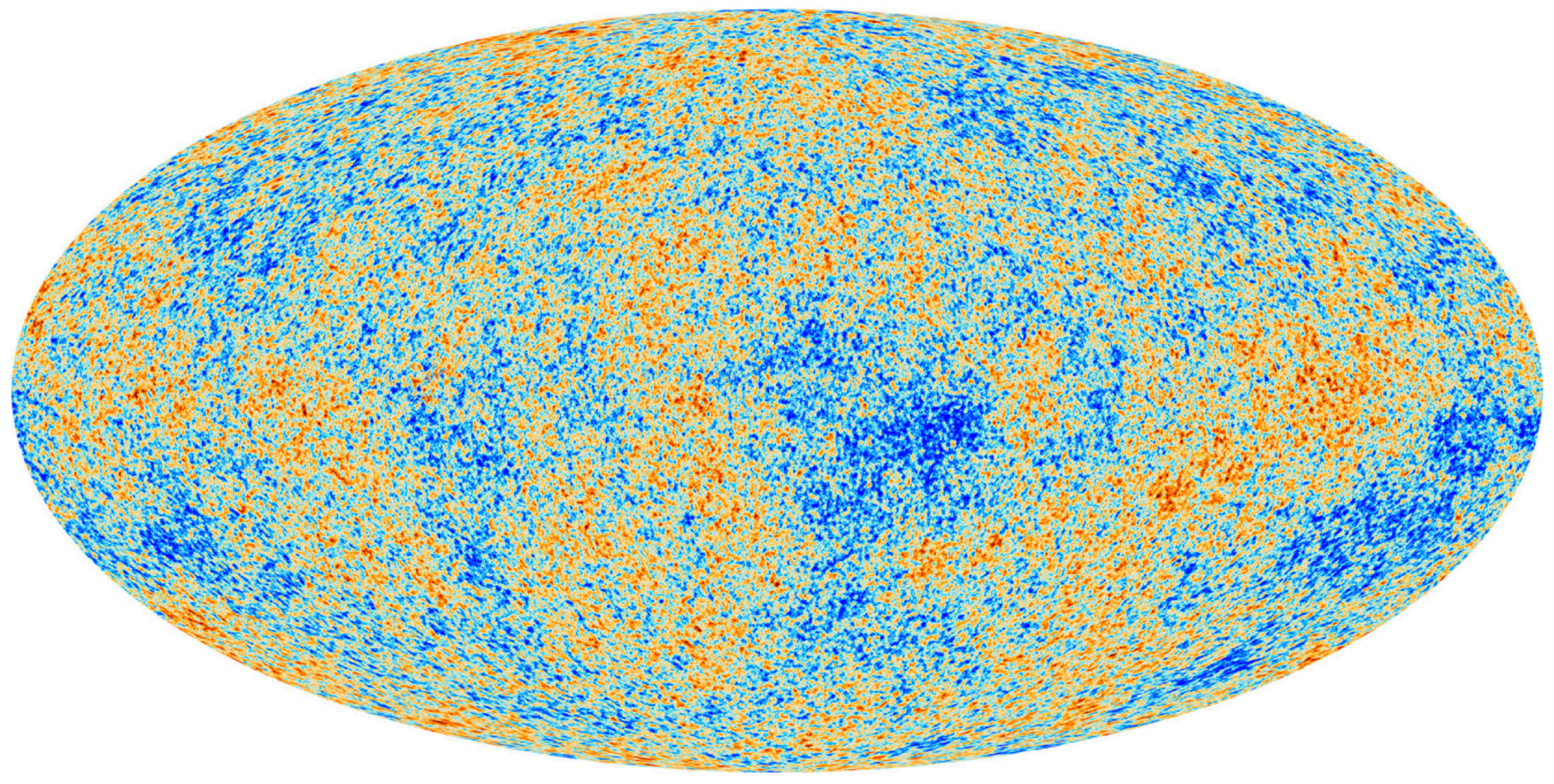


Figure 6: Randomly generated skies containing only a single multipole ℓ. Staring from top left: $\ell = 1$ (dipole only), 2 (quadrupole only), 3 (octupole only), 4, 5, 6, 7, 8, 9, 10, 11, 12. Figure by Ville Heikkilä.



Spherical harmonics provide a convenient way to decompose the fluctuations observed on the sky

$$\frac{\Delta T}{T}(\theta, \phi) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} A_{\ell m} Y_{\ell m}$$

with Fourier transform

$$A_{\ell m} = \int_{\text{sky}} \frac{\Delta T}{T}(\theta, \phi) Y^*_{\ell m} d\Omega$$

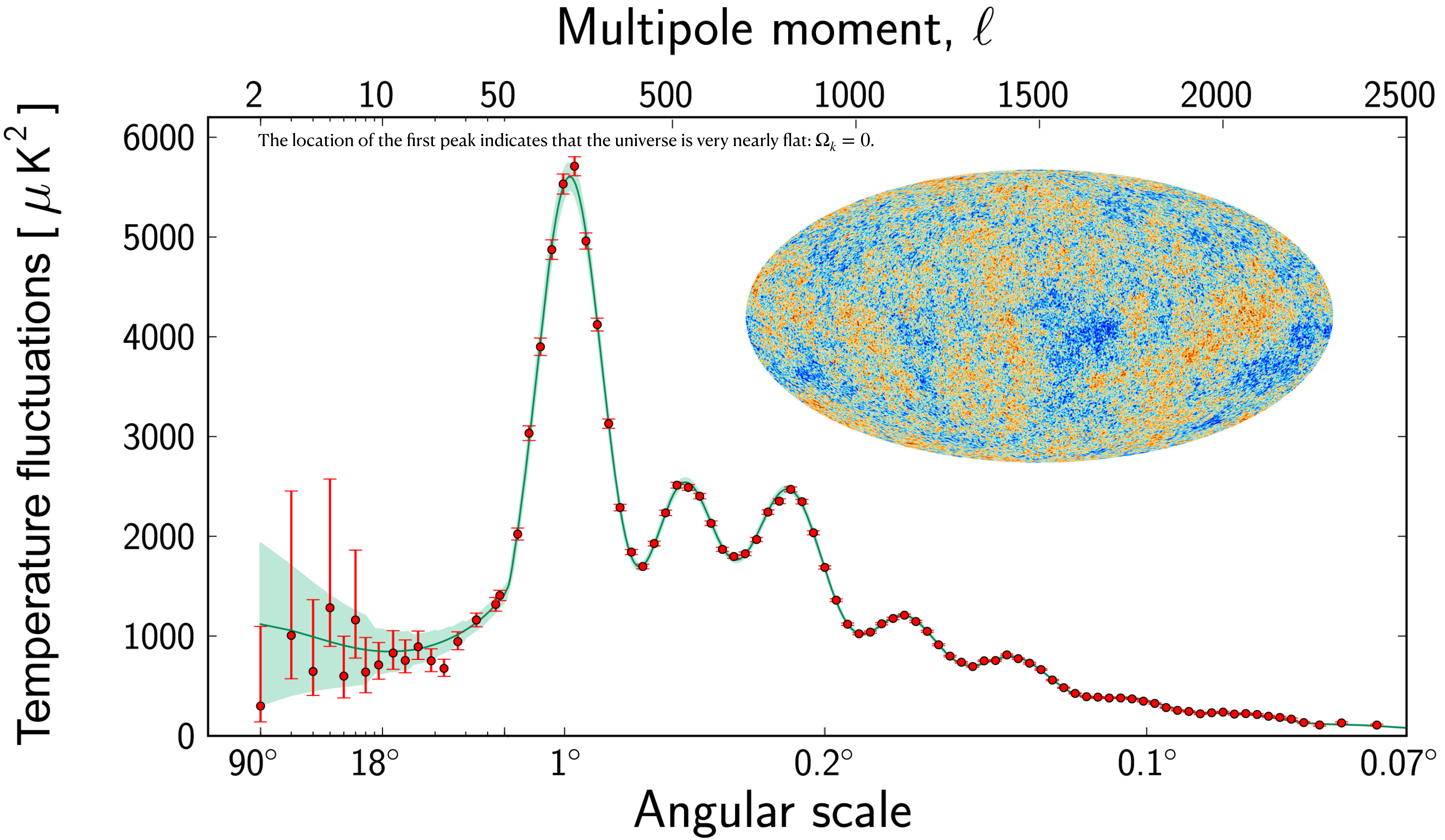
giving the power in fluctuations on an angular scale $\dfrac{\pi}{\ell}$

$$C_\ell = \frac{1}{2\ell + 1} \sum_m A_{\ell m} A^*_{\ell m} = \langle |A_{\ell m}|^2 \rangle$$

note: $1^\circ = 0.0175$ radians so one degree corresponds to $\dfrac{\pi}{\ell} = 0.0175$, hence $\ell = 180$.

