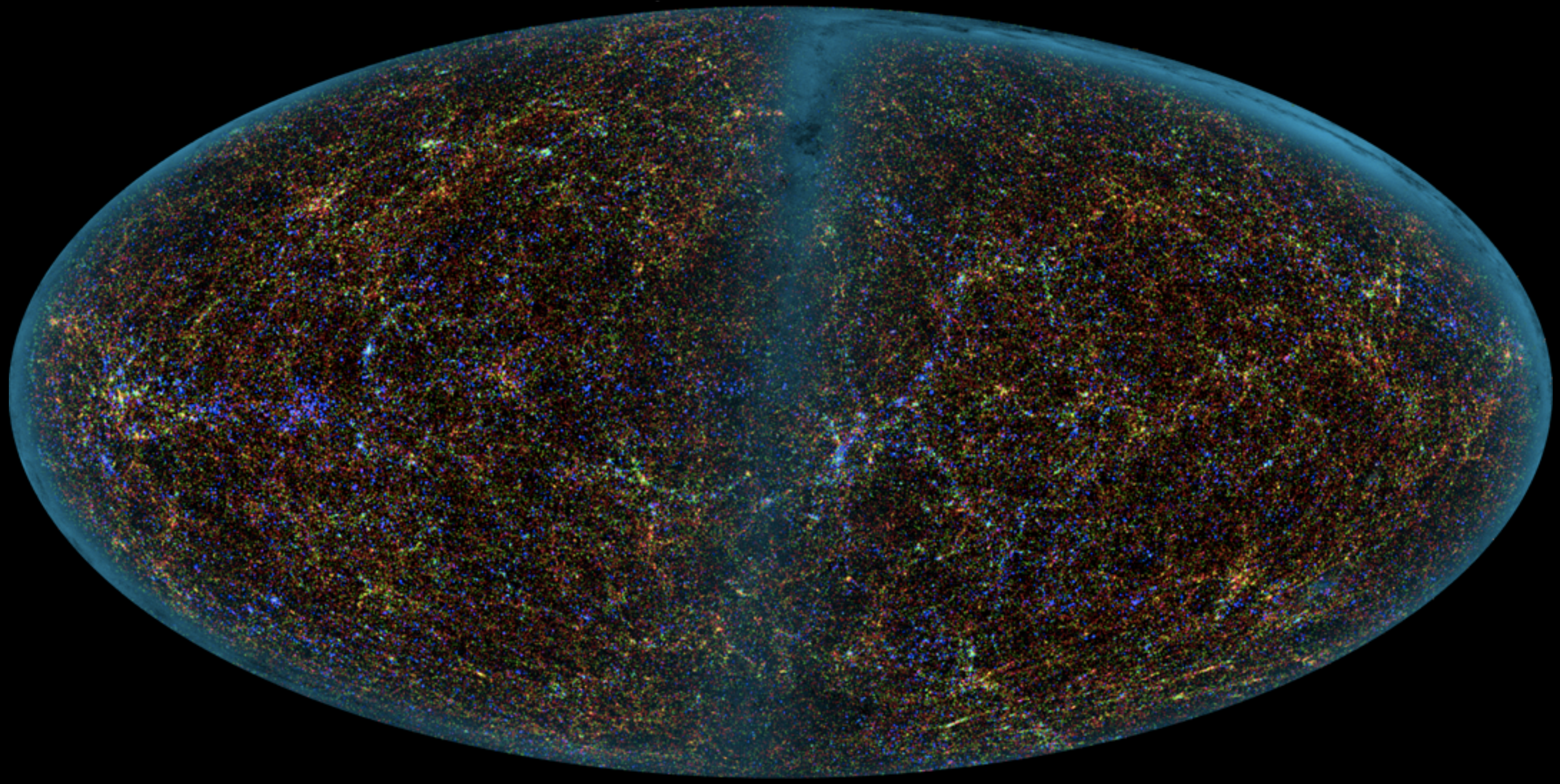
Cosmology

and Large Scale Structure



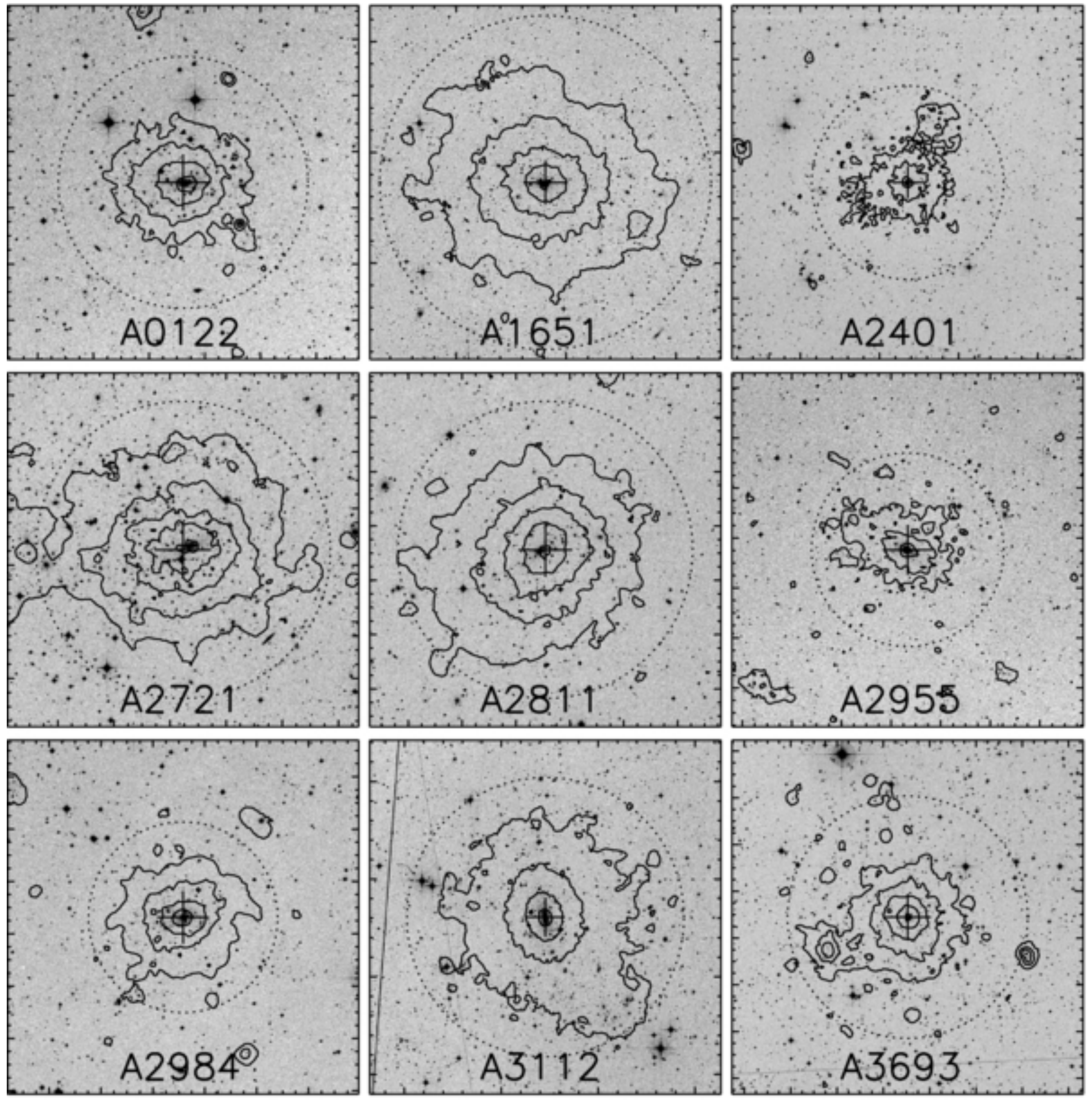
Today

Large Scale Structure

The Power Spectrum

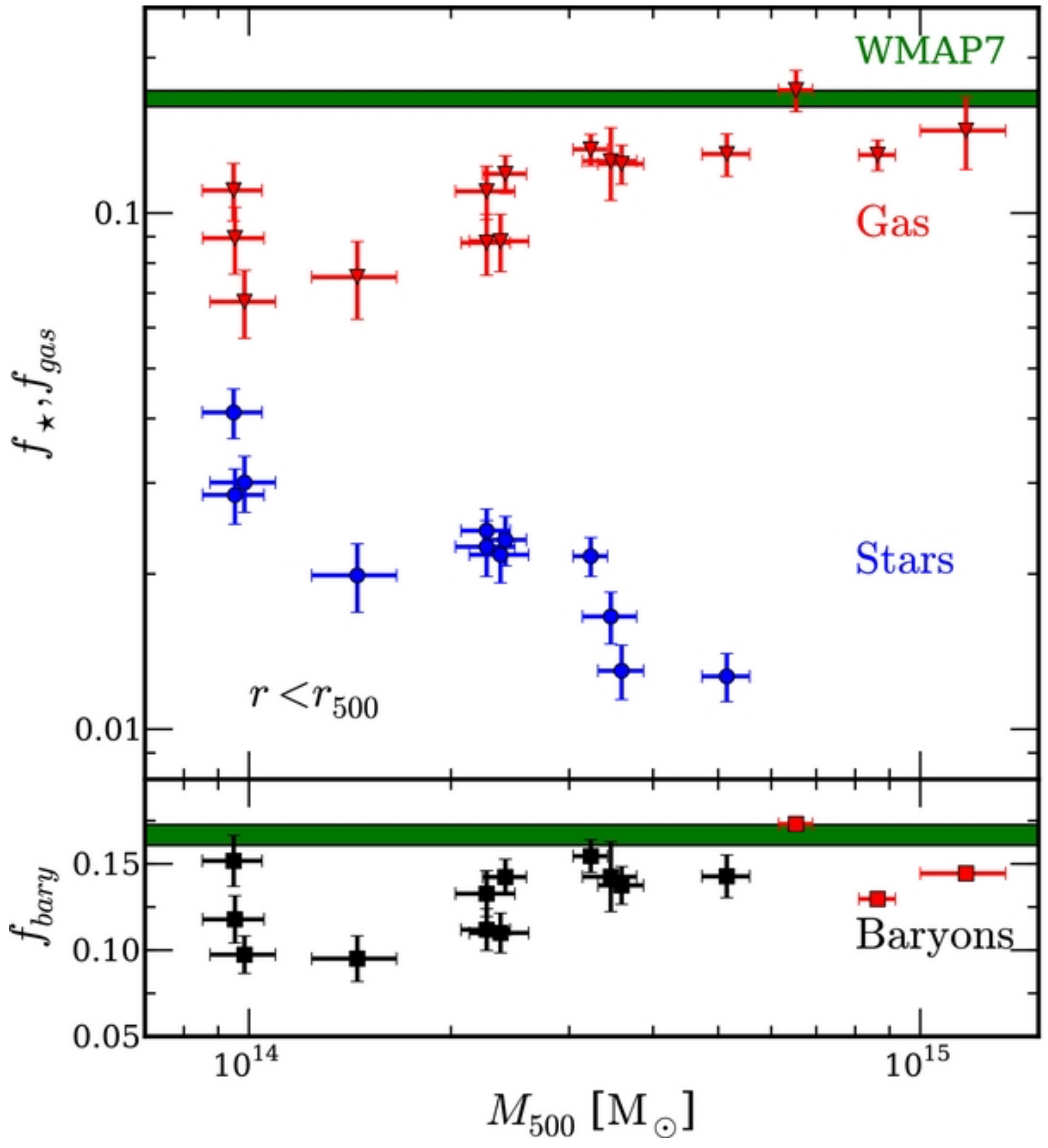
29 November 2022

http://astroweb.case.edu/ssm/ASTR328/



