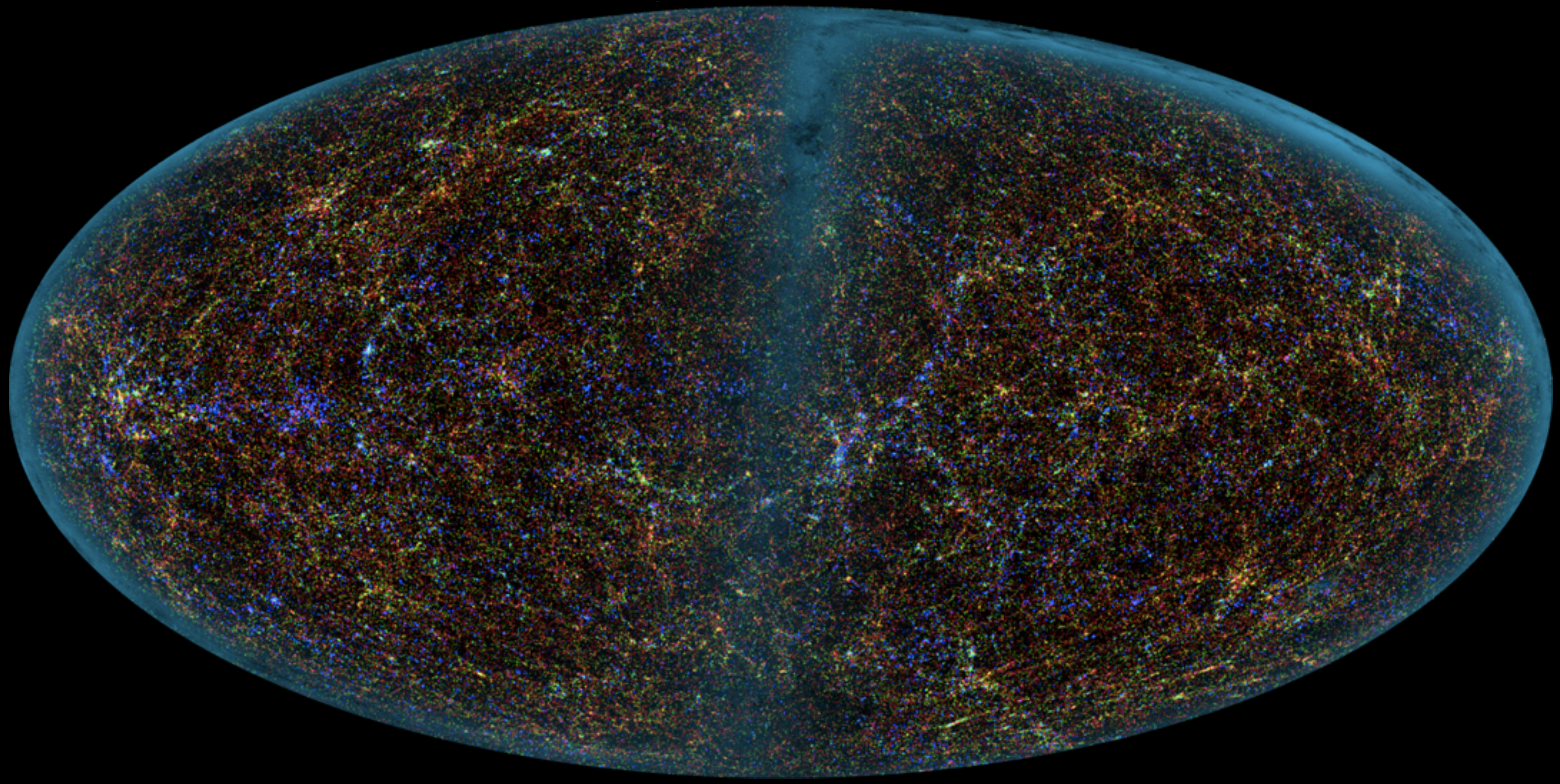
Cosmology

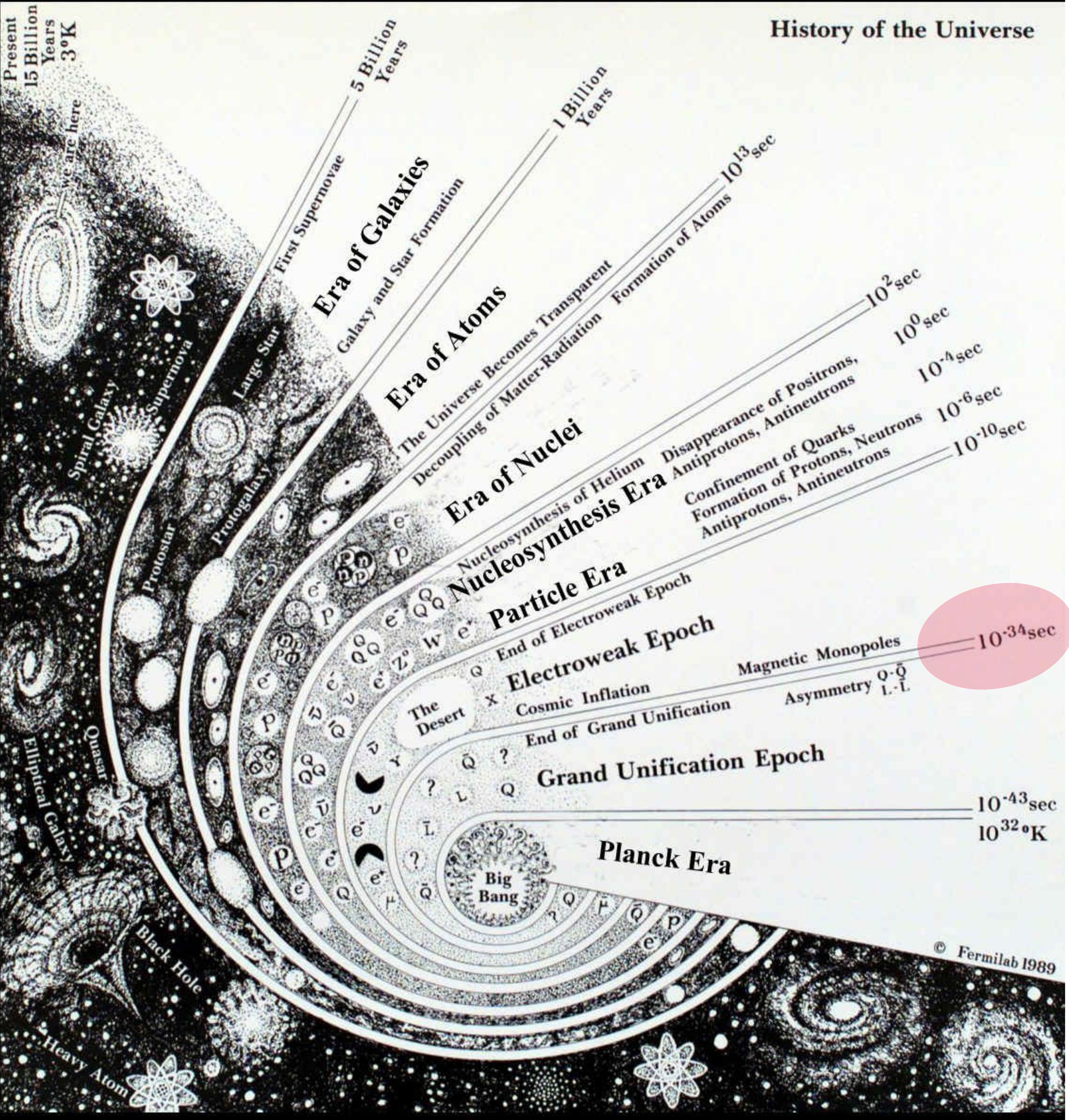
and Large Scale Structure



Today
Inflation

22 November 2022

http://astroweb.case.edu/ssm/ASTR328/



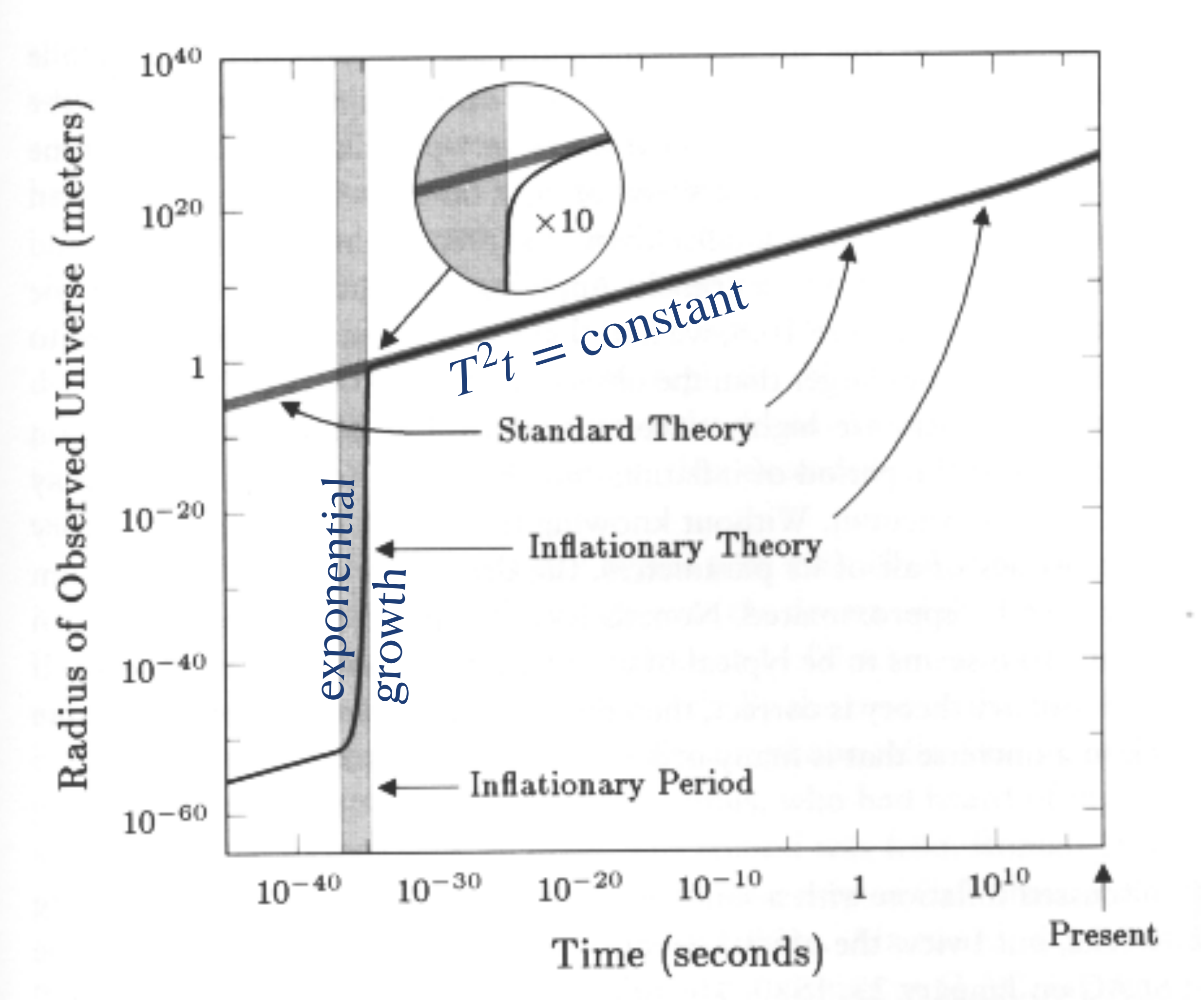
Time	Event
$t \sim 10^{-38}$ s	GUT scale (*speculative*)

GUT stands for Grand Unified Theory; this is the hypothetical scale at which the strong nuclear force becomes indistinguishable from the electroweak force.