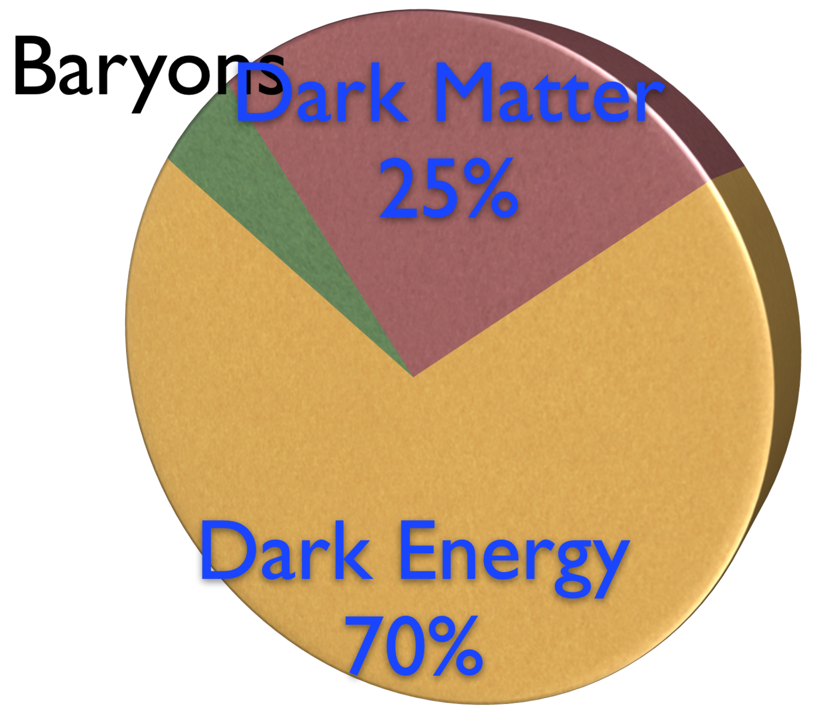
Multipole ℓ varies inversely with angular scale.

The acoustic power spectrum of the CMB is a Fourier transform of temperature fluctuations observed on the sky.
This quantifies the frequency with which structures of a characteristic scale appear.
These correspond to standing waves at the time of last scattering.



Current mass-energy content of the universe

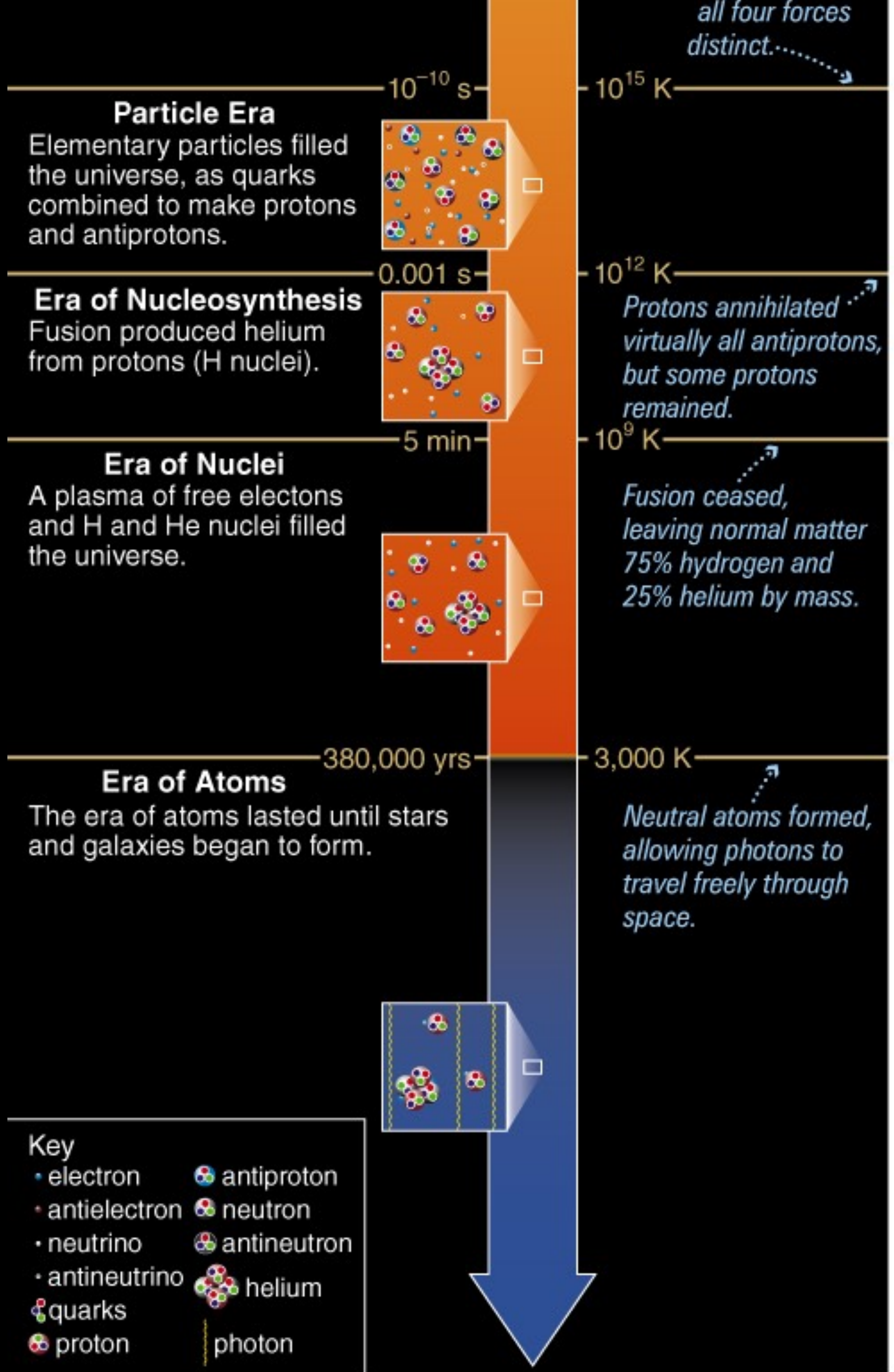
$$\Omega_m = \Omega_b + \Omega_{DM}$$



mass density	Ω_m		0.3	give or take a bit
normal matter		Ω_b	0.05	baryons - known from BBN
mass that is *not* normal matter		Ω_{CDM}	0.25	cold dark matter
cosmic background radiation		Ω_r	5×10^{-5}	photons
			4×10^{-5}	neutrinos
dark energy	Ω_Λ		0.7	energy density of vacuum

$$n_\nu = \frac{9}{11} n_\gamma$$

for 3 neutrino flavors



Early Universe

particle soup

< millisecond

T ~ 10^{14} K

nucleosynthesis (BBN)

~ 3 minutes

T ~ 10^{10} K

BBN occurs during radiation domination

$a(t) \propto t^{1/2}$

time Temp

recombination

~380,000 year

T ~ 3000 K

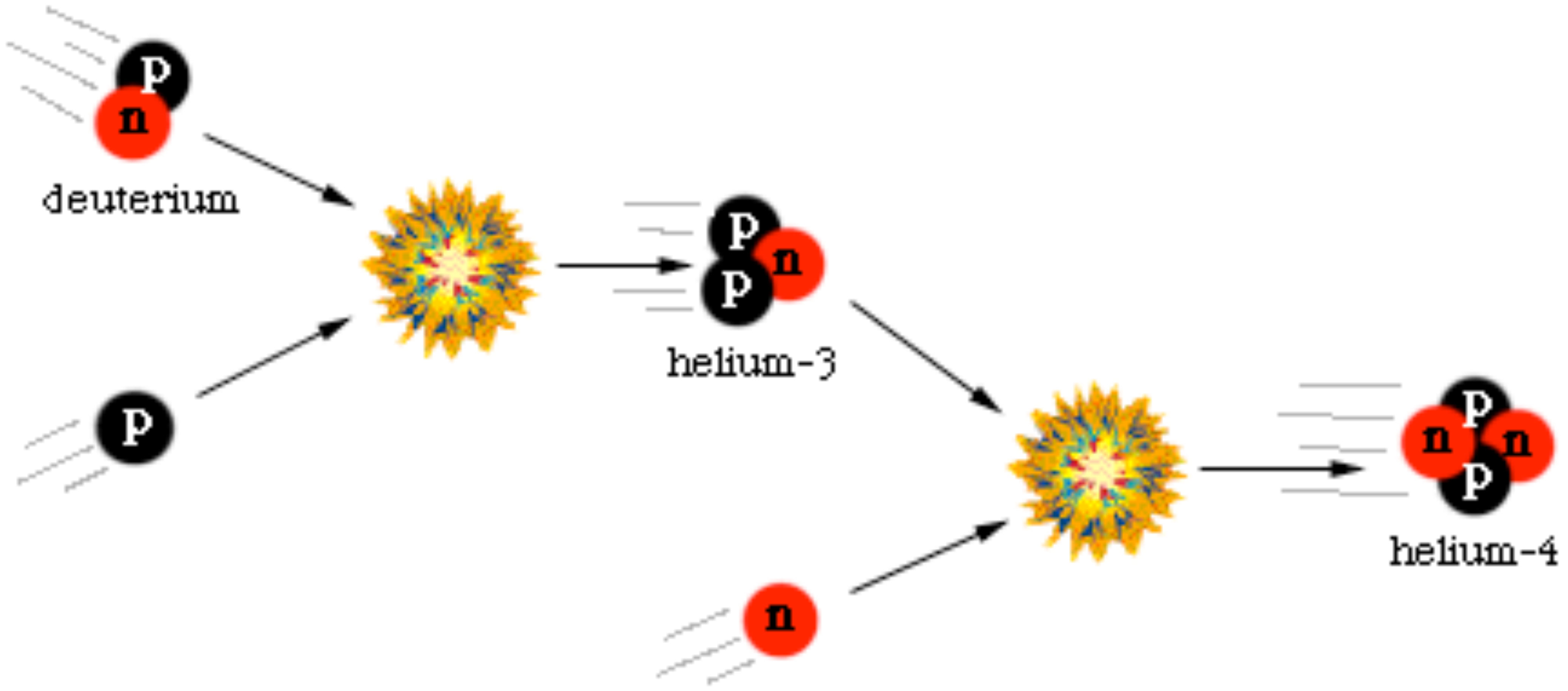
emission of CMB:

surface of last scattering - transition from opaque plasma to transparent neutral gas