– but first - a word from our sponsor about cluster baryon fractions

Gonzalez et al. (2013)



– cluster baryon fractions

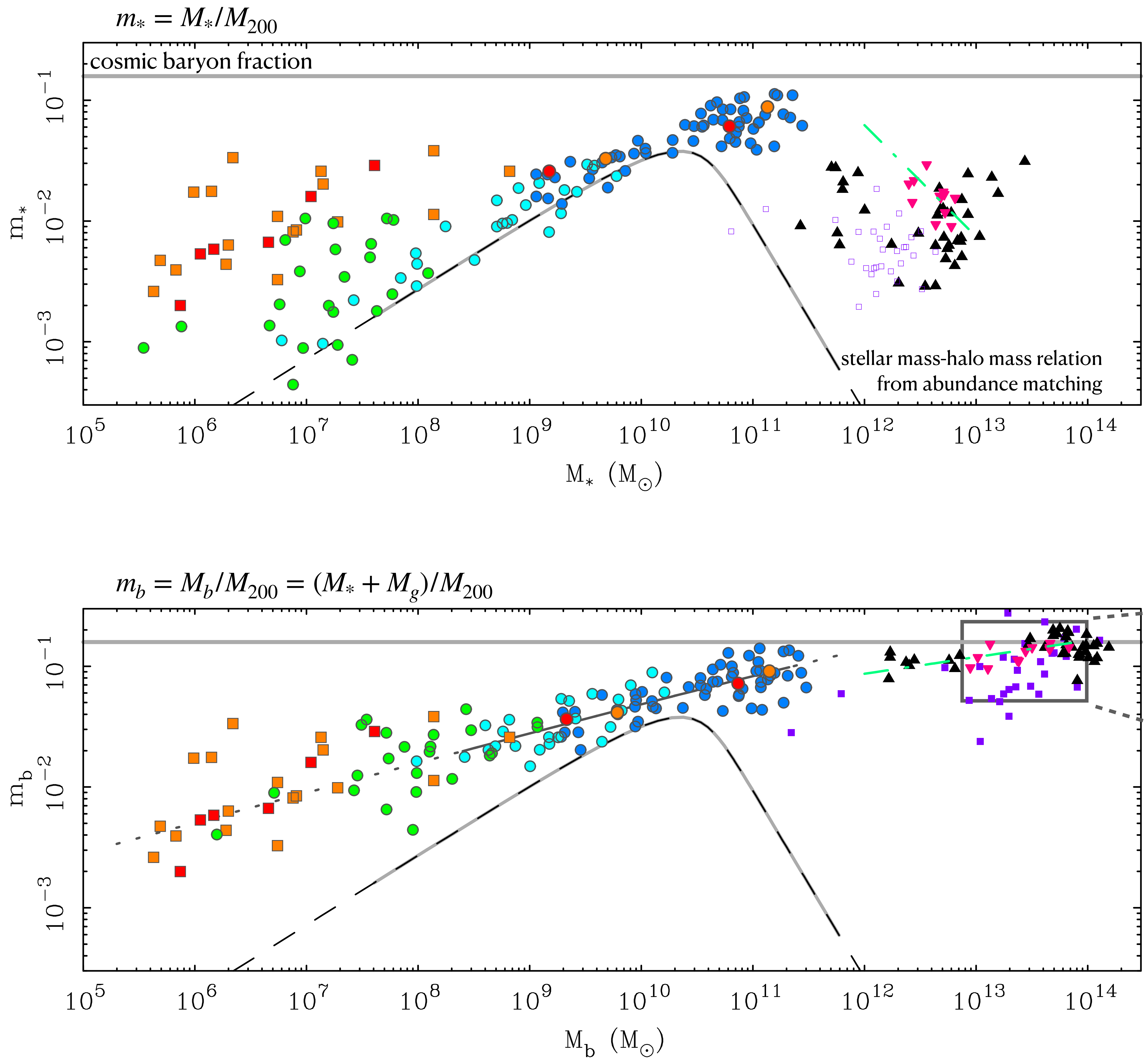
$$f_b = \frac{M_b}{M_{tot}} \quad \circ\!\longrightarrow \quad \Omega_m = \frac{\Omega_b}{f_b}$$