$t \sim 10^{-35}$ s Inflation (*speculative*)

Period of exponential growth: $a \sim e^{Ht}$

precedes radiation domination when $a \sim t^{1/2}$
so $T^2 t = \text{constant}$



Inflation

An epoch of early, exponential expansion $a \sim e^{H_I t}$

- Invoked to solve the
  - Flatness problem
  - Horizon problem
  - Magnetic monopole problem
- provides seeds for the formation of large scale structure

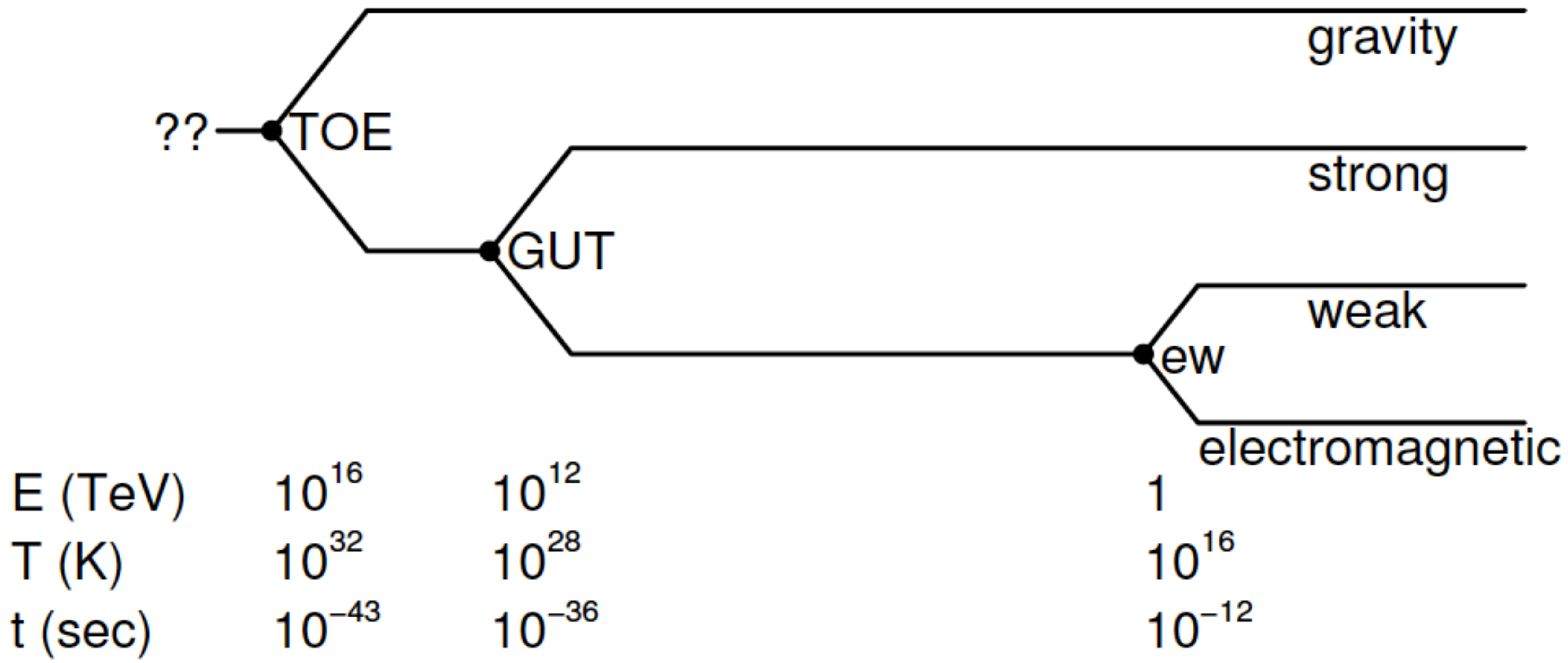


Figure 11.2: The energy, temperature, and time scales at which the different force unifications occur.

GUT = Grand Unified Theory
TOE = Theory of Everything

Inflation

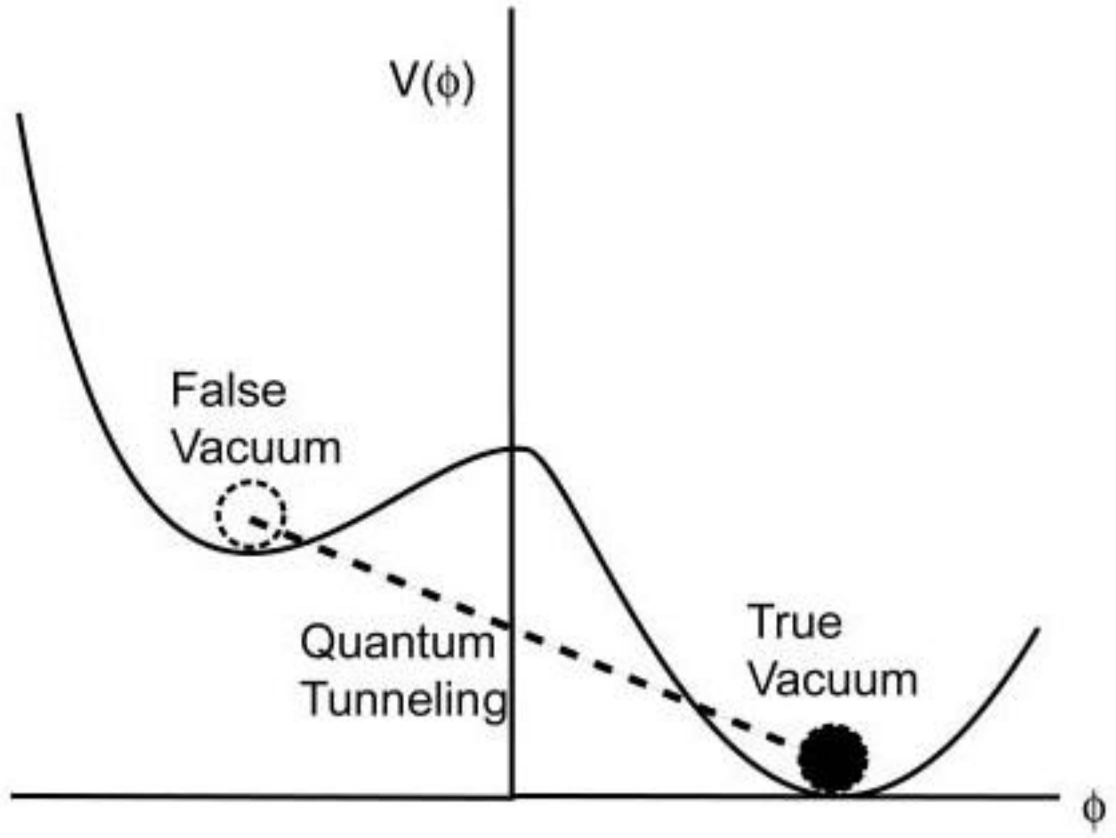
An epoch of early, exponential expansion

$$a \sim e^{H_I t}$$

Inflation emerged from the MIT "bag model" of dense nuclear matter, which has an *effective* equation of state $P = -\rho$ like what we now call dark energy. Solution of the Friedmann equation is the same, with

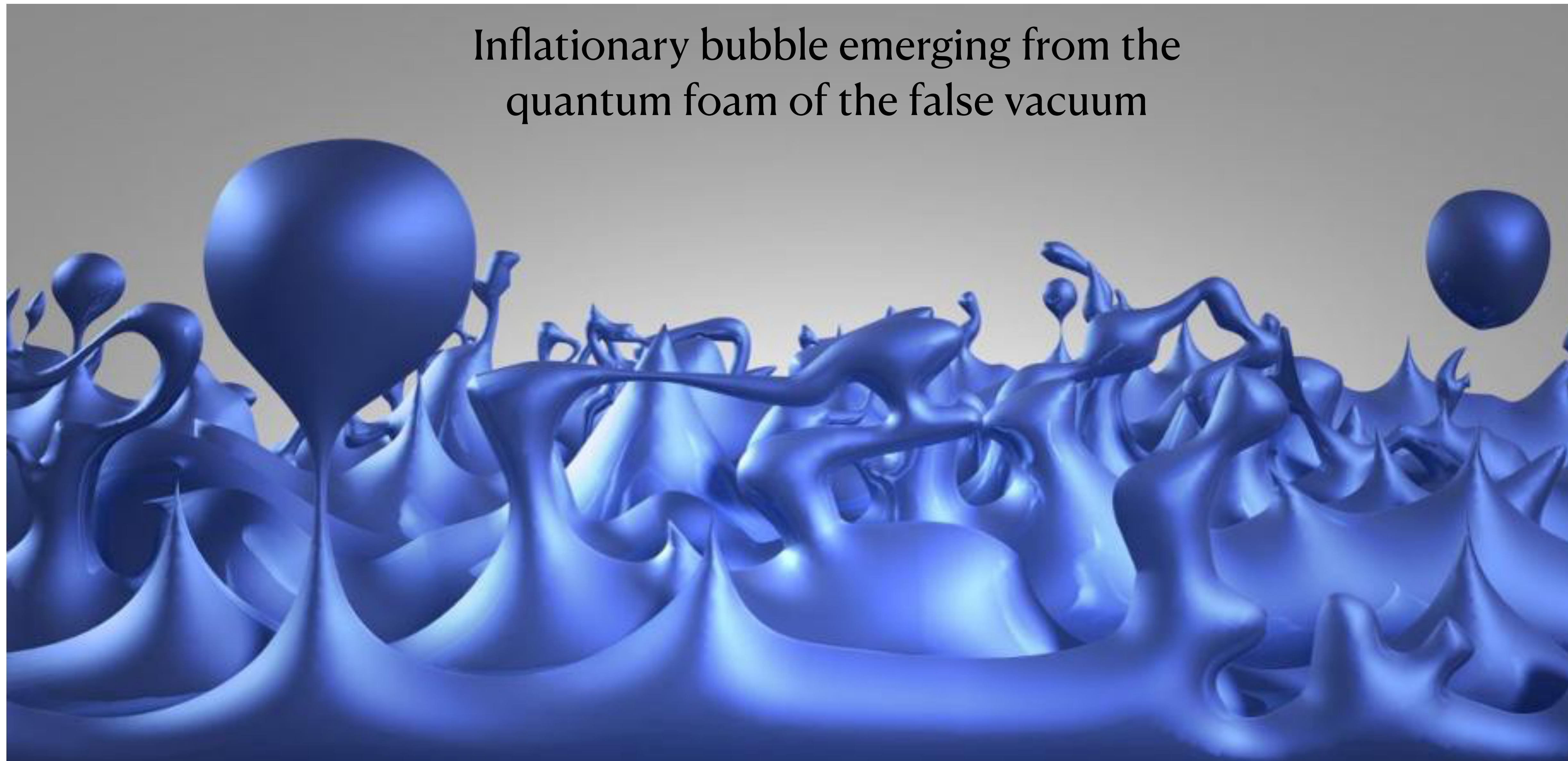
$$H_I = \sqrt{\frac{\Lambda_I}{3}}$$

where the subscript *I* denotes Inflation. This early "energy of the vacuum" is much greater than the current dark energy.