Big Bang Nucleosynthesis (BBN)


Gamow

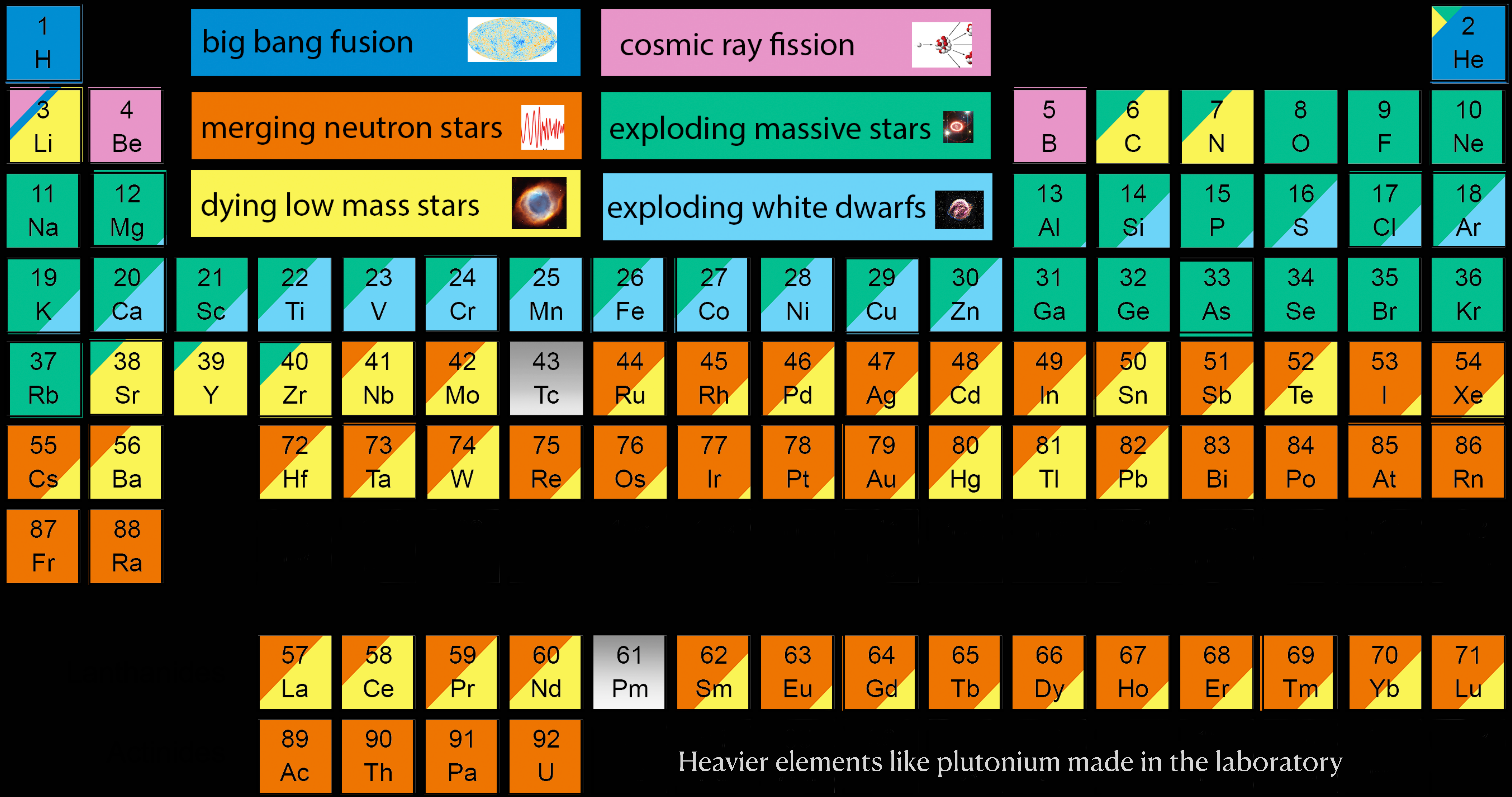
When the universe is just a few minutes old,
the temperature and density are right
for it to be one big nuclear furnace: $T \sim 10^{10}$ K



The light elements Hydrogen, Helium, and Lithium and their isotopes are made at this time.

Alpher & Gamow initially thought that they could make *all* of the elements through neutron capture.
This was wrong; only the light elements are made because of the helium bottleneck.
Heavier elements are made in stars and supernovae.

The Origin of the Solar System Elements

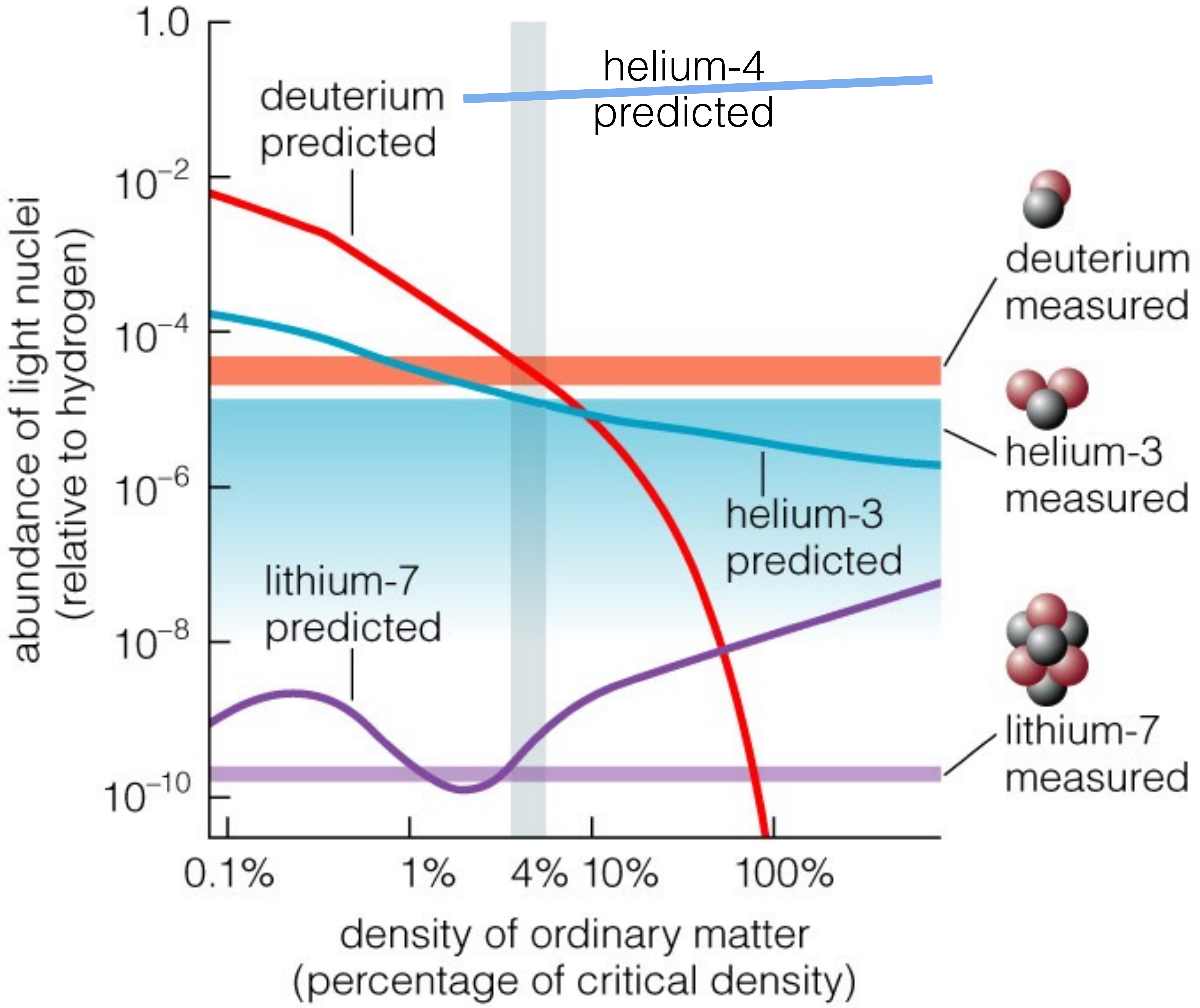


BBN products:

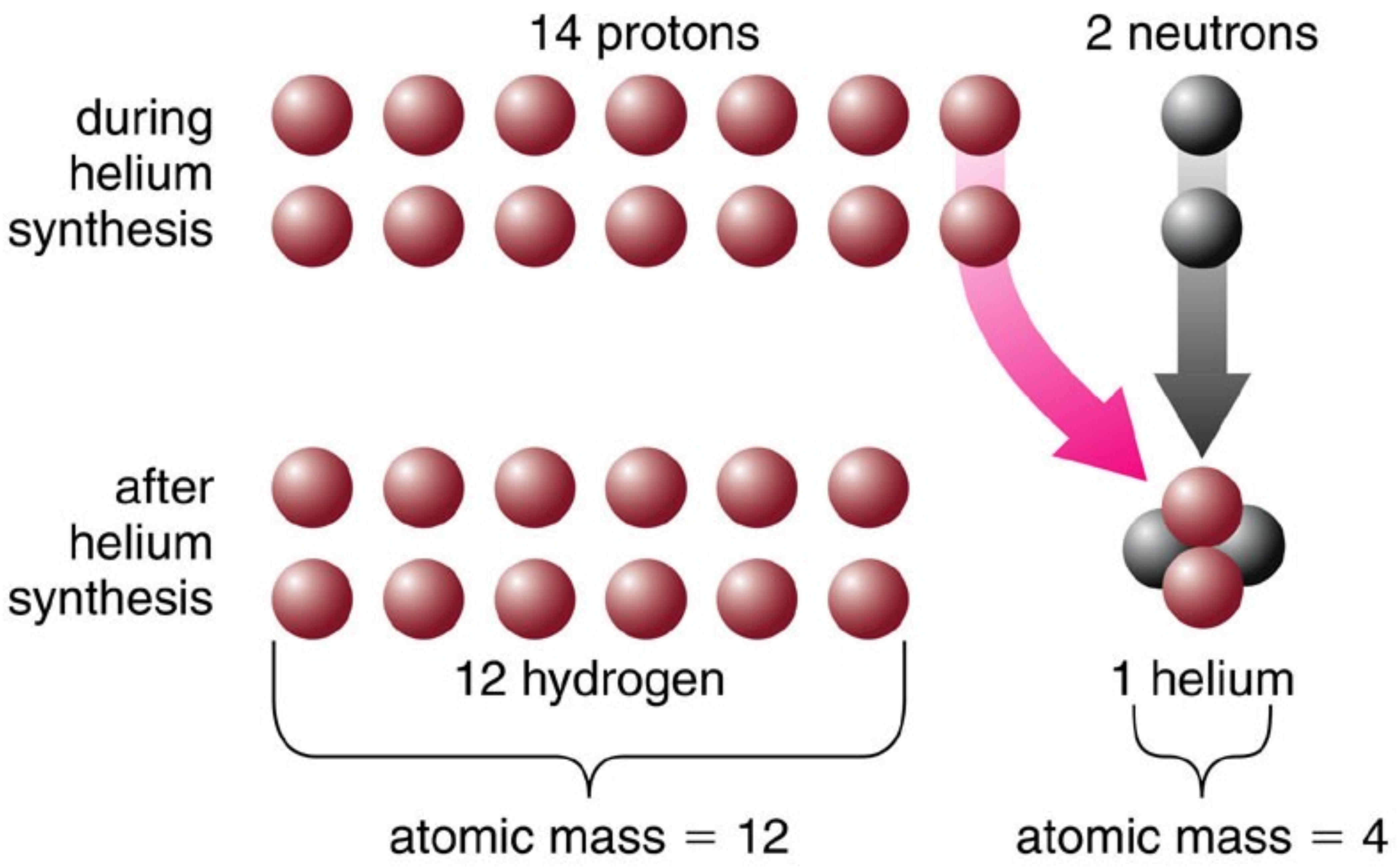
- 3/4 Hydrogen
- 1/4 Helium
- Traces of
  - deuterium
  - tritium
  - helium 3
  - lithium
  - beryllium

Beryllium decays into lithium after a few months.

Abundances depend on the density of matter. The higher the density parameter (Ω_b), the more helium.



To first order, BBN is just book-keeping: most of the available neutrons wind up in helium



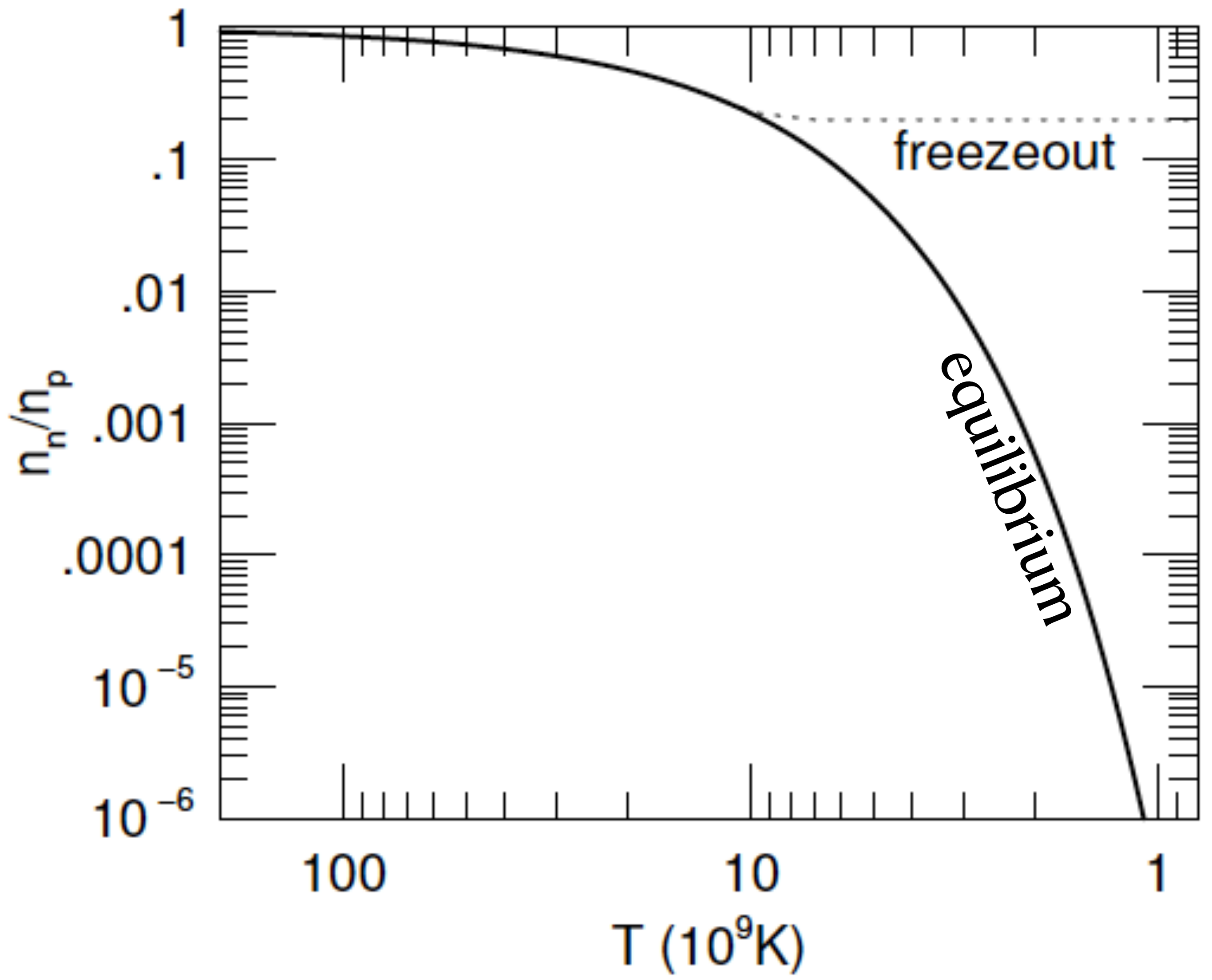
Big Bang theory prediction: 3/4 H, 1/4 He (by mass)

Matches observations of nearly primordial gases

There are fewer neutrons than protons at the time of BBN for several reasons…

High temperatures in the early universe mean all species start in thermal equilibrium