Measure cluster baryonic mass M_b from luminosity of X-ray gas (contours)
plus stars in galaxies (black)

Measure cluster dynamical mass M_{tot} from X-ray temperature (contours)
or weak lensing
or velocity dispersion

Most of the baryonic mass in rich clusters is in the hot, X-ray emitting gas of the ICM (intracluster medium). Only the most massive clusters approach the cosmic fraction found in fits to the acoustic power spectrum of the CMB. Lower mass clusters suffer a missing baryon problem.

– beyond cluster baryon fractions



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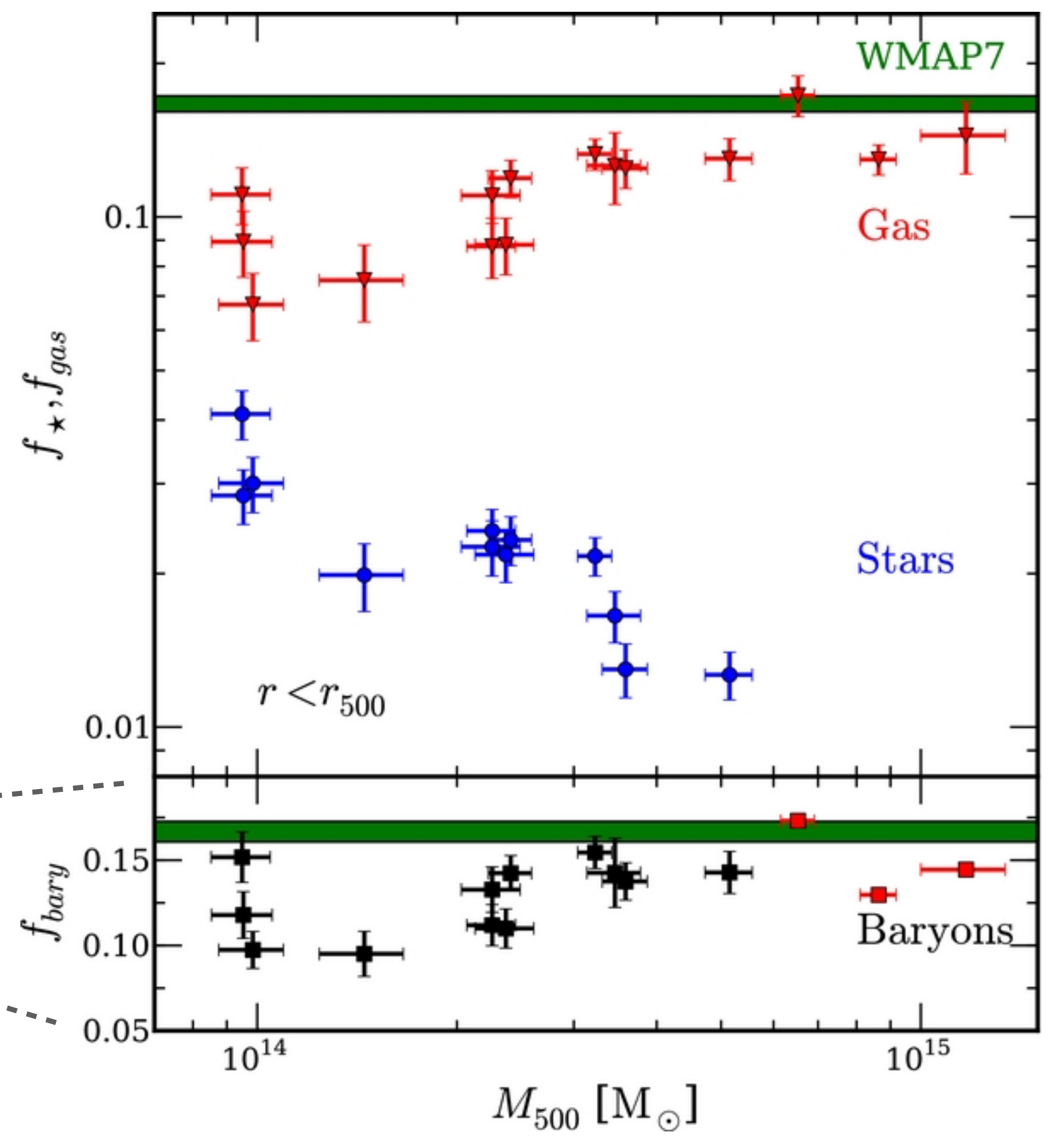