"False" vacuum of Inflation $\varepsilon_{\Lambda_I} = \frac{c^2}{8\pi G}\Lambda_I \sim 10^{120}\ \mathrm{GeV\,cm^{-3}}$

Current vacuum energy density $\varepsilon_{\Lambda_0} \sim 3.4\ \mathrm{GeV\,cm^{-3}}$


Inflationary bubble emerging from the quantum foam of the false vacuum

Inflation

An epoch of early, exponential expansion

- Invoked to solve the
  - Magnetic monopole problem

Expect monopoles to emerge in the GUT symmetry breaking, but they've never been detected. This can be avoided if they're sufficiently massive that they freeze out very early, then a period of Inflation dilutes their numbers.

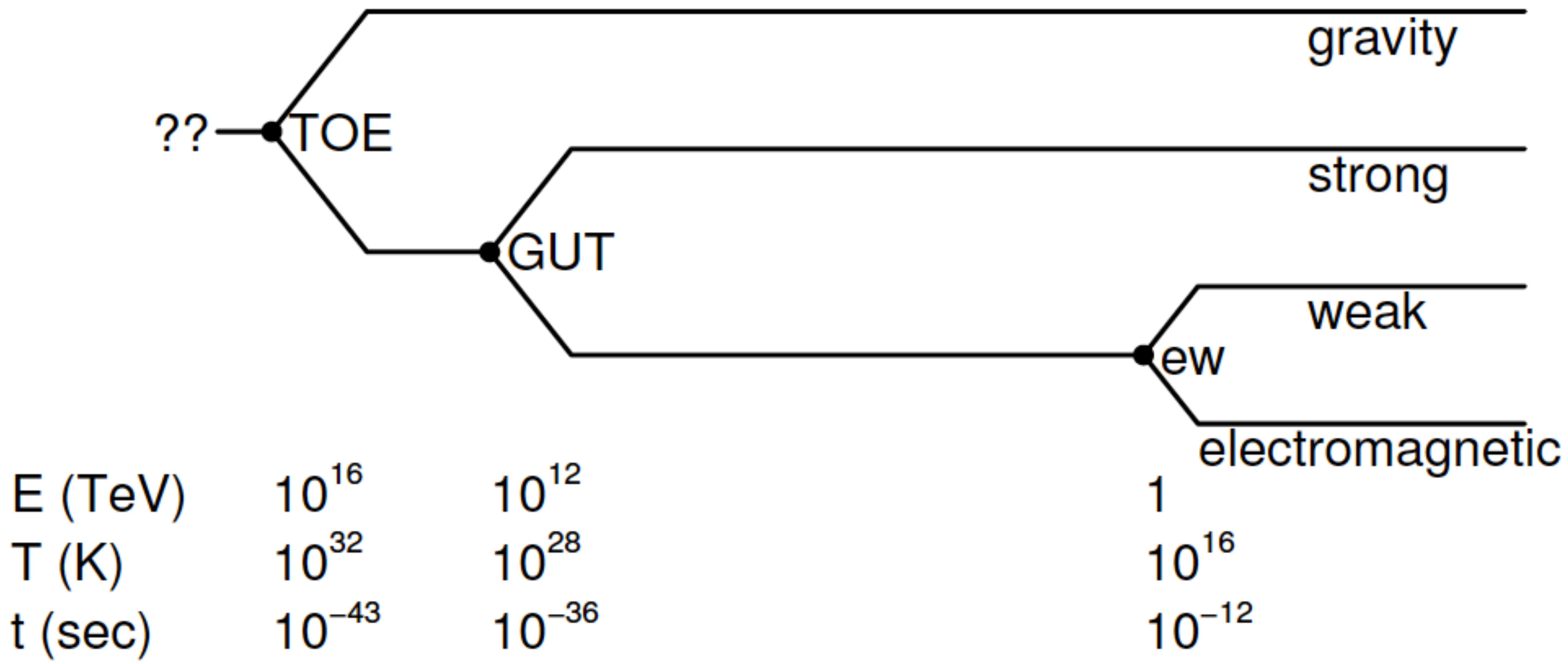


Figure 11.2: The energy, temperature, and time scales at which the different force unifications occur.

Motivating Inflation

See Ryden, chapter 10
see also Ned Wright's cosmology tutorial
https://www.astro.ucla.edu/~wright/cosmo_04.htm

Two observations, in particular, are extremely puzzling under the Big Bang model.

The flatness (or coincidence) problem
Why is $\Omega_{m_0} \approx 1$? Why not 106? Why not 42? Why not 0.0021034011031?
The density of the Universe changes with time, as the Universe expands. So Omega$_M$, the ratio of the actual density to the critical density also changes:

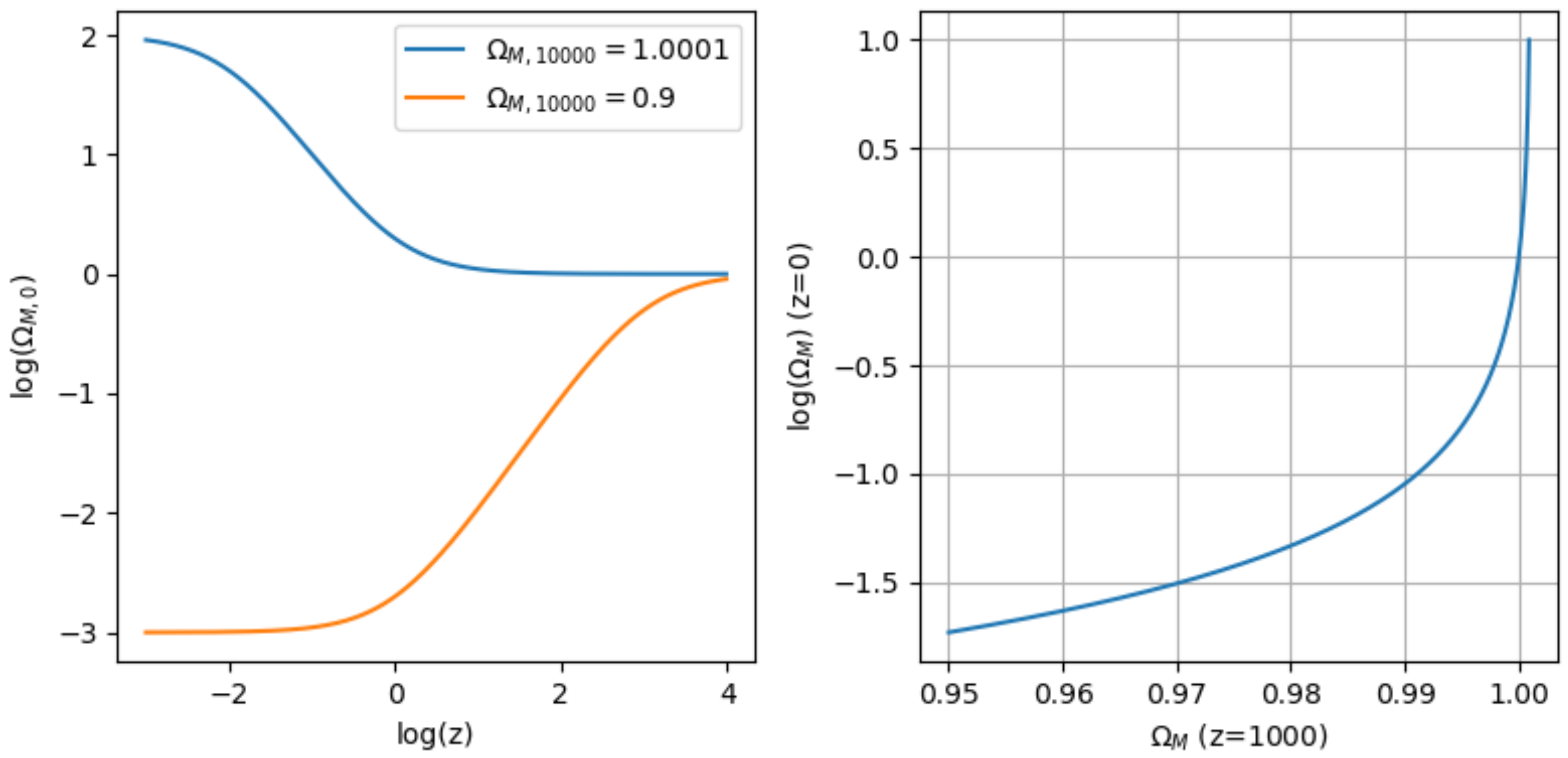
$$\Omega(z) = \Omega_0 \left[\frac{1+z}{1+\Omega_0 z}\right]$$

(Strictly speaking, this holds only for a matter dominated universe. But it's only recently that dark energy has started affecting the expansion of the universe, so early on the universe behaved like a matter dominated universe....)

Let's look at two examples.

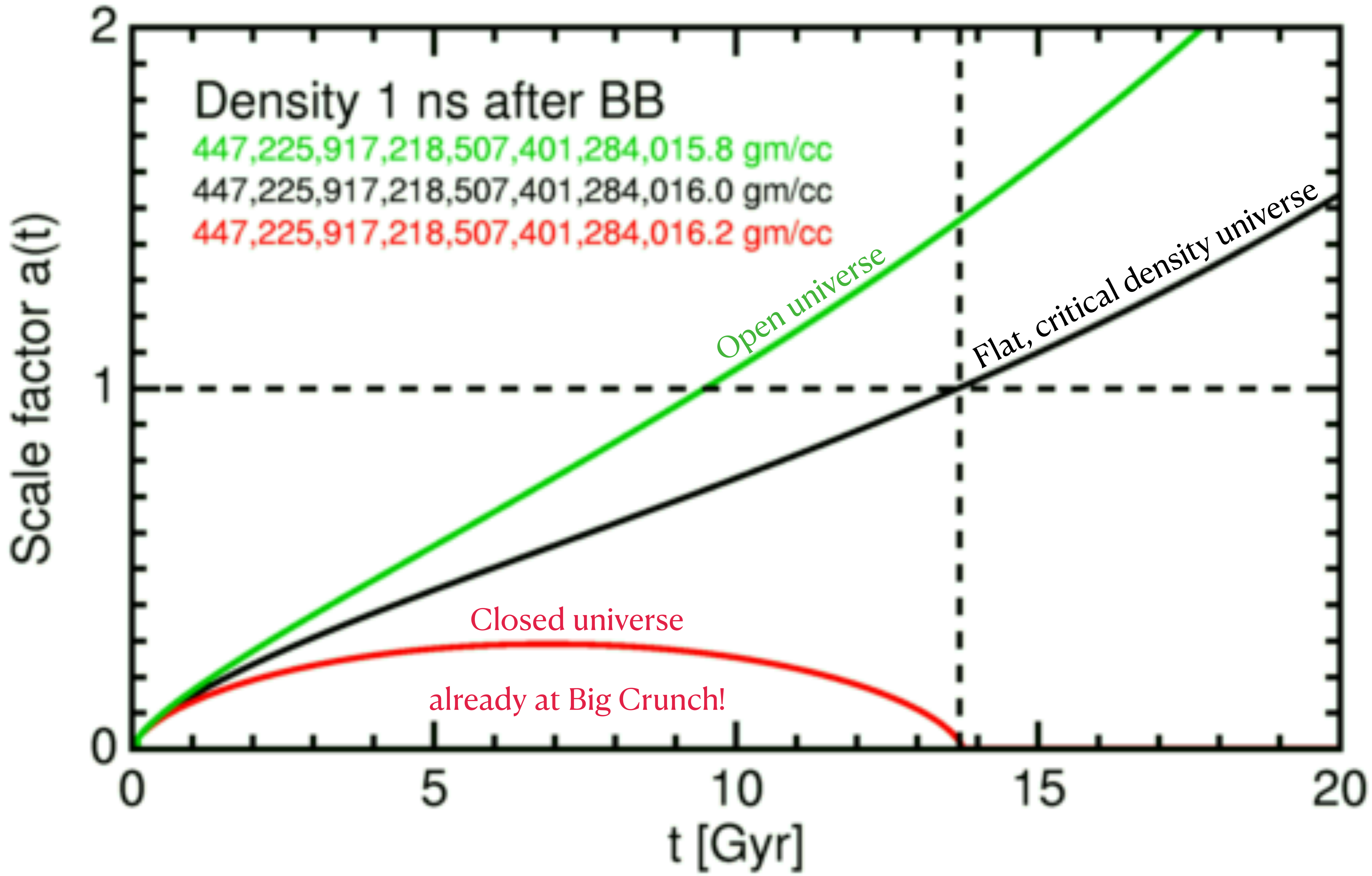
- At a redshift of z=10,000 (ie when the universe was 10^4 times smaller than now), it had a density parameter of 1.0001
- At a redshift of z=10,000, it had a density parameter of 0.9

In the universe that is slightly over-dense at z=10^4, the density parameter today (at z=0) would be 100. In the universe that is slightly under-dense at early times, we ought to measure a density parameter today of 0.001. Omega very quickly diverges from 1, unless it is exactly equal to 1.