$$\frac{n_n}{n_p} \approx e^{-\frac{\Delta mc^2}{kT}}$$



$\Delta mc^2 = 1.29$ MeV

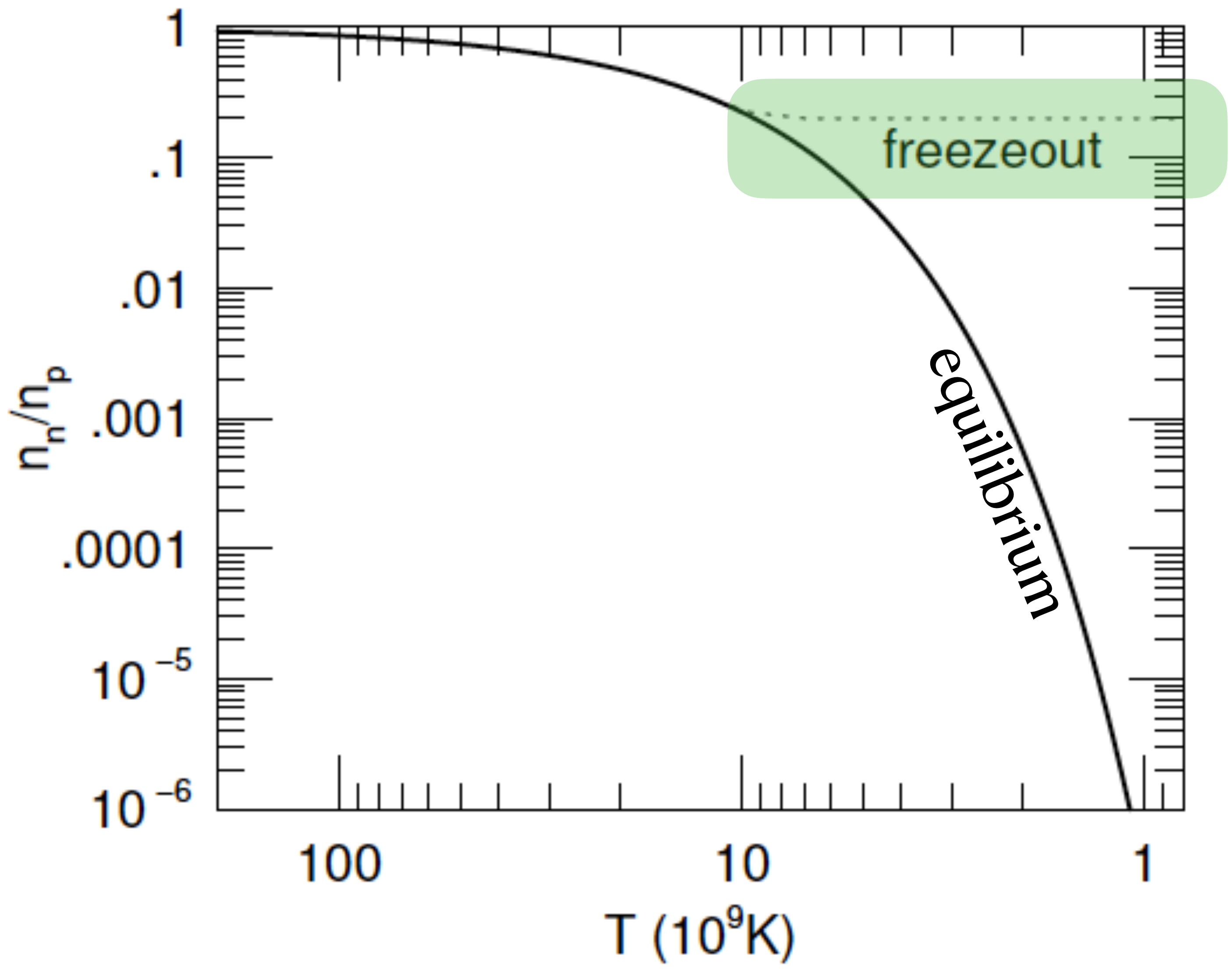
Neutrons are a little heavier than protons, so are disfavored as the universe cools

There are fewer neutrons than protons at the time of BBN for several reasons...

The neutron-proton equilibrium is mediated by the weak nuclear force.
These interactions "freeze out" when the expansion rate of the universe out-competes the interaction rate.

$$\frac{n_n}{n_p} \approx e^{-\frac{\Delta mc^2}{kT}}$$

$\Delta mc^2 = 1.29$ MeV



Freeze out occurs when

weak interaction rate $\quad n_\nu c\sigma_W \approx H \quad$ expansion rate

$$kT_{freeze} = 0.8 \text{ MeV}$$

$$\frac{n_n}{n_p} \approx e^{-\frac{1.29}{0.8}} \approx 0.2$$

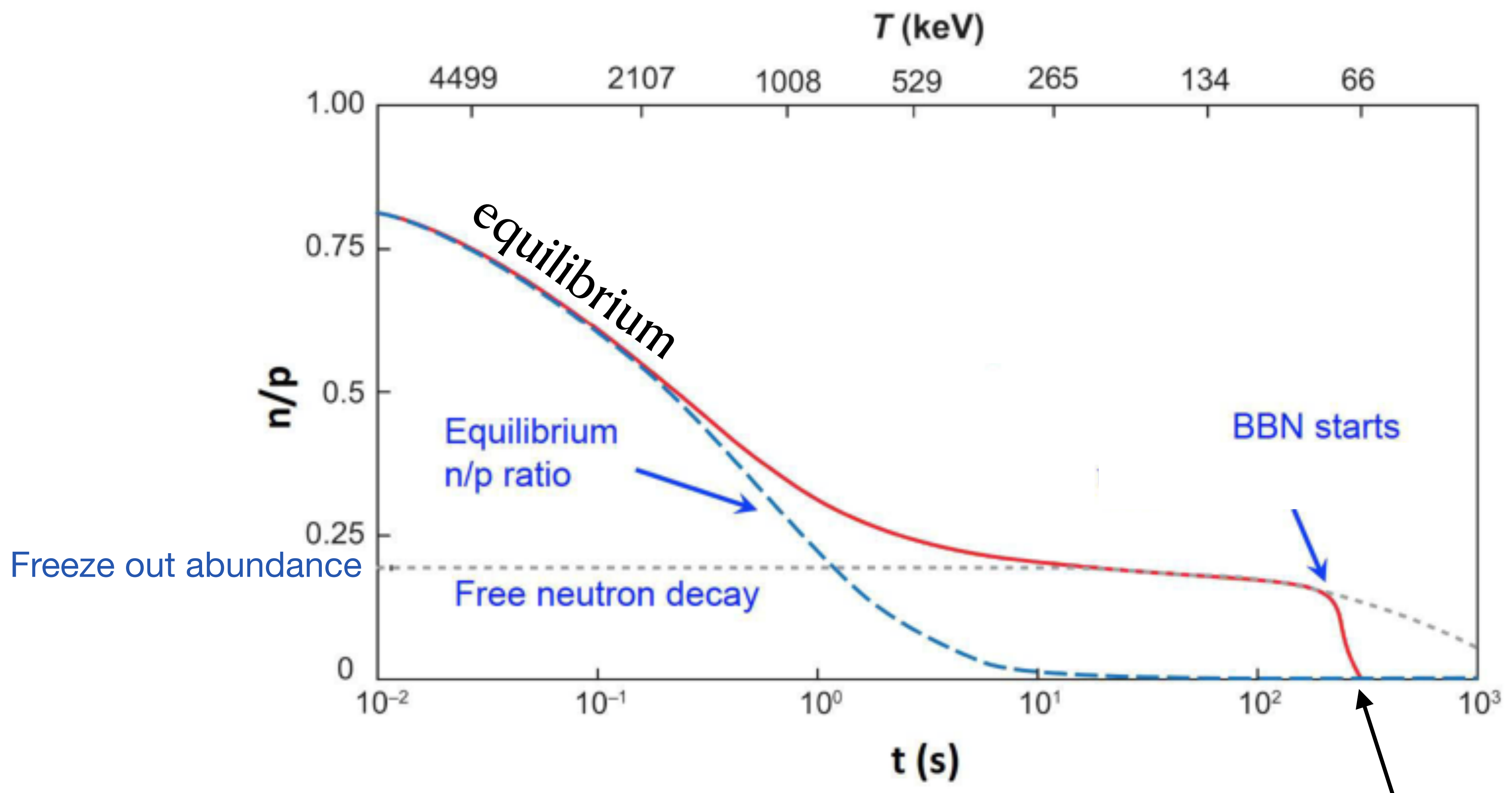
is the neutron-proton ratio after freeze out.

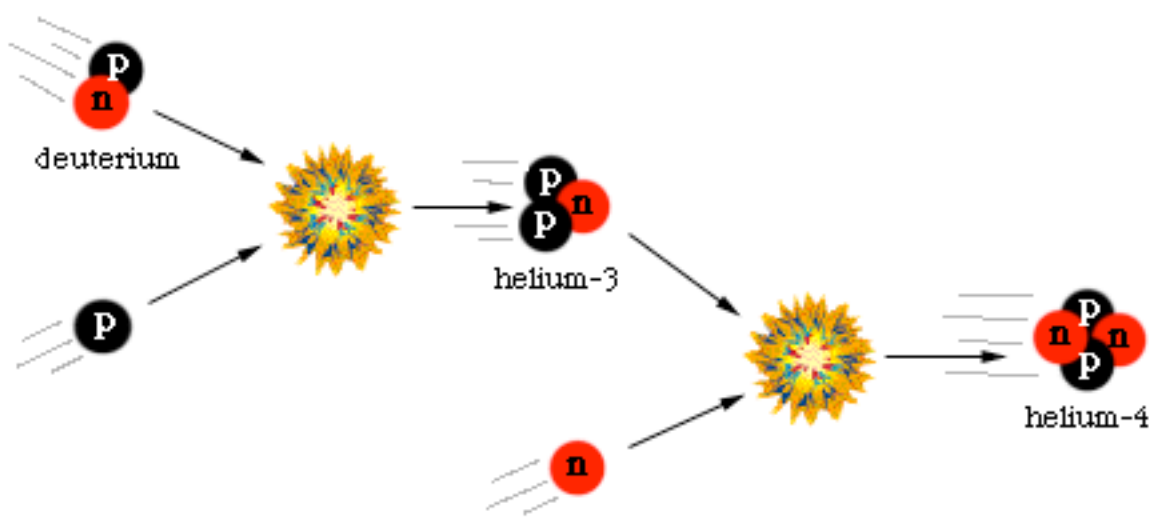
Only get this right if the universe behaves as expected for radiation domination: $a(t) \sim t^{1/2}$