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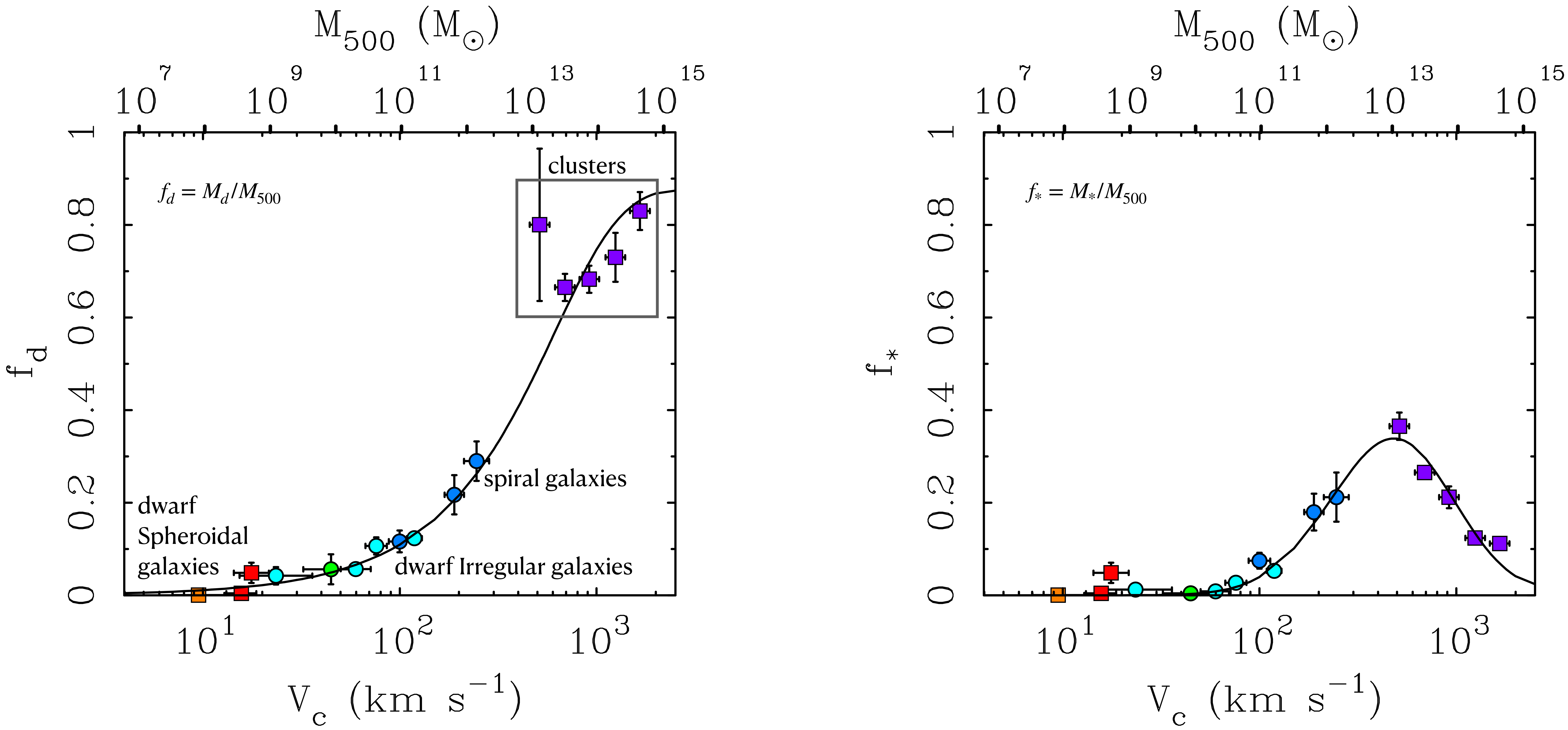
– beyond cluster baryon fractions

McGaugh et al. (2010)

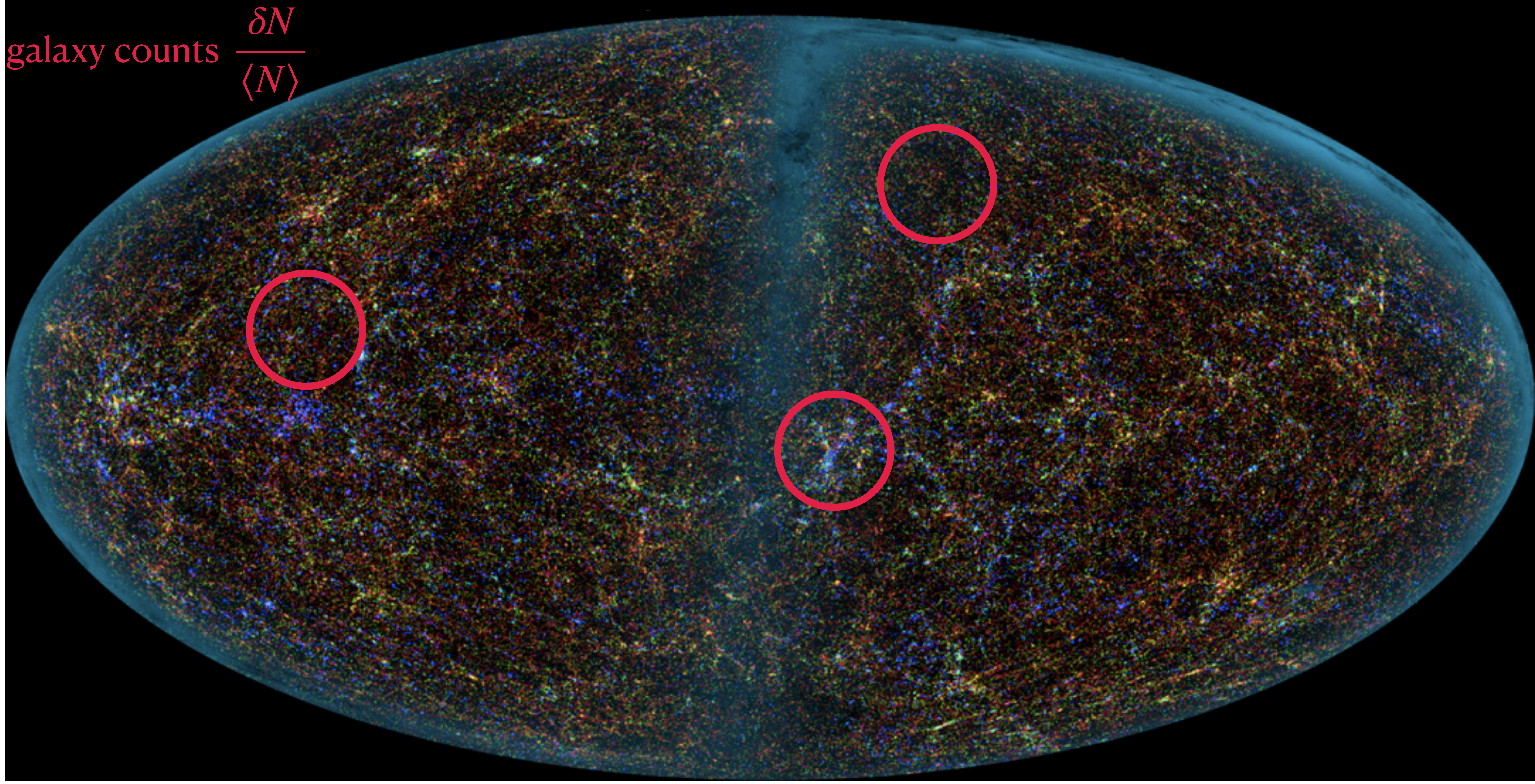


Structure formation basics:

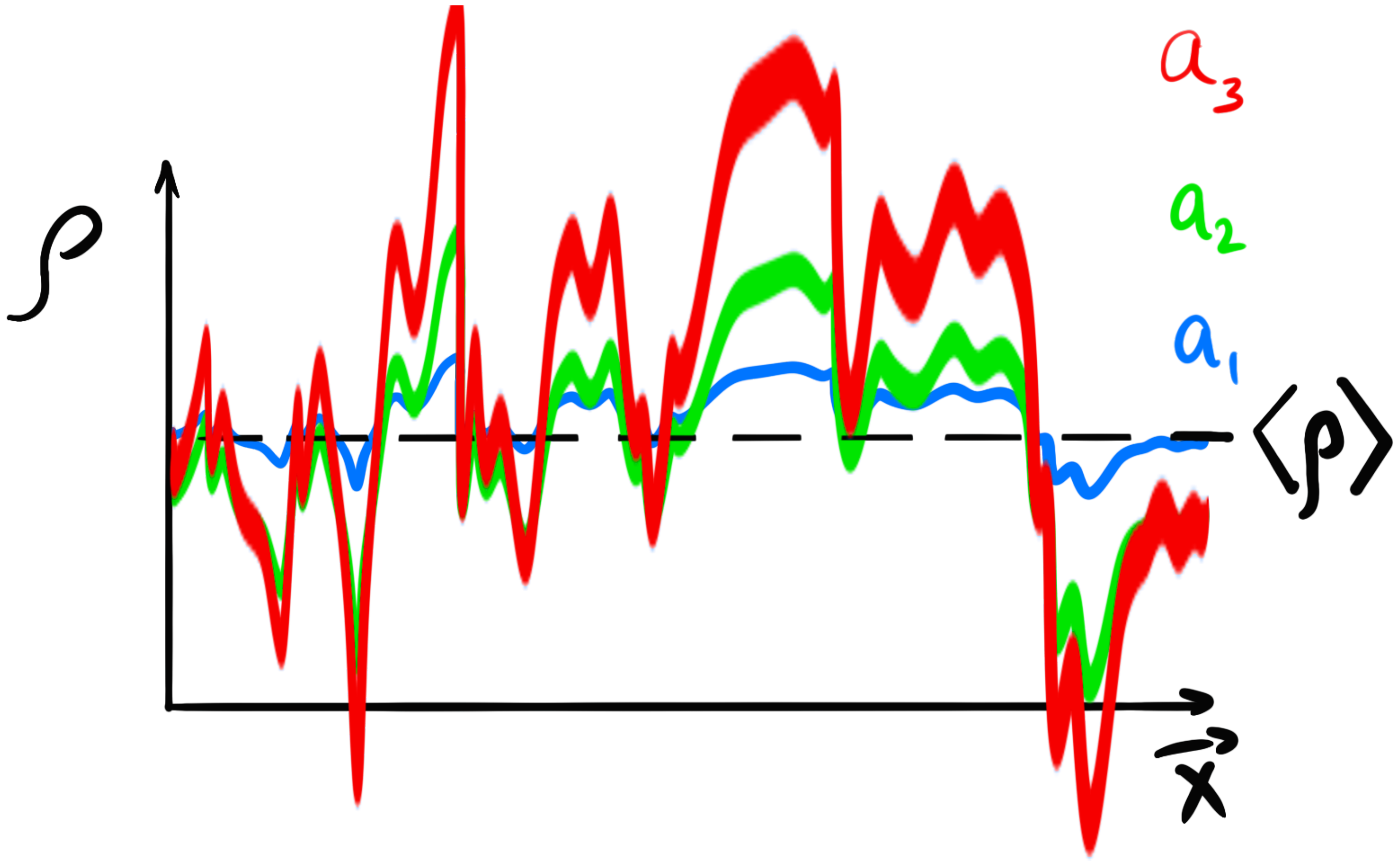
Density perturbations $\delta = \dfrac{\rho - \langle\rho\rangle}{\langle\rho\rangle}$

grow as $\delta(t) \sim a(t)$.

In the early universe, $\langle\rho\rangle = \rho_{\text{crit}}$.



over-density δ
grows with time



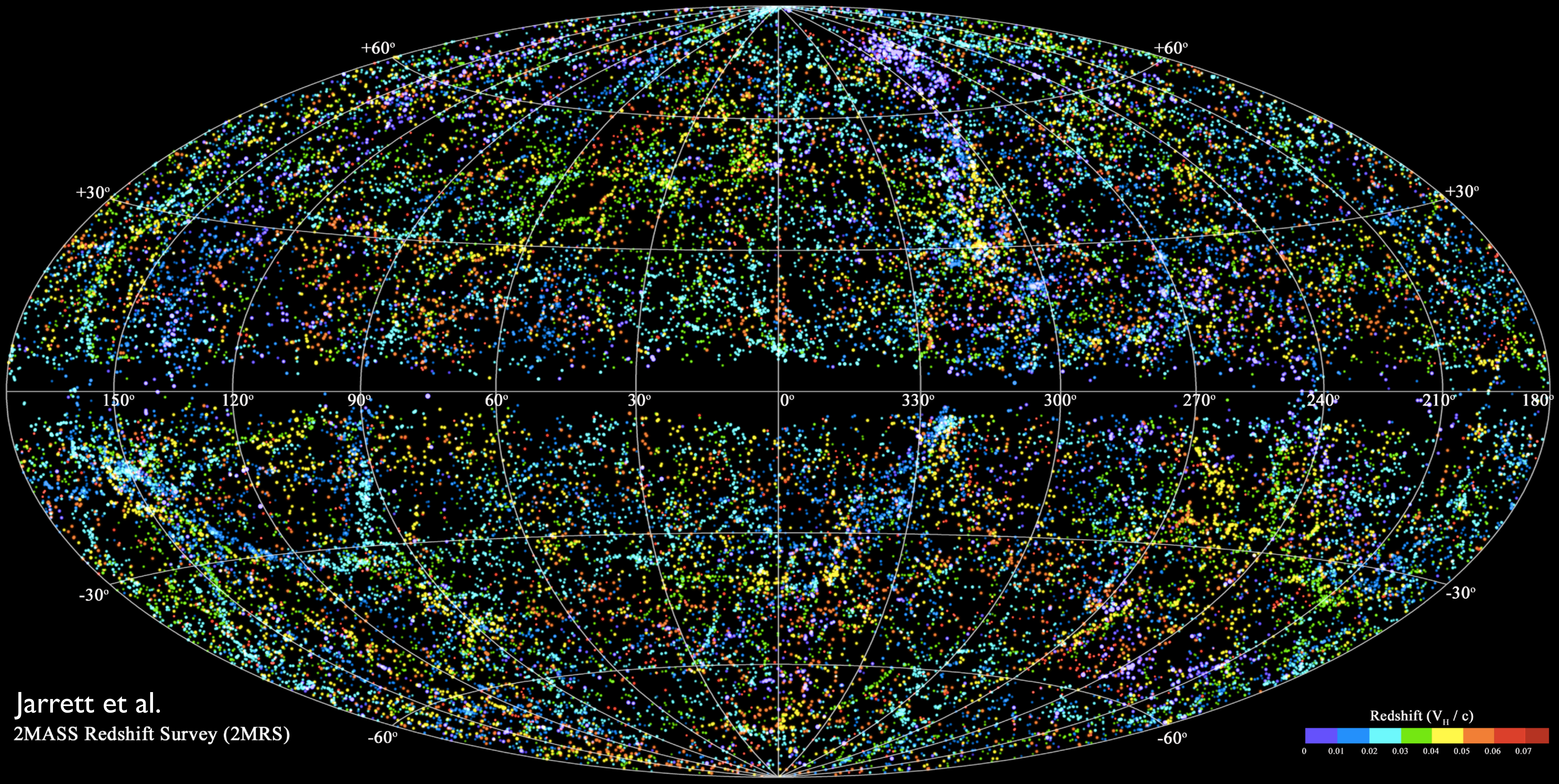
You can't get here from there

The factor of 100 offset in density and temperature fluctuations is a prime motivation for non-baryonic **cold dark matter** — a substance for which perturbations δ can grow sufficiently large while not leaving an imprint of corresponding magnitude on the CMB.