The flatness problem

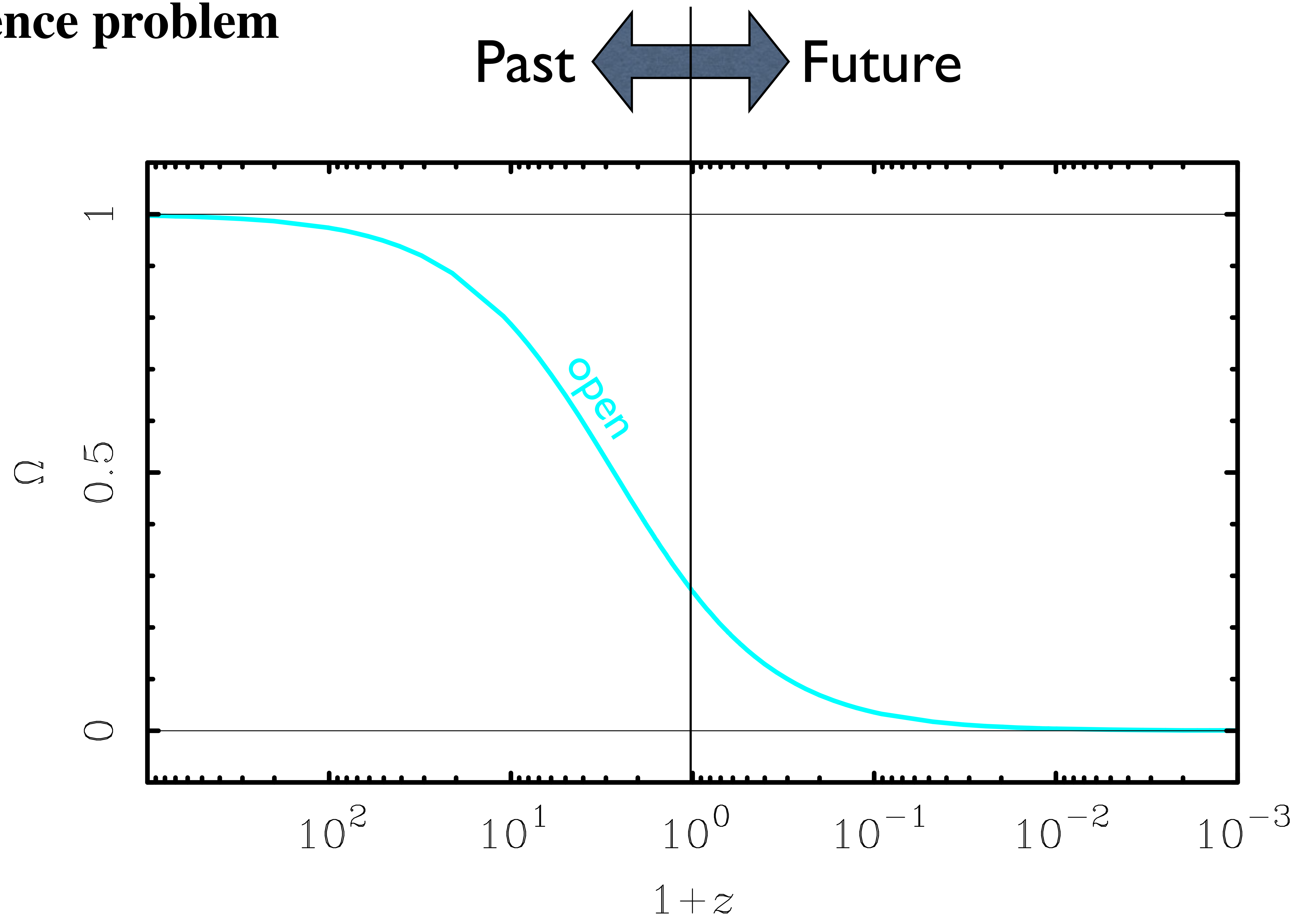
Look at this figure (taken from Ned Wright's Cosmology Tutorial; see also the textbook of Kolb & Turner) this argument made Inflation fly in the '80s:
if the density of the universe had been ever so slightly non-critical 1 nanosecond after the big bang, we would have a drastically different universe:



Why 1 ns?

How can this be? That we live in a universe anywhere near to $\Omega = 1$ but not exactly that is incredible fine-tuned - to a part in 10^{25} !
Hence we infer that some mechanism drove Omega to exactly one - **Inflation**.

The coincidence problem



Another way to look at it: how can $\Omega_m \approx 0.3$ today when it will spend all eternity asymptotically approaching $\Omega_m \to 0$?

The flatness/coincidence problem

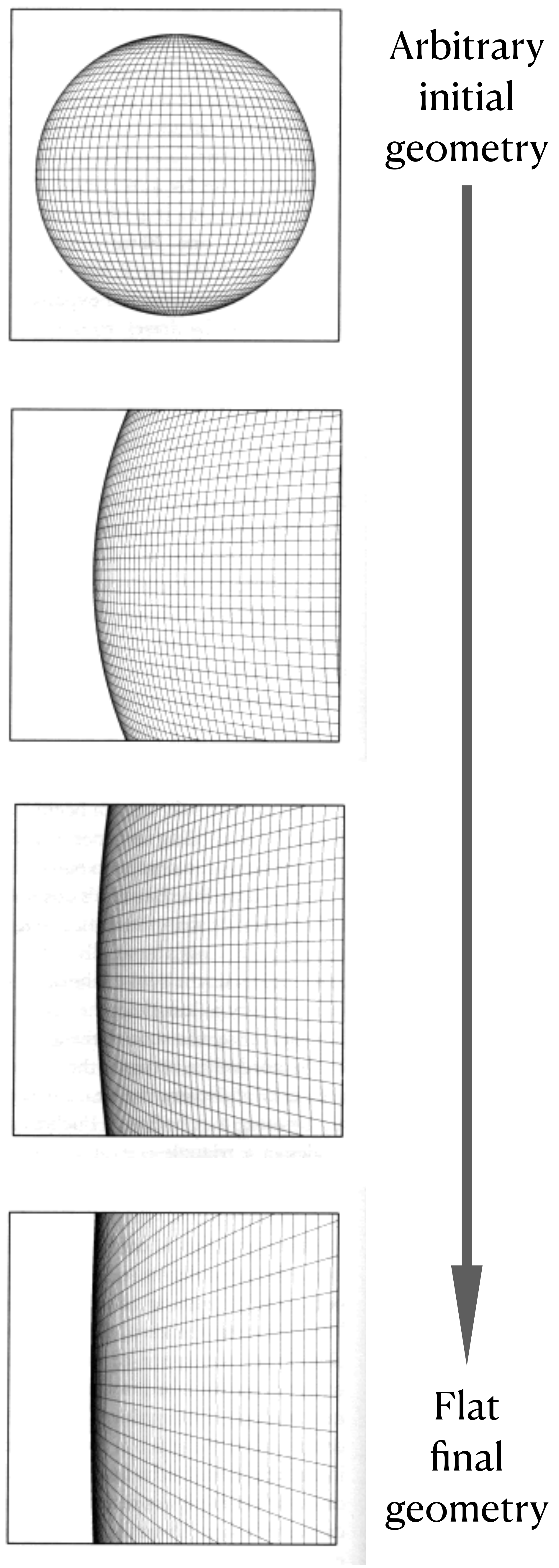
The Inflationary solution:

$$\Omega_k = 1 - \sum_{\text{not } k} \Omega = \left(\frac{c}{R_0 a(t) H(t)} \right)^2$$

During Inflation, $a(t) \sim e^{H_I t}$ with $H_I = \sqrt{\Lambda_I / 3}$ = constant, so $|\Omega_k| \rightarrow e^{-2H_I t}$

The initial condition is irrelevant; the exponential expansion drives the universe to be flat.

To get the observed flatness, $|\Omega_k| < 0.005$, requires that Inflation persists for > 60 e-foldings.



The Horizon (or Smoothness) Problem

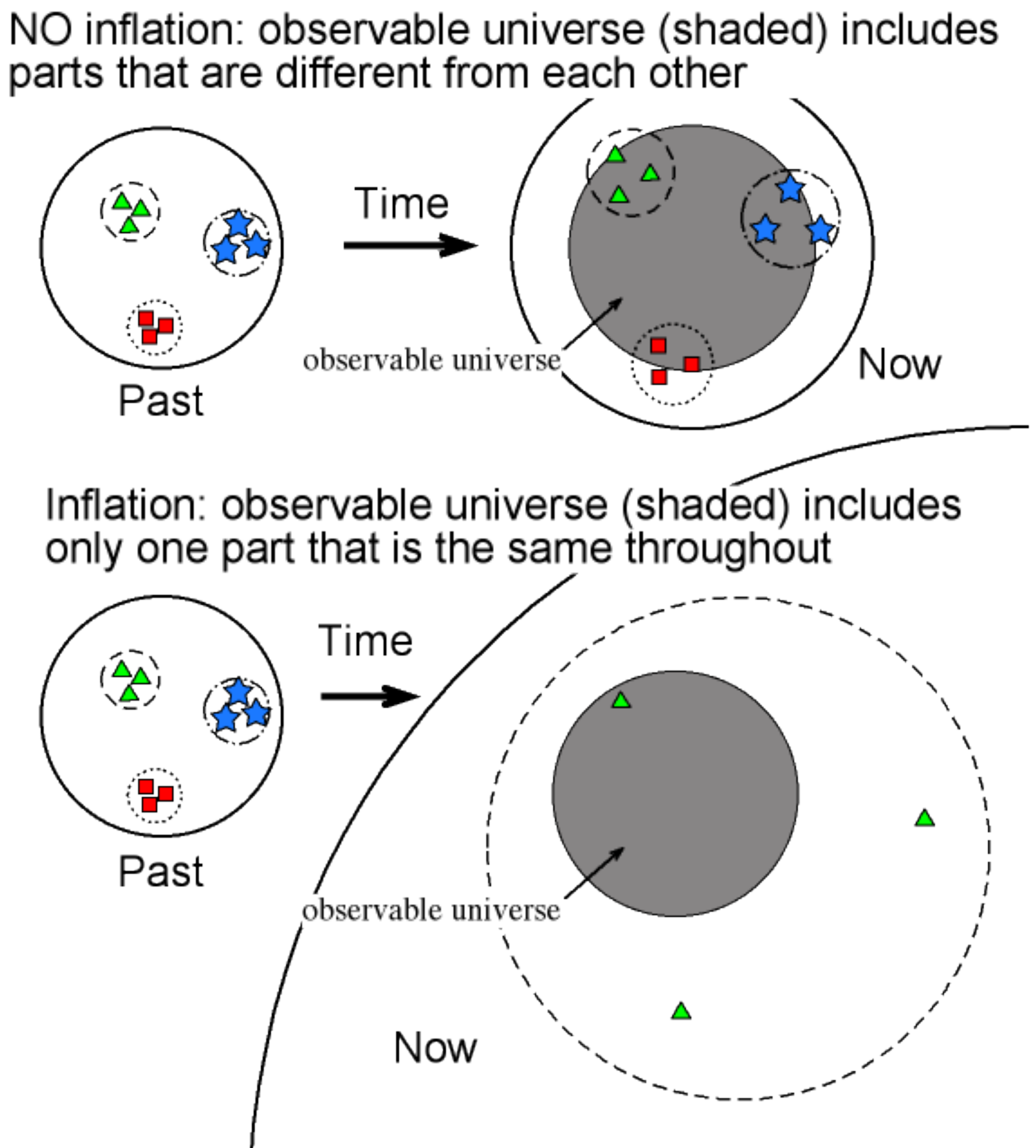
Looking at the microwave background, it is very smooth to 1 part in 10^5. Everywhere. But at the time of recombination, regions of the universe which are now separated by more than 2 degrees on the sky were never in causal contact.