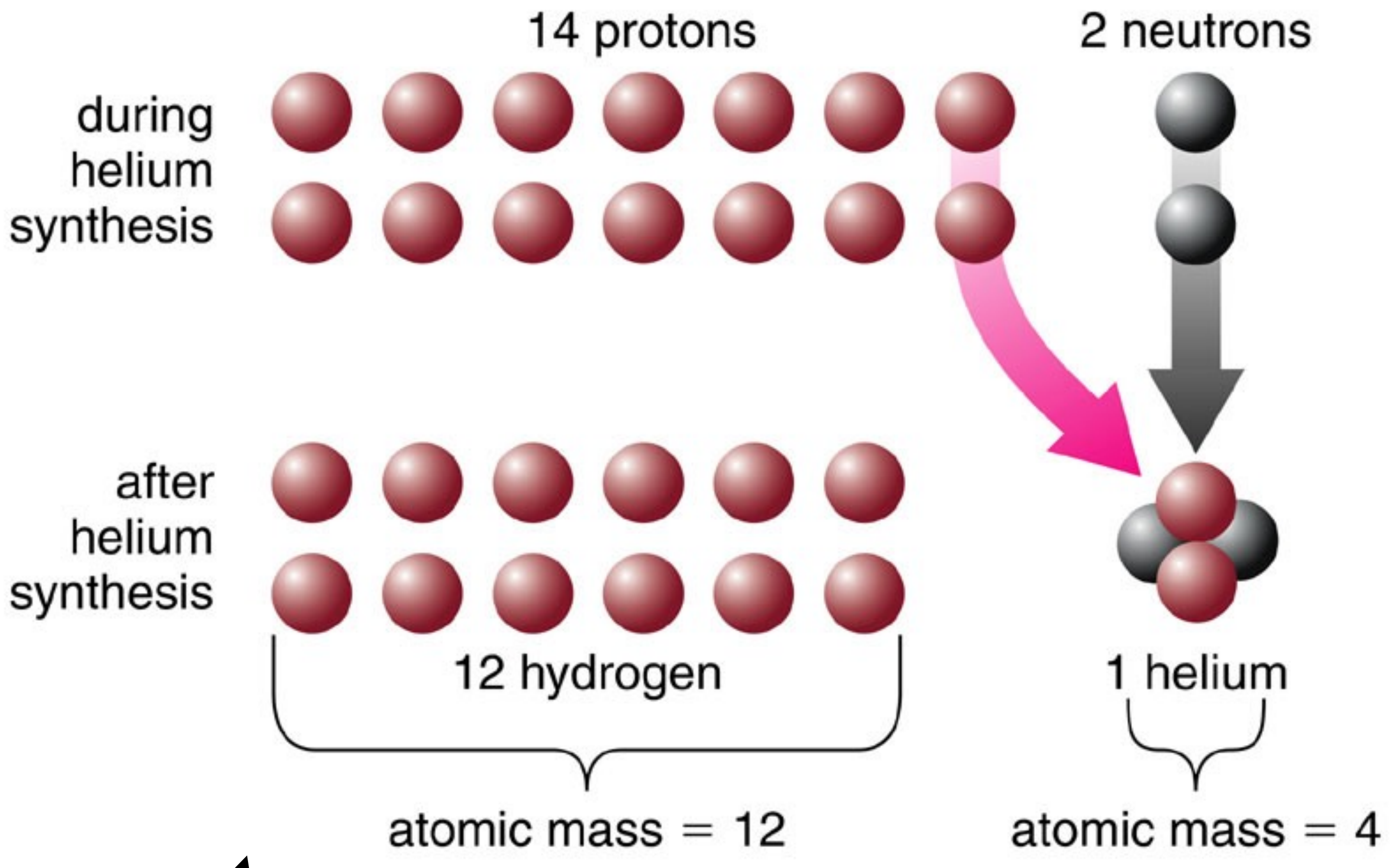
There are fewer neutrons than protons at the time of BBN for several reasons...

In addition, neutrons in free space are unstable, decay with an e-folding time of $\tau_n = 611\ \mathrm{s}$ – a little over 10 minutes.

Neutrons have just started to decay when BBN happens, so the uncertainty in the half-life is important.

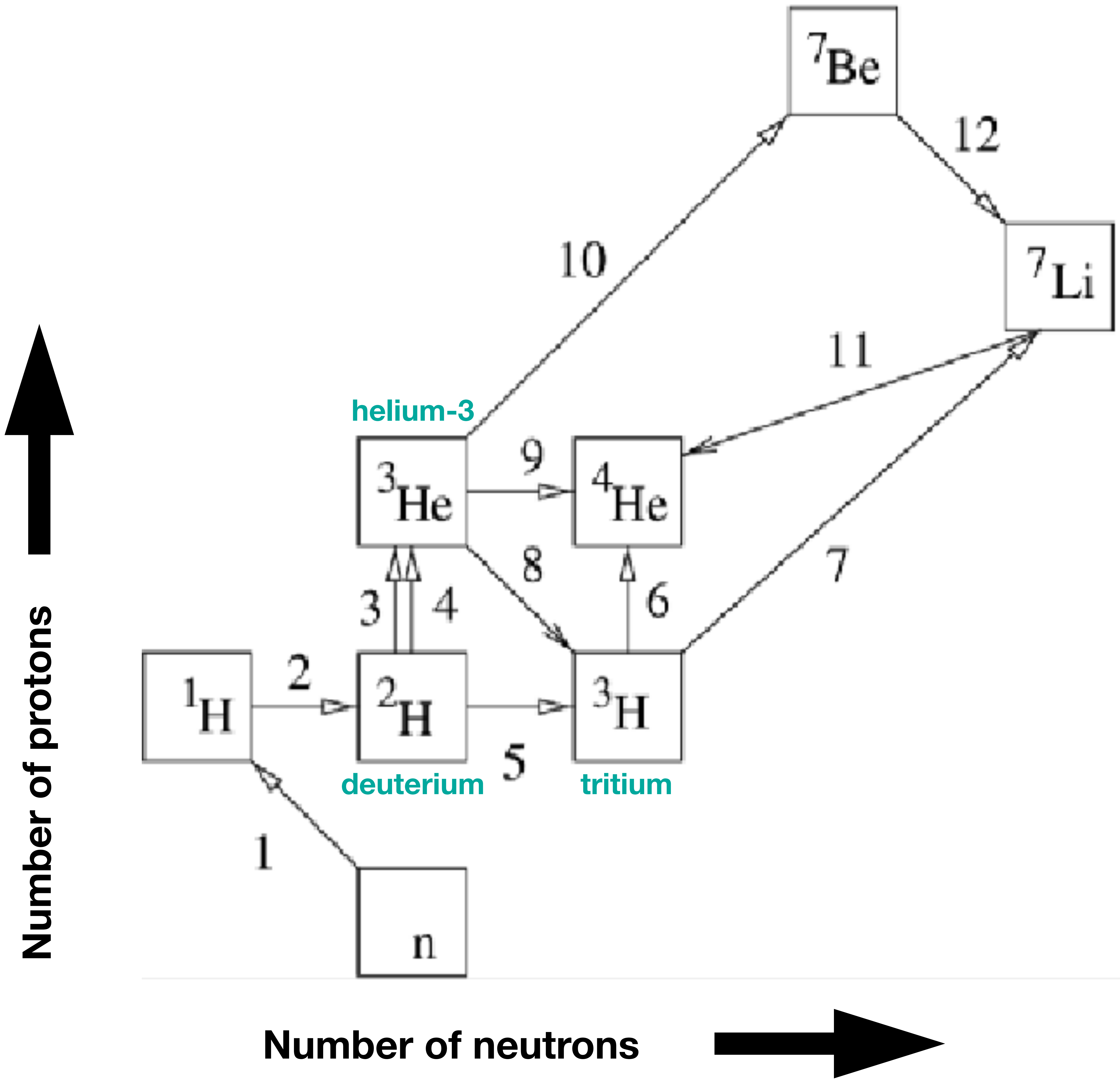






Remaining neutrons are mostly gobbled up by helium

In detail, need to keep track of all relevant nuclear reactions



1. $p \longleftrightarrow n$	proton-neutron equilibrium
2. $p(n,\gamma)d$	proton-neutron fusion to form deuterium
3. $d(p,\gamma)^3\mathrm{He}$	deuterium-proton fusion to form helium 3
4. $d(d,n)^3\mathrm{He}$	deuterium-deuterium fusion to form helium 3
5. $d(d,p)t$	deuterium-proton fusion to form tritium
6. $t(d,n)^4\mathrm{He}$	tritium-deuterium fusion to form helium 4
7. $t(\alpha,\gamma)^7\mathrm{Li}$	tritium-helium fusion to form lithium
8. $^3\mathrm{He}(n,p)t$	
9. $^3\mathrm{He}(d,p)^4\mathrm{He}$	helium 3-deuterium fusion to form helium 4
10. $^3\mathrm{He}(\alpha,\gamma)^7\mathrm{Be}$	helium 3-helium 4 fusion to form beryllium
11. $^7\mathrm{Li}(p,\alpha)^4\mathrm{He}$	
12. $^7\mathrm{Be}(n,p)^7\mathrm{Li}$	beryllium decay into lithium

BBN reactions

The following stages occur during the first few minutes of the Universe:

Less than 1 second after the Big Bang, the reactions shown at right maintain the neutron:proton ratio in thermal equilibrium. About 1 second after the Big Bang, the temperature is slightly less than the neutron-proton mass difference, these weak reactions become slower than the expansion rate of the Universe, and the neutron:proton ratio *freezes out* at about 1:6.	$p + e^- \longleftrightarrow n + \nu$ $n + e^+ \longleftrightarrow p + \bar{\nu}$
After 1 second, the only reaction that appreciably changes the number of neutrons is neutron decay, shown at right. The half-life of the neutron is 615 seconds. Without further reactions to preserve neutrons within stable nuclei, the Universe would be pure hydrogen.	$n \longrightarrow p + e^- + \bar{\nu}$
The reaction that preserves the neutrons is deuteron formation. The deuteron is the nucleus of deuterium, which is the heavy form of hydrogen (H^2). This reaction is exothermic with an energy difference of 2.2 MeV, but since photons are a billion times more numerous than protons, the reaction does not proceed until the temperature of the Universe falls to 1 billion K or kT = 0.1 MeV, about 100 seconds after the Big Bang. At this time, the neutron:proton ratio is about 1:7.	$p + n \longleftrightarrow d + \gamma$
Once deuteron formation has occurred, further reactions proceed to make helium nuclei. Both light helium (He^3) and normal helium (He^4) are made, along with the radioactive form of hydrogen (H^3). These reactions can be photoreactions as shown here. Because the helium nucleus is 28 MeV more bound than the deuterons, and the temperature has already fallen so far that kT = 0.1 MeV, these reactions only go one way.	$d + n \longrightarrow H^3 + \gamma$ $H^3 + p \longrightarrow He^4 + \gamma$ $d + p \longrightarrow He^3 + \gamma$ $He^3 + n \longrightarrow He^4 + \gamma$
The reactions at right also produce helium and usually go faster since they do not involve the relatively slow process of photon emission.	$d + d \longrightarrow He^3 + n$ $d + d \longrightarrow H^3 + p$ $H^3 + d \longrightarrow He^4 + n$ $He^3 + d \longrightarrow He^4 + p$
The net effect is shown at right. Eventually the temperature gets so low that the electrostatic repulsion of the deuterons causes the reaction to stop. The deuteron:proton ratio when the reactions stop is quite small, and essentially inversely proportional to the total density in protons and neutrons. Almost all the neutrons in the Universe end up in normal helium nuclei. For a neutron:proton ratio of 1:7 at the time of deuteron formation, 25% of the mass ends up in helium.	$d + d \longrightarrow He^4 + \gamma$

Source: Ned Wright: http://www.astro.ucla.edu/~wright/BBNS.html

Reaction rates depend on the temperature & number density, both of which decrease as the universe expands.
The absence of stable A = 5 & A = 8 nuclei causes a bottleneck.

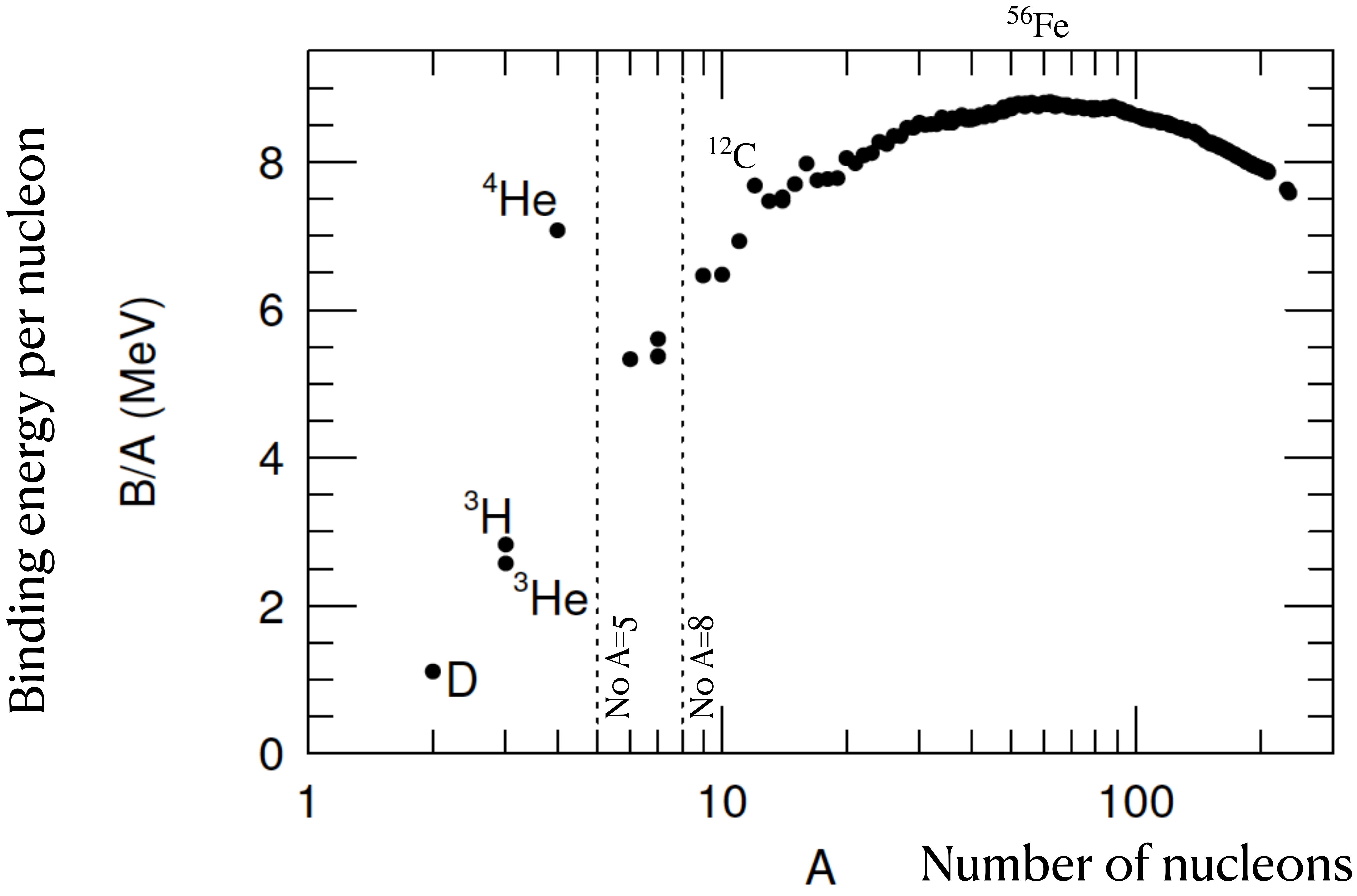


Figure 10.1: The binding energy per nucleon (B/A) as a function of the number of nucleons (protons and neutrons) in an atomic nucleus. Note the absence of nuclei at $A = 5$ and $A = 8$.