Radiation and baryon plasma tightly coupled at recombination, so a fluctuation in density is reflected by one in temperature: $\dfrac{\delta\rho}{\rho} \propto \dfrac{\Delta T}{T}$.

- Power spectrum of galaxies

Measure positions and redshift of galaxies; use this spatial distribution to construct the 3D correlation function



Jarrett et al.
2MASS Redshift Survey (2MRS)

Large Scale Structure

Quantified with the **correlation function** $\xi(r)$ which is the Fourier transform of the **power spectrum** $P(k)$.

The correlation function is the excess probability of finding a galaxy near another galaxy over that in a random distribution.

$$\frac{dN}{N} = [1 + \xi(r)]dV$$

$$\xi(r) = \frac{V}{(2\pi)^3} \int P(k) e^{-\vec{k}\cdot\vec{r}}\, d^3k$$

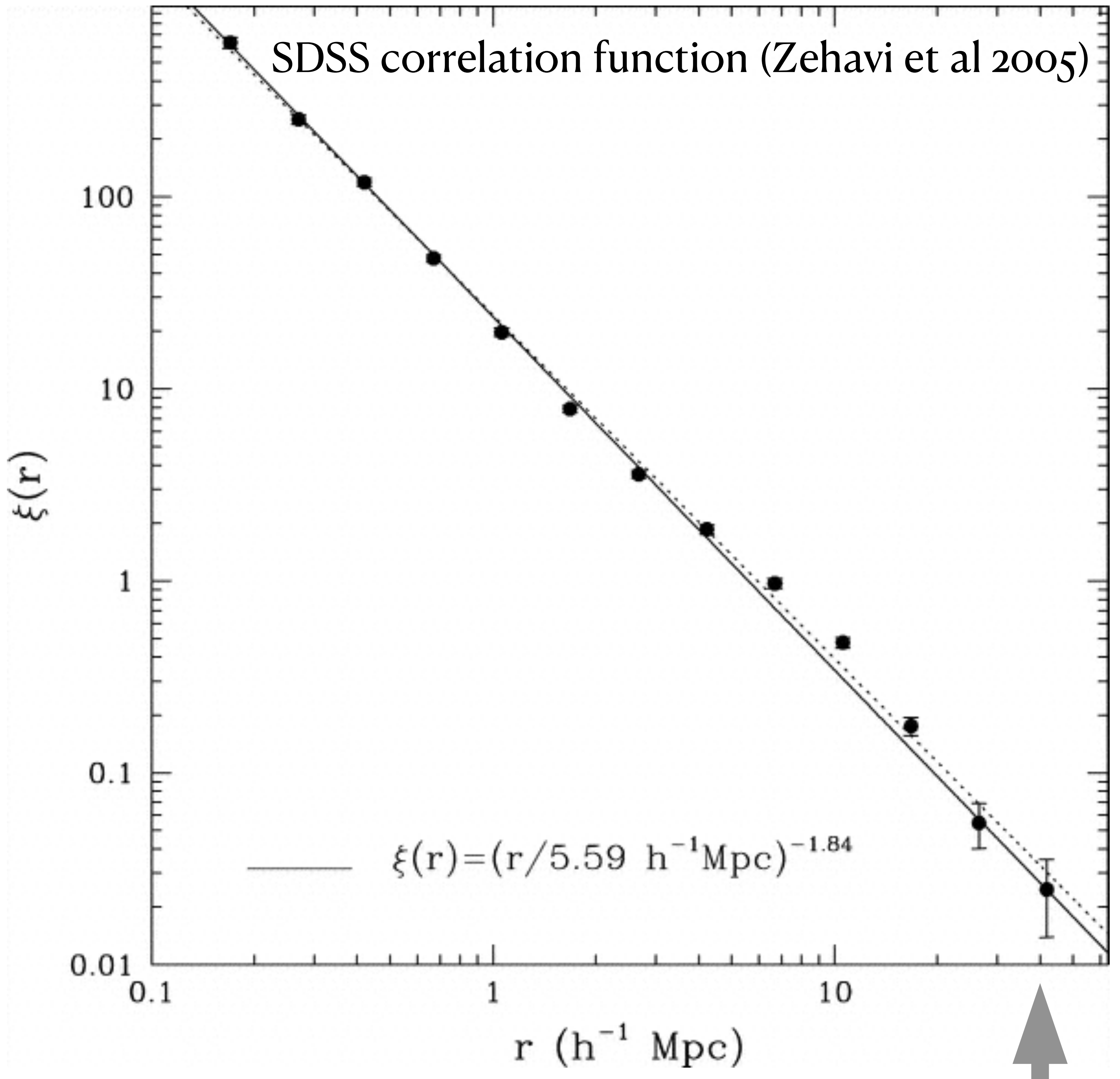
$$P(k) \propto |\delta(k)|^2 \propto k^n$$

$$\xi(r) \propto r^{-(n+3)}$$

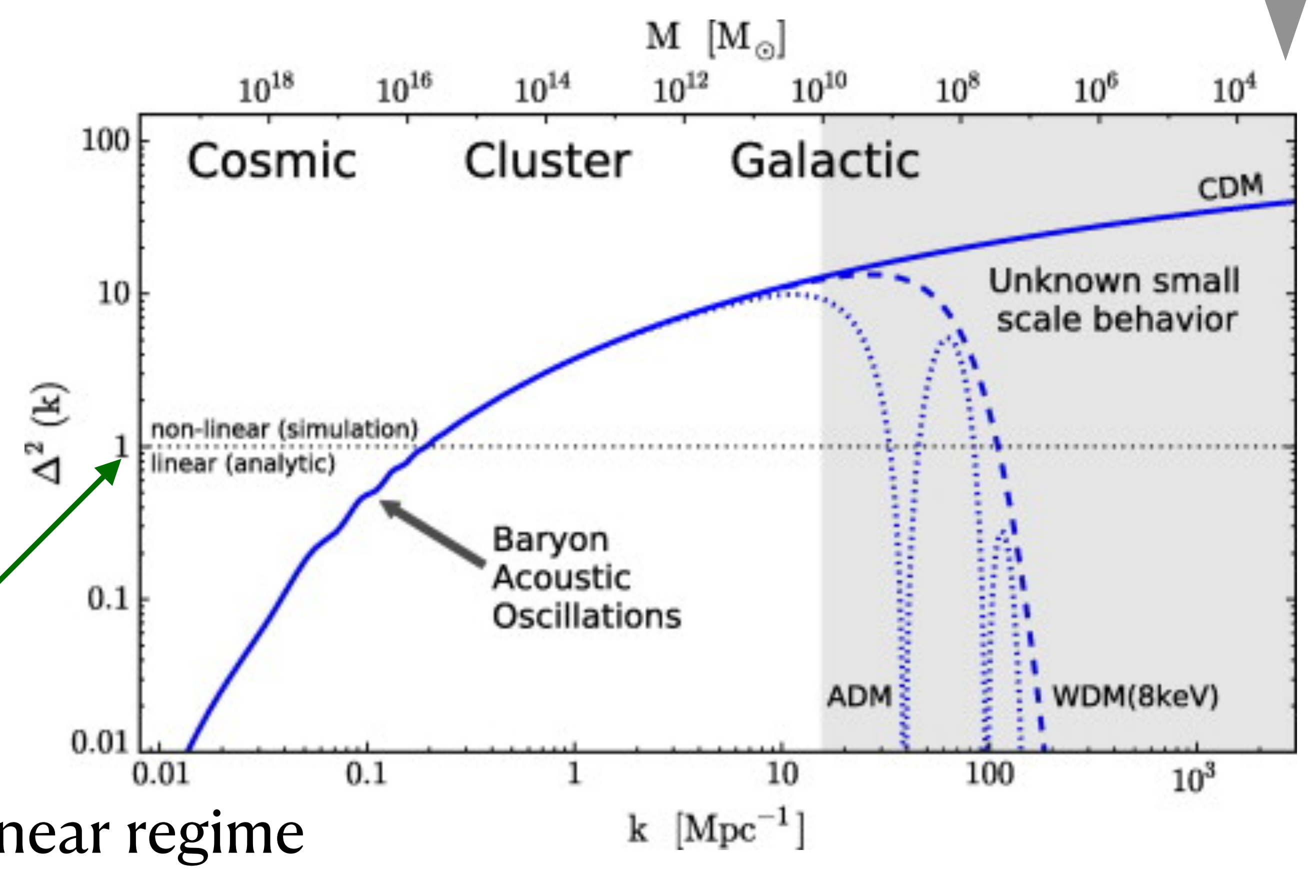
Harrison-Zeldovich spectrum has $n = 1$, which is a Gaussian random field.
Inflation predicts $n \approx 1$, but different flavors of Inflationary theory predict slightly different values depending on the shape of the Inflationary potential (the Inflaton).