Think about the horizon distance: $$d_h(t) = R(t)\int_0^t \frac{c\ dt}{R(t')}$$

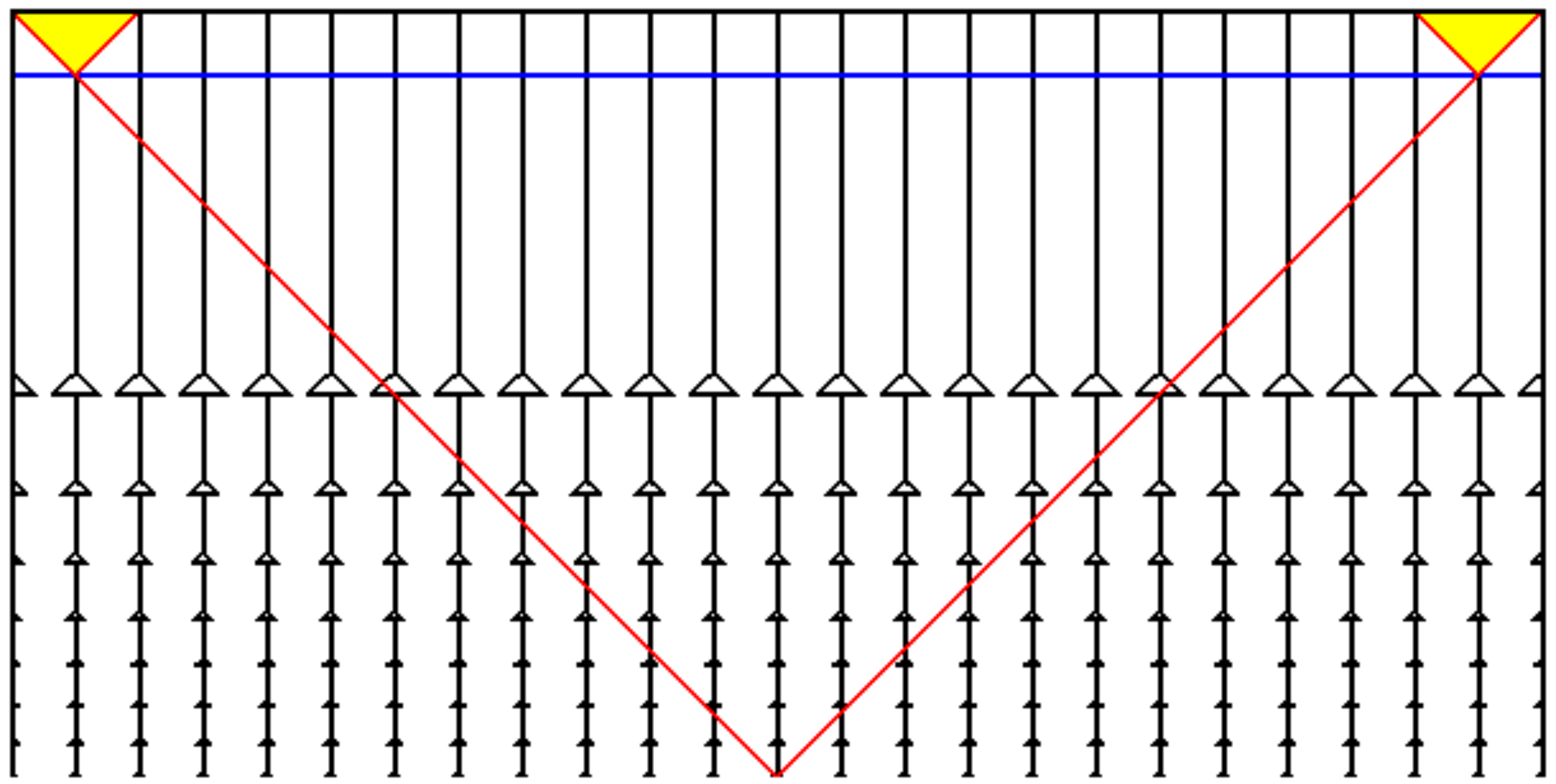
At the time of the CMB, the horizon scale was about 0.25 Mpc. The current horizon distance is ~ 14.6 Gpc, so the observable universe at a redshift of z=1100 was 14.6 Gpc/1100 ~ 13.2 Mpc in size - much larger than the horizon.

*How did regions out of causal contact **know** to all have the same temperature?*

Well, how about T^2t = constant

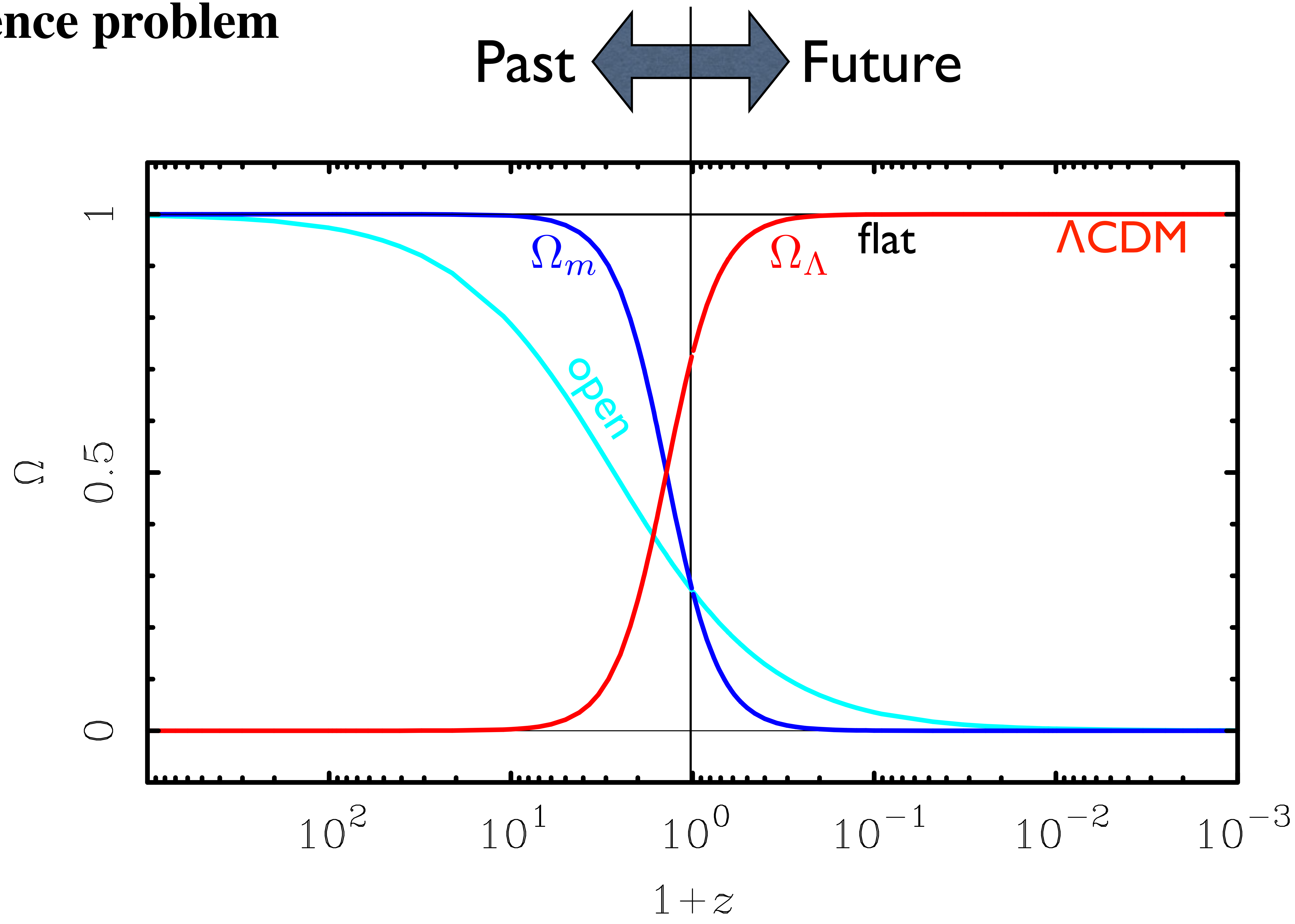


Yellow regions out of causal contact at recombination yet are observed to have the same temperature on the sky now



The conformal space-time diagram above has exaggerated this part even further by taking the redshift of recombination to be 1+z = 144, which occurs at the blue horizontal line. The yellow regions are the past lightcones of the events which are on our past lightcone at recombination. Any event that influences the temperature of the CMB that we see on the left side of the sky must be within the left-hand yellow region. Any event that affects the temperature of the CMB on the right side of the sky must be within the right-hand yellow region. These regions have no events in common, but the two temperatures are equal to better than 1 part in 10,000. How is this possible? This is known as the "horizon" problem in cosmology. (Image credit: Ned Wright).

The coincidence problem



The coincidence problem gets ***worse*** in LCDM. The geometry may be flat, but we still live at a special time when the universe transitions from matter domination to dark energy domination.

Inflation

An epoch of early, exponential expansion $a \sim e^{H_I t}$

- Invoked to solve the
  - Flatness problem **Coincidence problem not really solved**
  - Horizon problem $T^2 t =$ **constant, so is this really a problem?**
  - Magnetic monopole problem **Do monopoles even exist?**

Monopoles could be a figment of Grand Unified Theories that haven't panned out.

Graceful exit problem:

To get the observed flatness, $|\Omega_k| < 0.005$, requires that Inflation persists for > 60 e-foldings. Why does it ever stop?