no stable mass 5 or 8

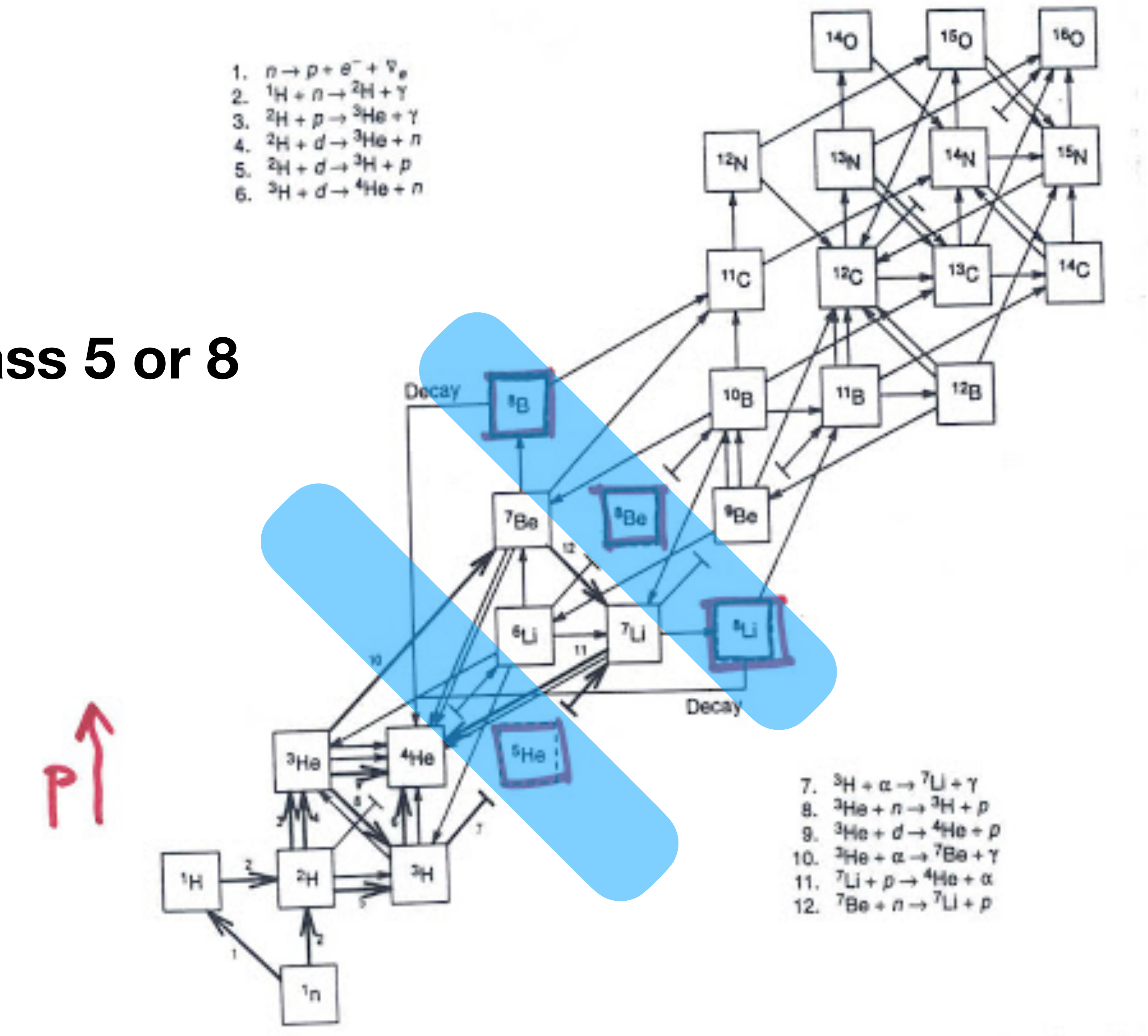
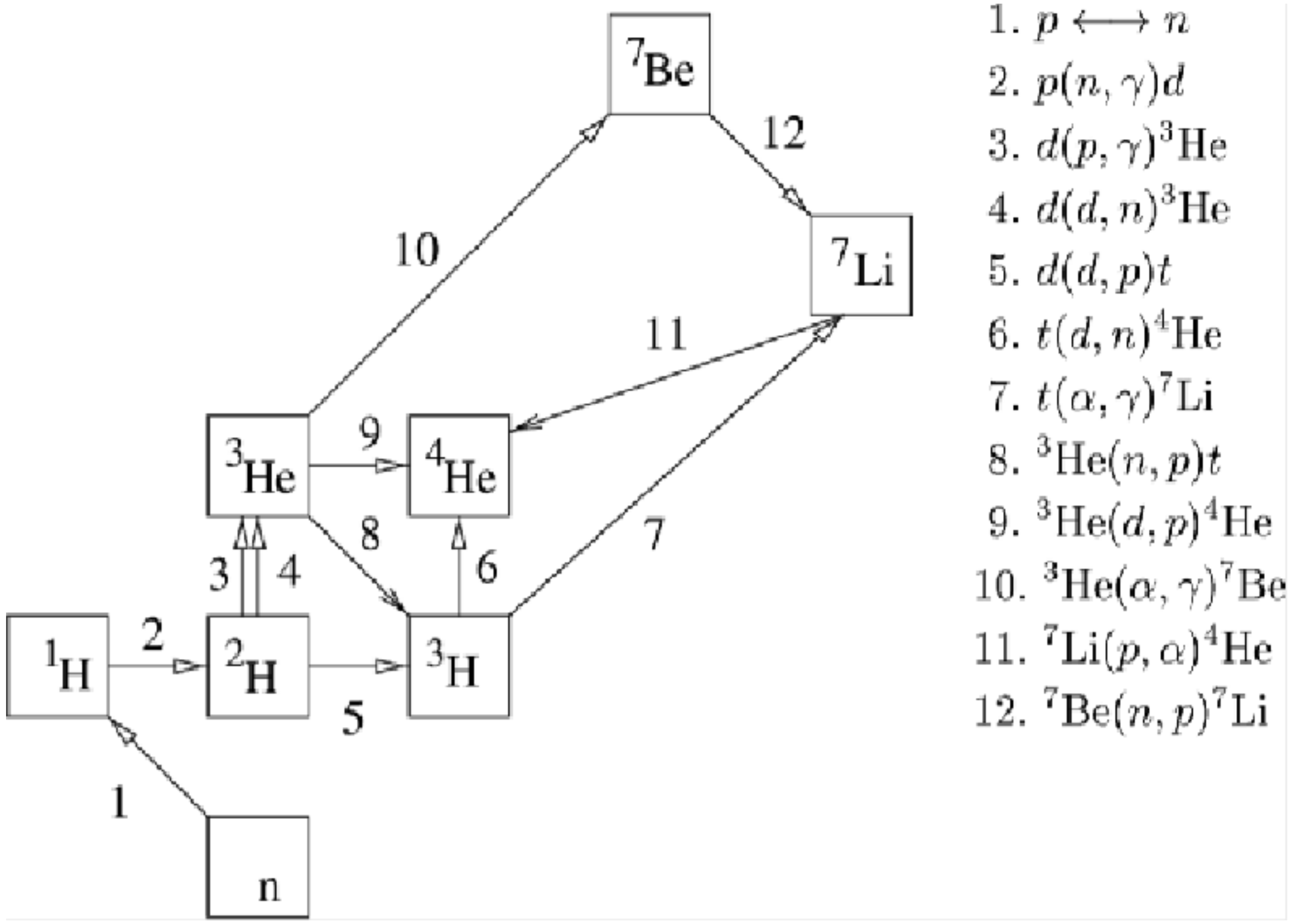


Fig. 2. The nuclear reaction network used for big-bang nucleosynthesis; the most important reactions are numbered and have bold arrows. The broken boxes for mass 5 and 8 indicate that all nuclides of this mass are very unstable.

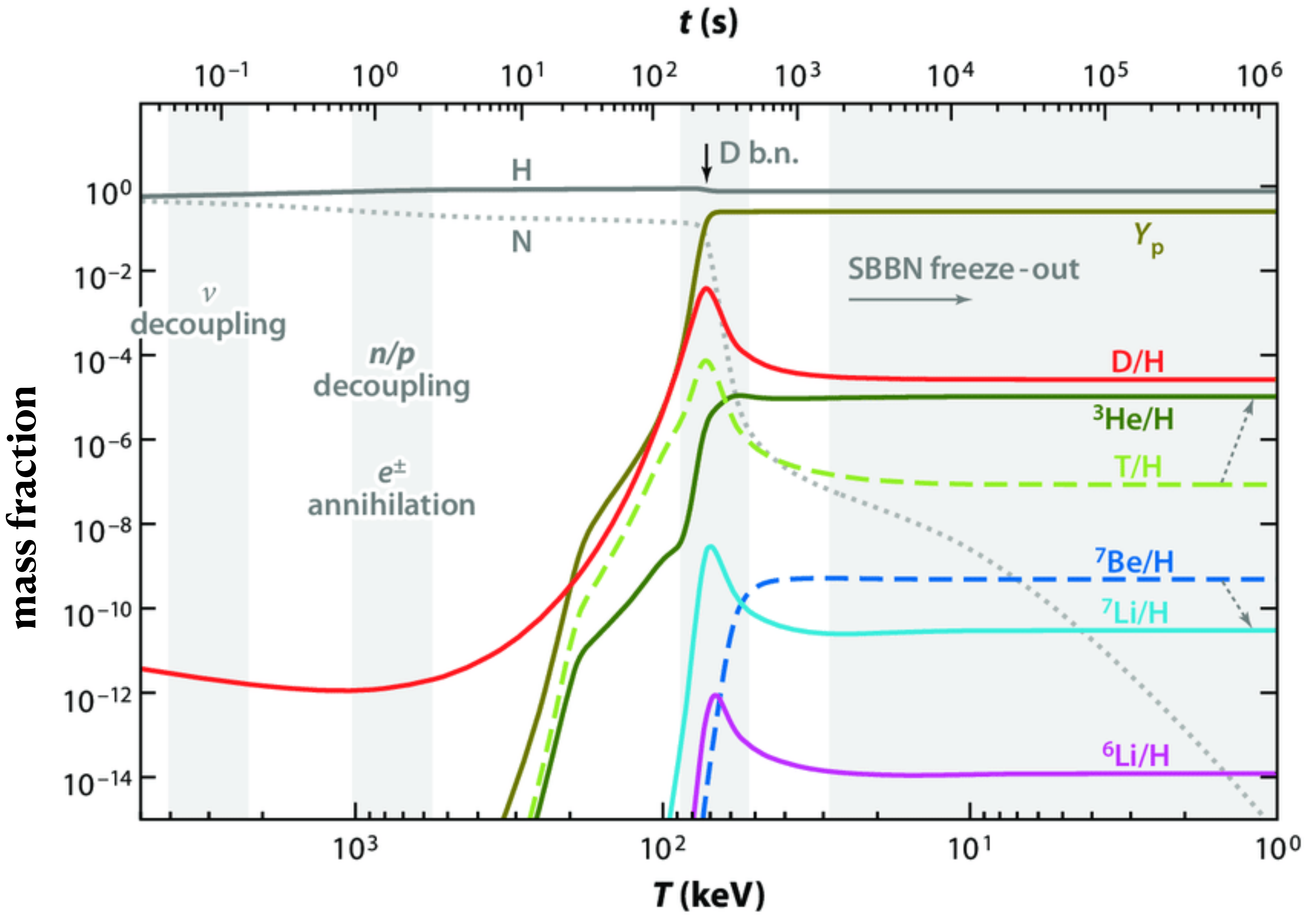
BBN restricted to the light elements by mass 5 & 8 bottlenecks.

Some stars skip over the mass bottleneck via the triple alpha reaction: $$3\,{}^{4}_{2}\mathrm{He} \to {}^{12}_{6}\mathrm{C}^*$$

Big Bang Nucleosynthesis occurs during the radiation dominated era



$$a(t) \propto t^{1/2}$$



Solve nuclear reaction chain as the universe expands and cools.
Must also keep track of neutron decay!

$$T(a)\ ;\ \rho_m(a)\ ;\ \rho_r(a)$$

$$\tau_N = 10.2 \text{ minutes}$$