Planck measures $n = 0.965 \pm 0.004$

The shape of the power spectrum is set by n
The amplitude of the power spectrum is set by σ_8



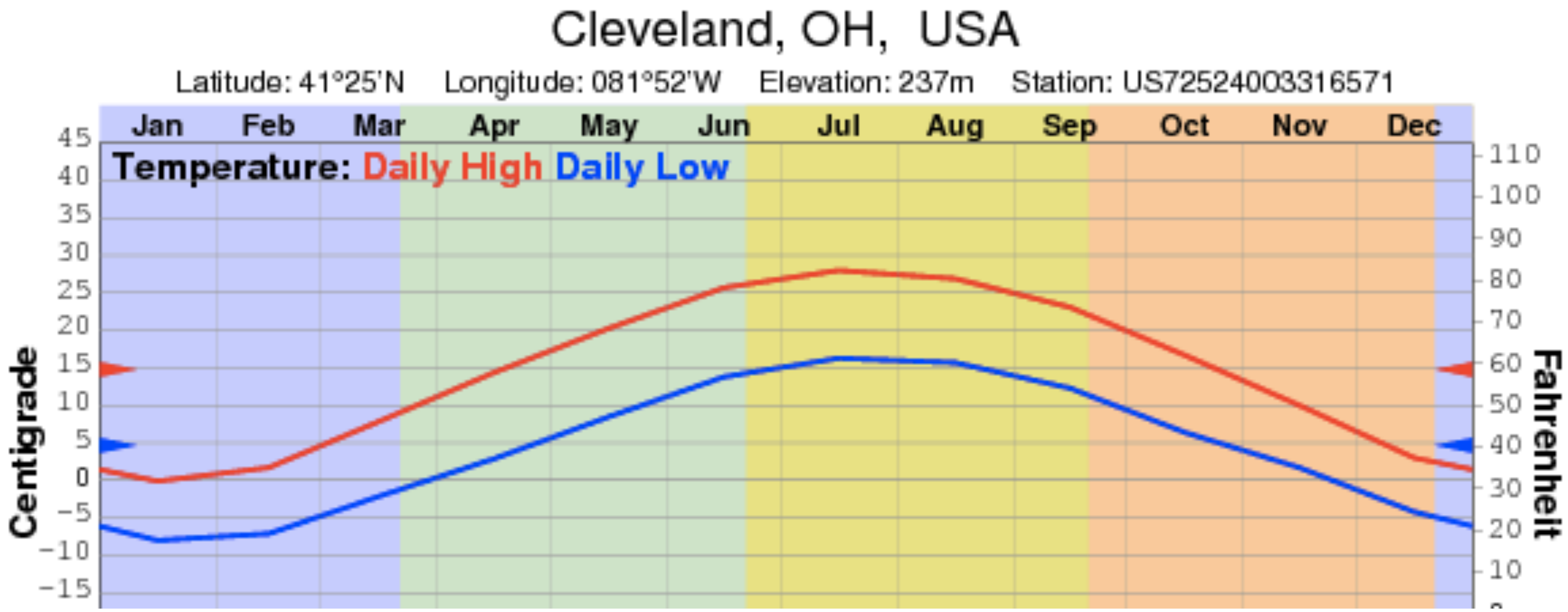
Power Spectrum

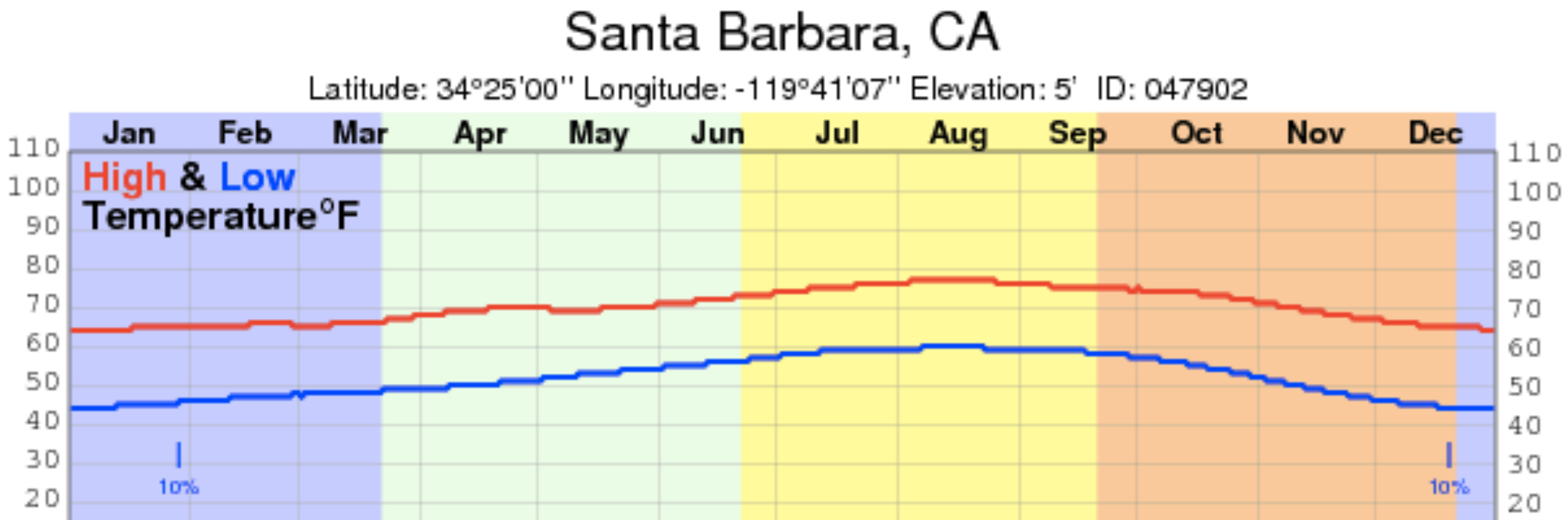


$\delta > 1$ marks the transition to the non-linear regime where perturbation theory no longer applies.