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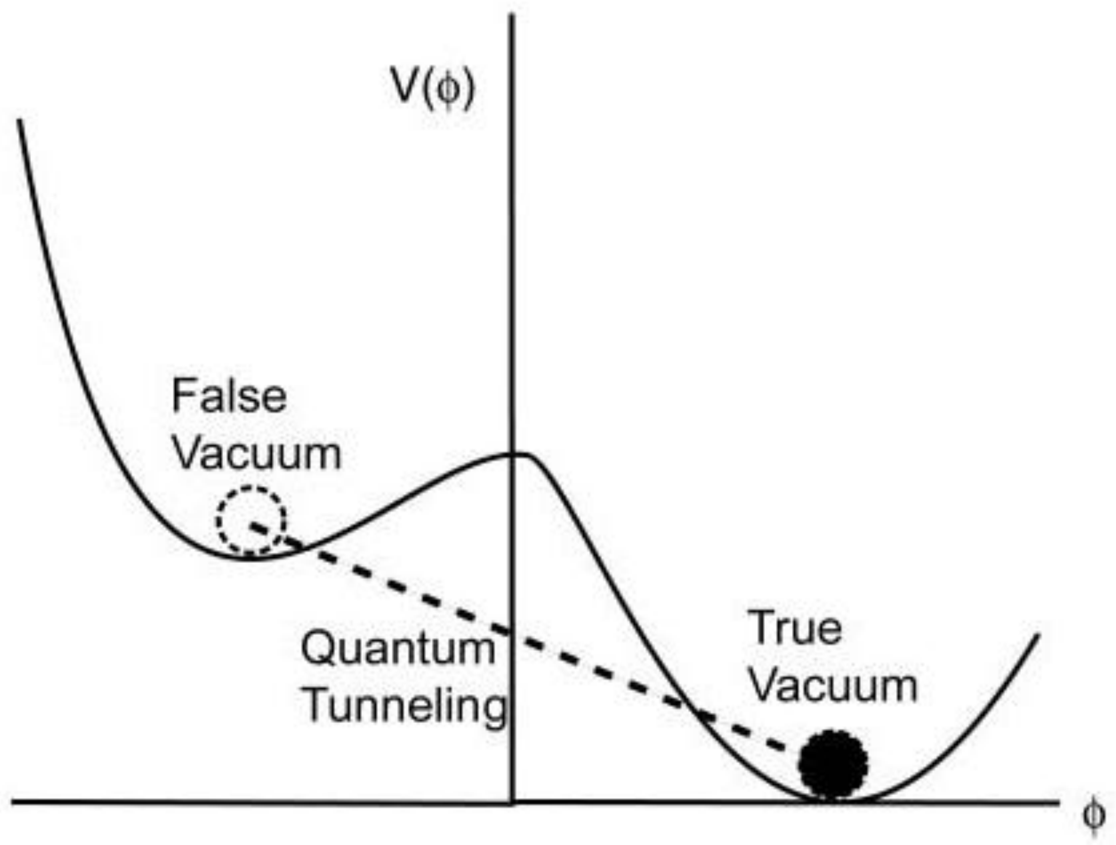
Why does it ever stop?

The graceful exit problem was solved by slow-roll Inflation (Albrecht & Steinhardt 1982)



old inflation

$$V_T(\varphi) = (2A - B)\sigma^2\varphi^2 - A\varphi^4 + B\varphi^4 \ln(\varphi^2/\sigma^2) + 18(T^4/\pi^2)\int_0^\infty dx\, x^2 \ln\{1 - \exp[-(x^2 + 25g^2\varphi^2/8T^2)^{1/2}]\}, \quad (1)$$

where the adjoint Higgs field, Φ, has been reexpressed as $\varphi(1,1,1,-\frac{3}{2},-\frac{3}{2})$ (the fundamental Higgs field will be irrelevant for this discussion); g is the gauge coupling constant; σ is chosen to be 4.5×10^{14} GeV; $B = 5625g^4/1024\pi^2$; and A is a free parameter. Equation (1) includes the one-loop quantum and thermal corrections to the effective potential. For a CW model, the coefficient of the quadratic term, $2A - B$, is set equal to zero and the Higgs mass is $m_{\rm CW} = 2.7\times 10^{14}$ GeV. We will also present results for non-CW models in which $2A - B$ is small and therefore the Higgs mass, $m_{\rm H}$, is such that $\Delta_{\rm H} \equiv (m_{\rm H}{}^2 - m_{\rm CW}{}^2)/m_{\rm CW}{}^2$ is small.

As for more general GUT models, the process of the first-order phase transition from the SU(5) *symmetric* phase to the SU(3)⊗SU(2)⊗U(1) *symmetry-breaking* phase for the CW model can be understood by studying the shape of the effective potential as a function of the scalar field for various values of the temperature, as shown in Fig. 1. For temperatures above the critical temperature ($T_{\rm GUT}$) for the transition, the symmetric phase ($\varphi = 0$) is the global stable minimum of the effective potential. At $T = T_{\rm GUT}$, the symmetric phase and the symmetry-breaking phase have equal energy densities. As the temperature drops below $T_{\rm GUT}$, the symmetric phase becomes *metastable*—it has a higher-energy density than *stable* symmetry-breaking phase but a potential barrier prevents it from becoming unstable.

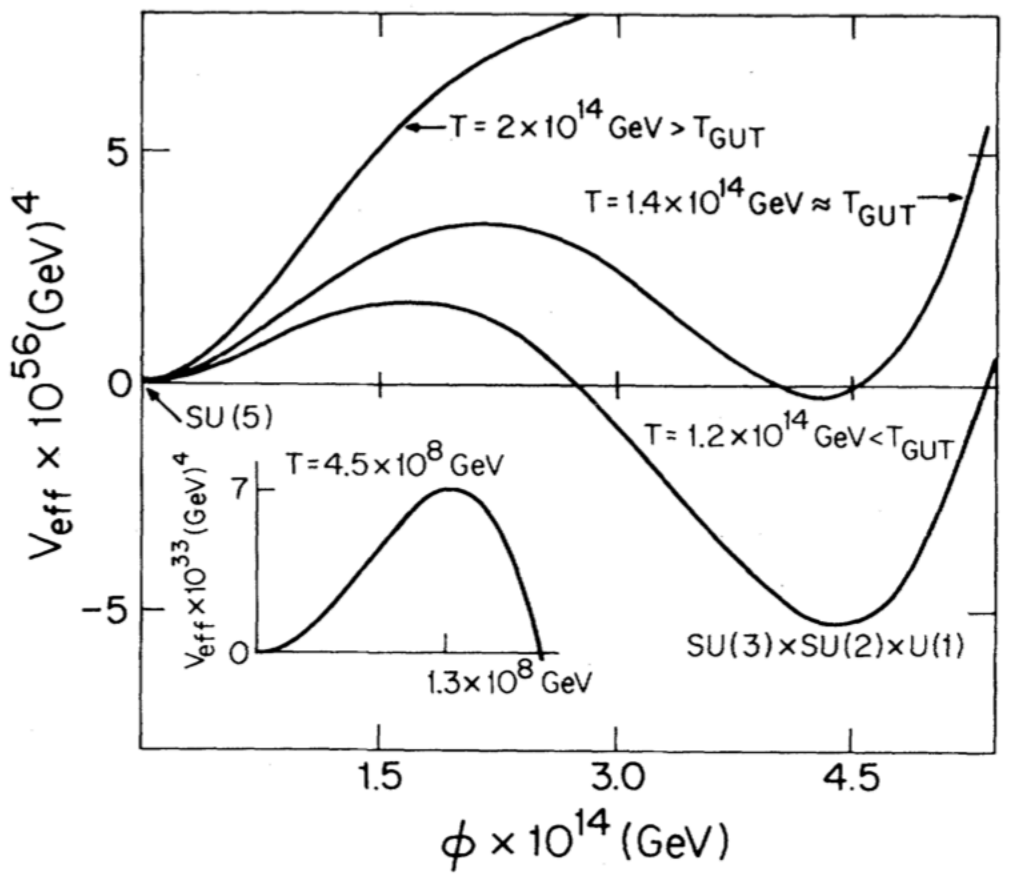


FIG. 1. Effective potential vs φ for various values of T.

new inflation

See Guth & Steinhardt (1984): http://astroweb.case.edu/ssm/USNA287/InflationGuthSteinhardt.html

Schramm (1993)

BBN

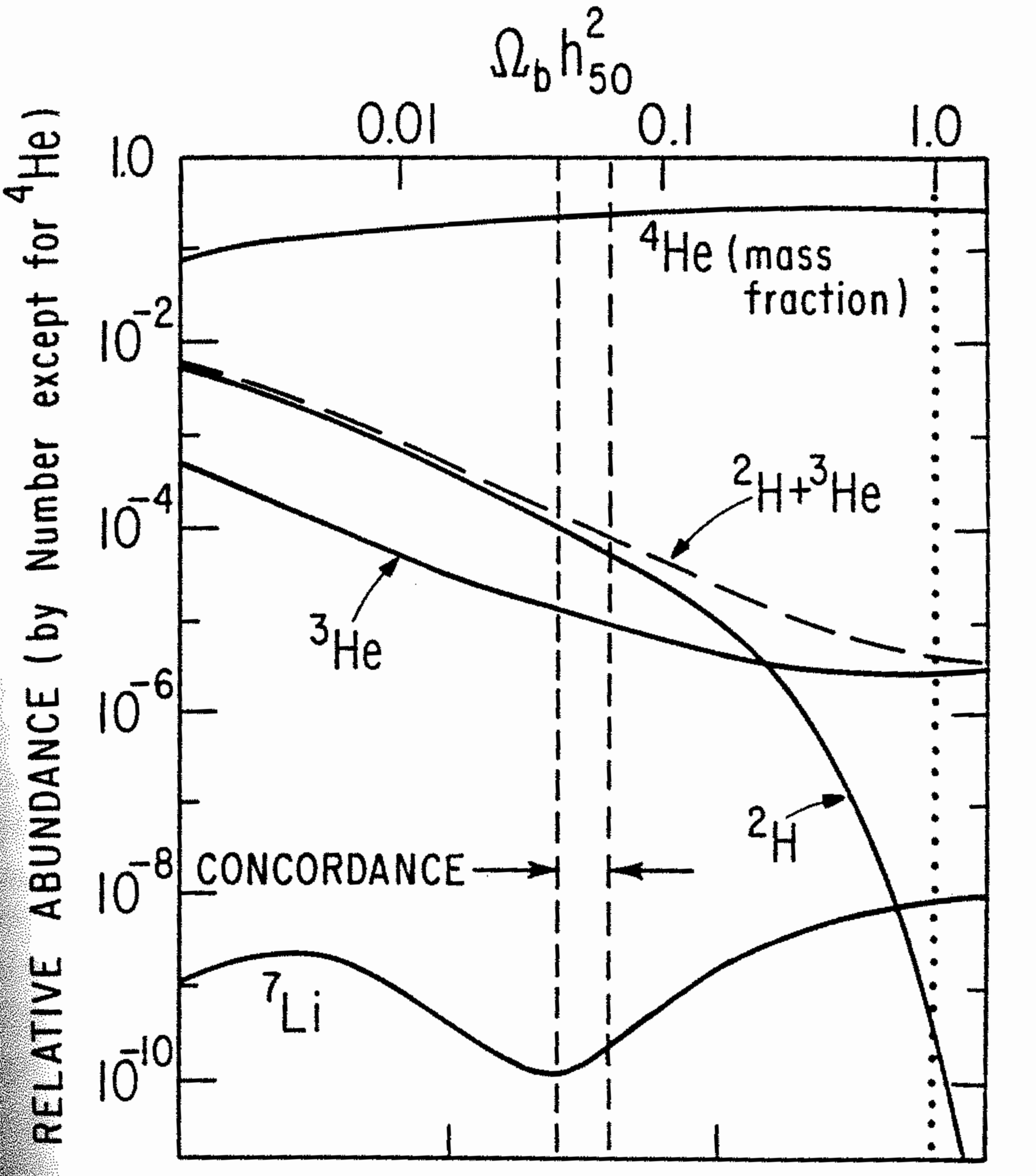


Figure 1. The abundances of light isotopes produced in the standard big bang model as a function of the fraction of the critical density in baryons, Ω_b. Note that the agreement with all abundance determinations is for $\Omega_b \sim 0.06$.

Mass density with scale

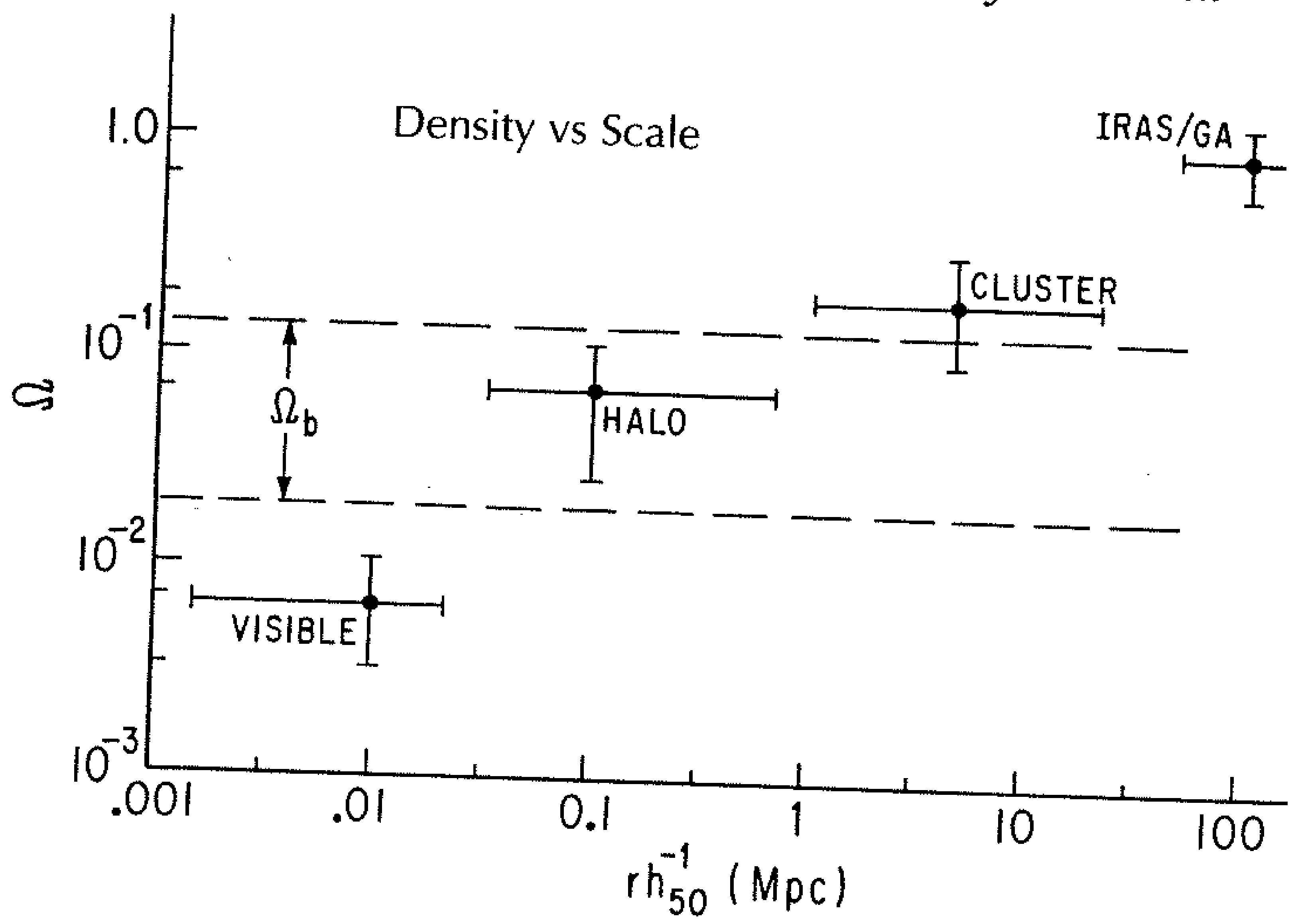


Figure 2. The inferred density in units of the critical density as a function of the scale on which it is "measured." Note the increase in Ω toward unity as larger scales are probed. Note also that Ω_b agrees with densities on the scale of galactic halos and is greater than the amount of visible matter.

Schramm (1993)

Growth of Structure

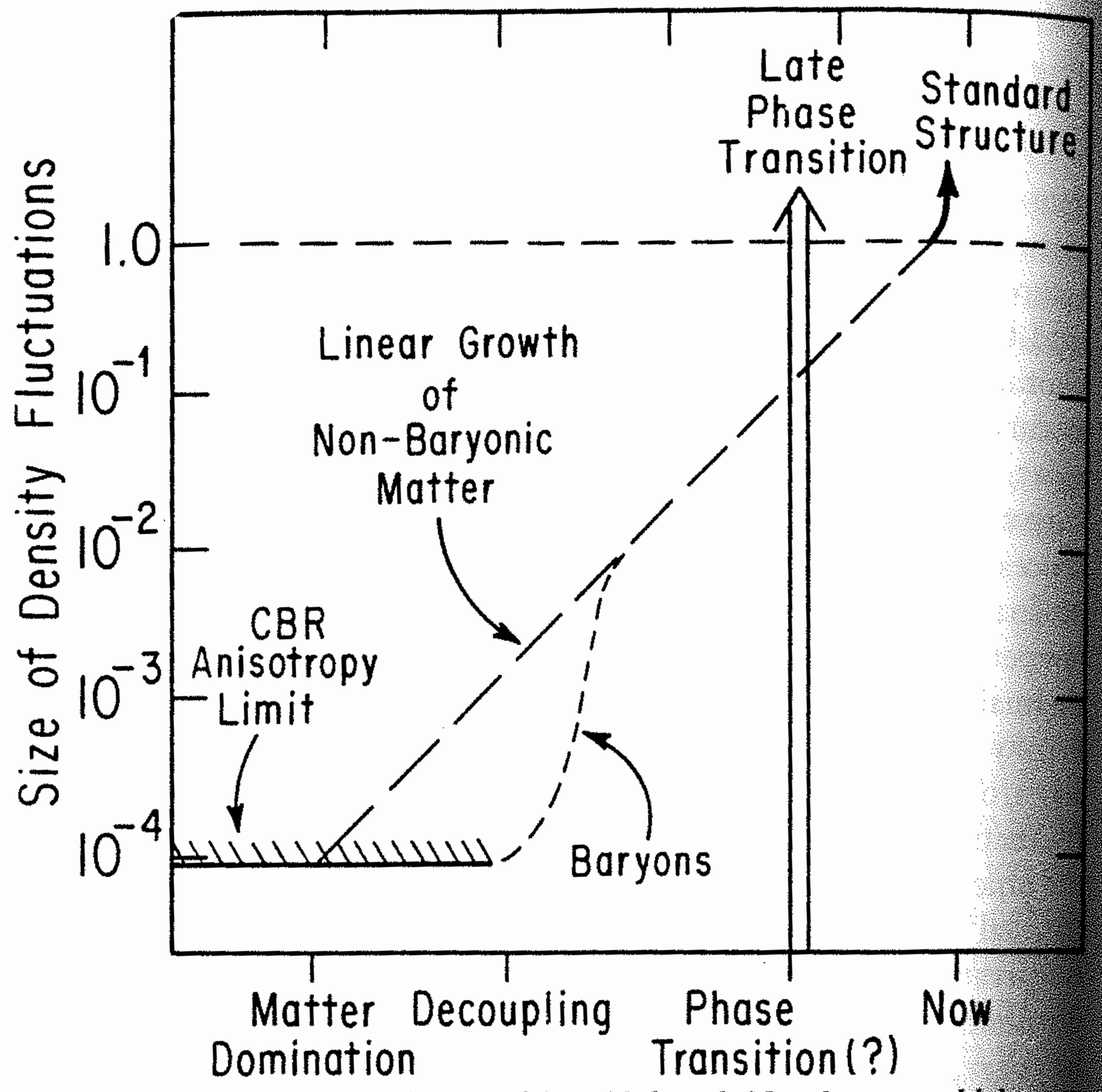


Figure 3. Structure growth as a function of the redshift epoch. Note that any model that starts with primordial seeds that are constrained by the isotropy of the cosmic background radiation produces most of its structure relatively late.

Mass Budget

Table 1. Matter

Baryonic ($\Omega_b \sim 0.06$)

- Visible
  - $\Omega_{vis} \leq 0.01$
- Dark
  - Halo
    - Jupiters
    - Brown dwarfs
    - Stellar black holes
  - Intergalactic
    - Hot gas at $T \sim 10^5$K
    - Stillborn Galaxies

NonBaryonic ($\Omega_{nb} \sim 0.94$)

- Hot
  - m_{ν_τ} 25eV
- Cold
  - WIMPS/Inos ~ 100GeV
  - Axions ~ 10^{-5}eV
  - Planetary mass black holes

Schramm (1993)

N(z) - lots more structure than expected

correlation function - lots more power on large scales than expected

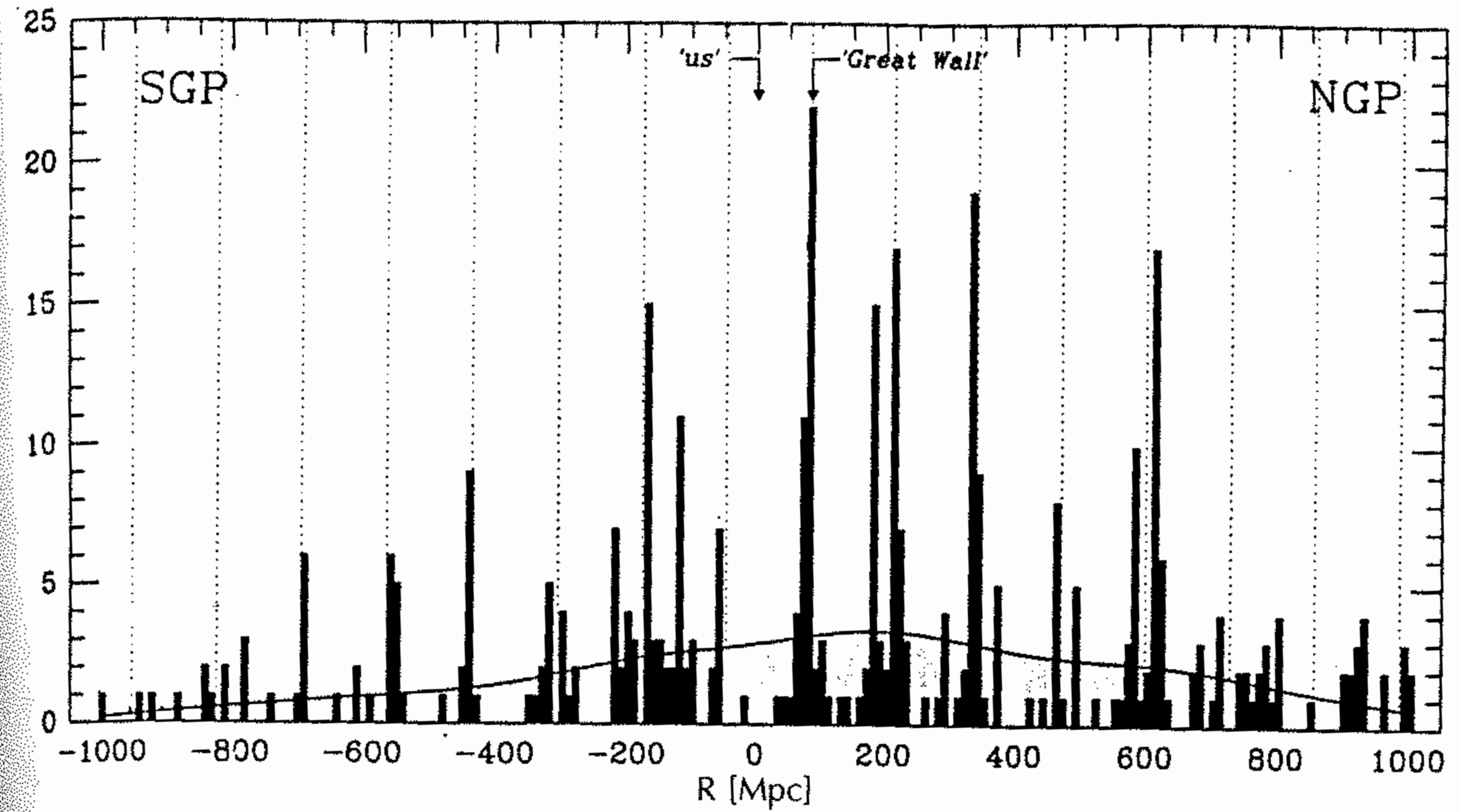


Figure 4. Data from the pencil beam surveys of Broadhurst, Ellis, Koo, and Szalay using the Anglo-Australian Telescope for the South Galactic Pole (SGP) pencil and the Kitt Peak Telescope for the North Galactic Pole (NGP) pencil. The plot shows numbers of galaxies versus distance. The solid line with a shaded region under it represents what we would find for a random distribution of galaxies. The spikes appear to show the pencil penetrating walls of galaxies. The average spacing of the spikes is about 130 Mpc.

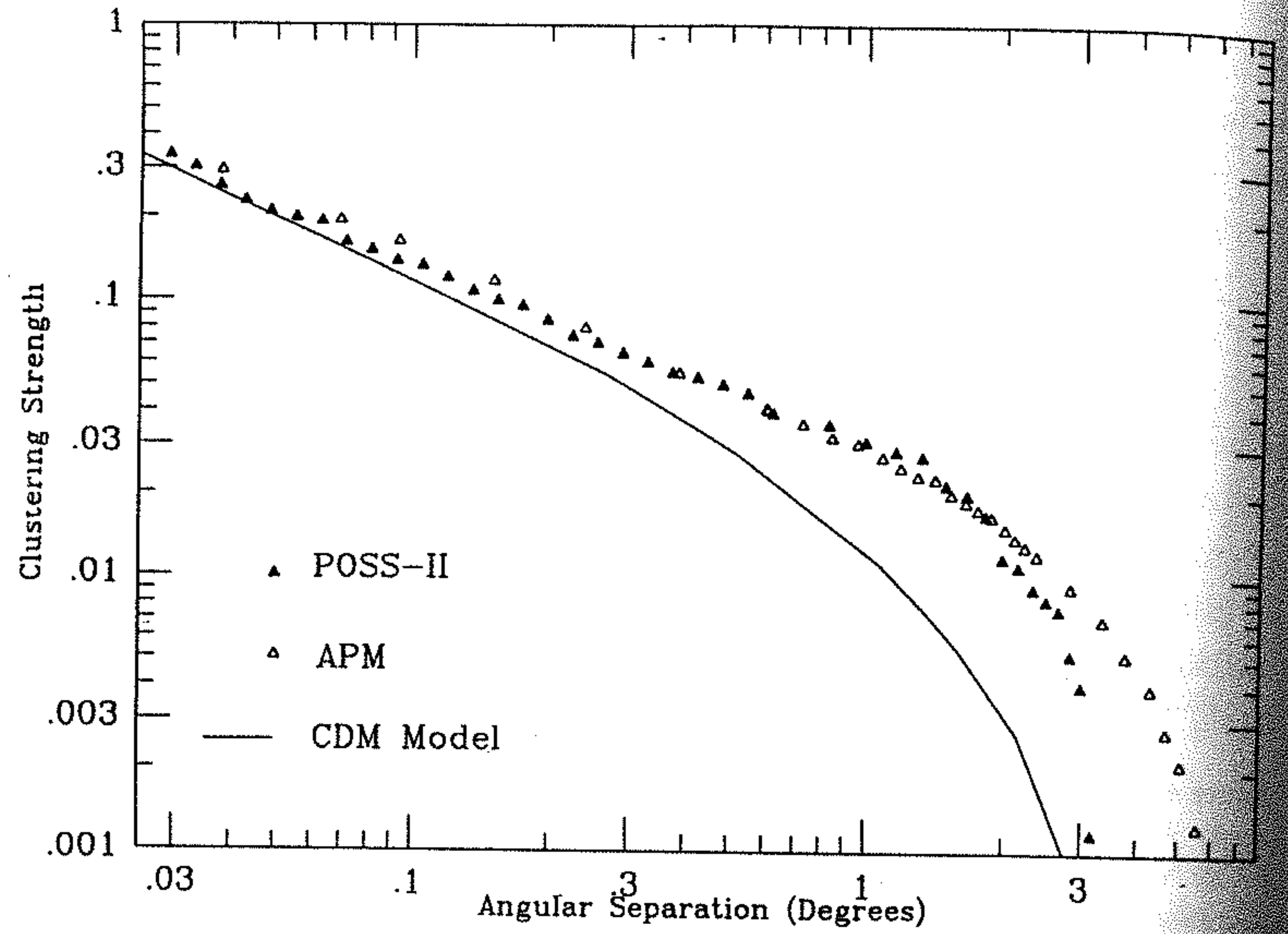


Figure 5. Galaxy clustering strength versus angular separation for the second Palomar Sky Survey (POSS II) and for the Automatic Plate Measuring (APM) Survey. The solid curve represents observations predicted from the cold dark matter theoretical model with a flat Gaussian initial fluctuation spectrum.

From an accident report in the *Boston Driver's Handbook*:

"The guy was all over the road. I had to swerve several times before I hit him."

The power spectrum of SCDM missed badly:
too much power on small scales;
too little power on large scales.

<u>SCDM</u> ("Standard" CDM)

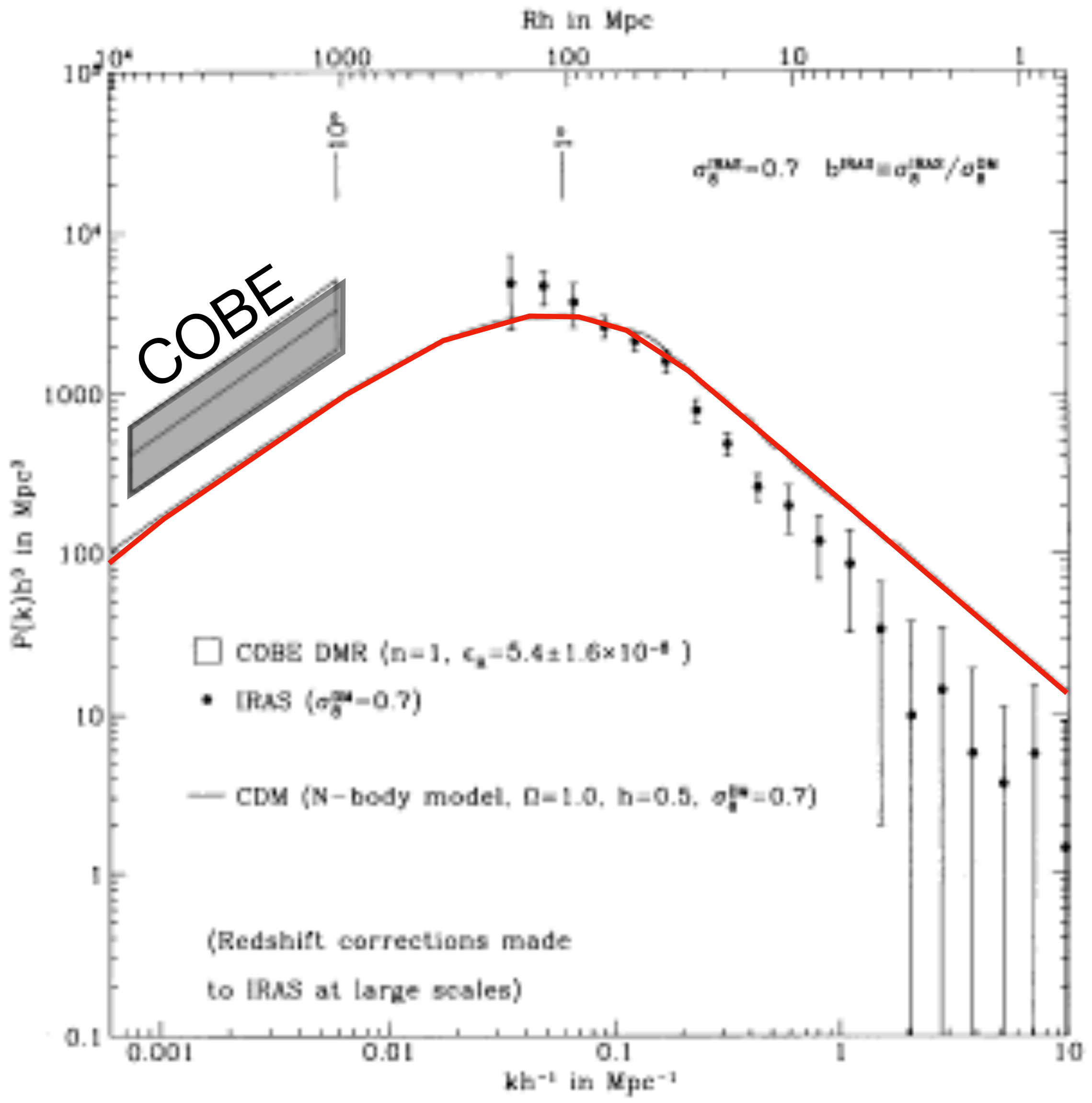
$$\Omega_m = 1$$

$$H_0 = 50$$

$\Omega_m h = 0.5$ expected

$\Omega_m h \approx 0.2$ observed

Schramm (1993) also expressed concern about the existence of quasars at $z \approx 4$ (!)



SCDM
$\Omega_m h = 0.5$
$\sigma_8 = 0.7$

FIG. 10.—Solid curve is the real space power spectrum of the full nonlinear CDM *N*-body simulation (as in Fig. 3) normalized to the real space variance of *IRAS* galaxies ($\sigma_8 = 0.7$). The points are the *IRAS* redshift space $\tilde{P}(k)$ from Fig. 4, rescaled by eq. (17) with $\Omega = 1$ and $b = 1$; this is then, apart from the effects of the convolution in eq. (14), an approximation to the power spectrum of *IRAS* galaxies in *real* space on large scales if the *IRAS* galaxies are unbiased. The box indicates the power spectrum inferred from the *COBE* DMR measurements, assuming a $n = 1$ spectral index and $\epsilon_H = (5.4 \pm 1.6) \times 10^{-6}$ (Smoot et al. 1992; Wright et al. 1992). Note that when the CDM model is normalized to the *IRAS* variance, it produces excessive power on small scales while simultaneously failing to produce sufficient power on large scales to match the *COBE* results.

Fisher et al. (1993) ApJ, 402, 42

All this is solved by LCDM - provided that we are no longer concerned about the flatness/coincidence problem.