Power Spectrum

Example: weather in Cleveland and Santa Barbara
More power on long time scales in Cleveland (seasonal variation)

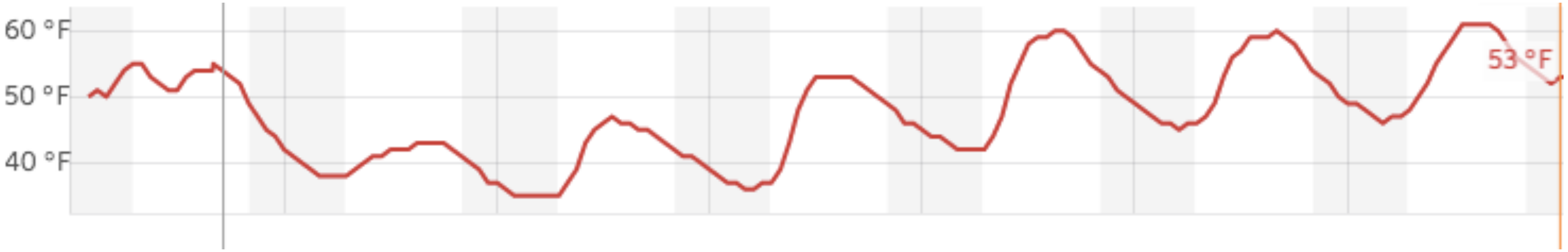




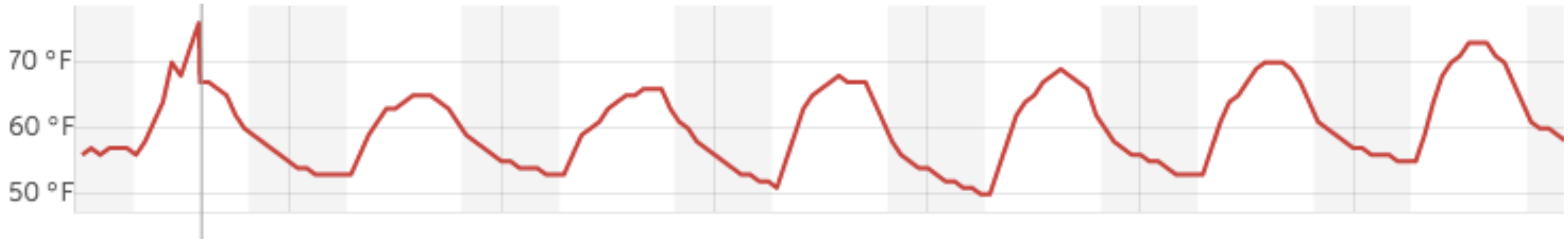
Power Spectrum

Example: weather in Cleveland and Santa Barbara
Similar power on short time scales in Santa Barbara (diurnal variation)

Cleveland forecast



Santa Barbara forecast



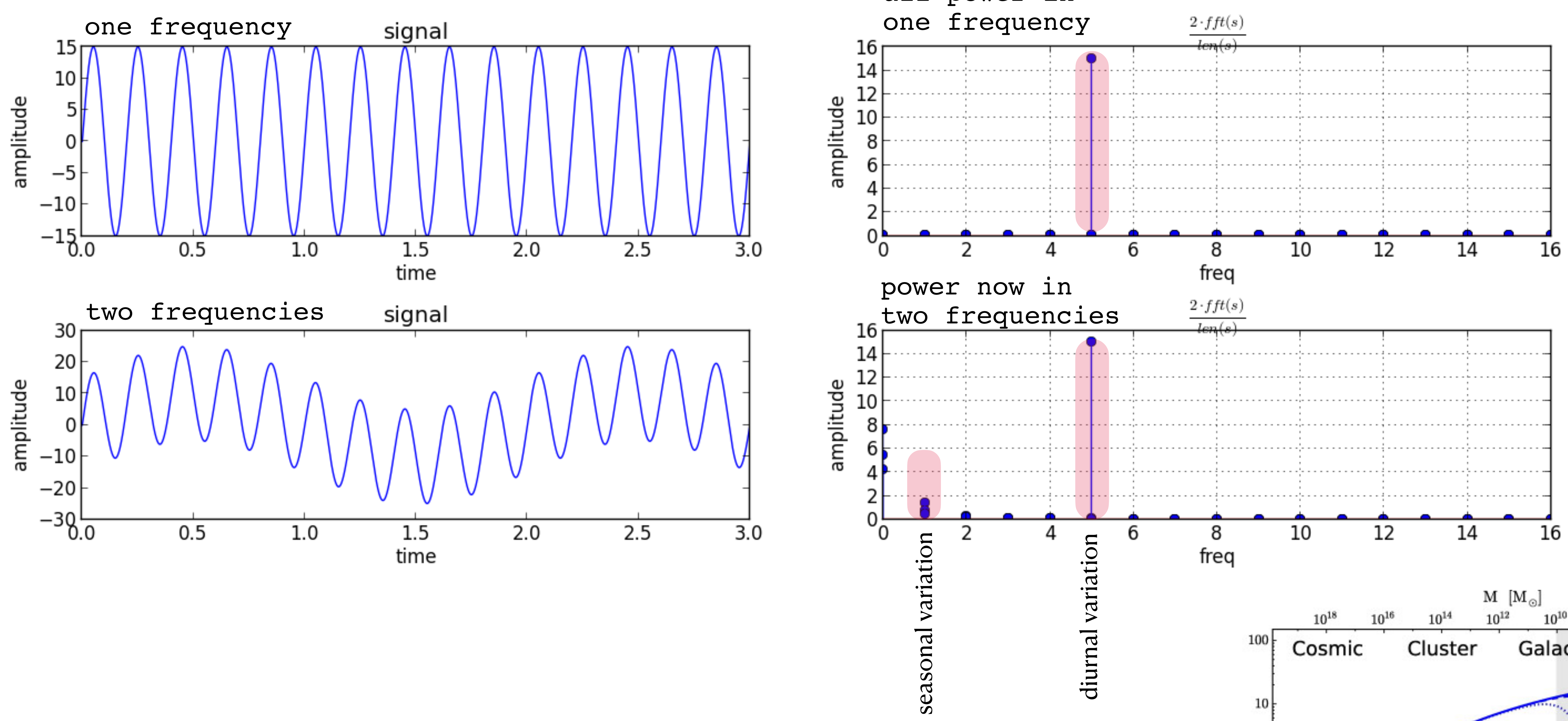
A power spectrum is a Fourier transform that quantifies the relative variability on different scales

Superposition of two sinusoids

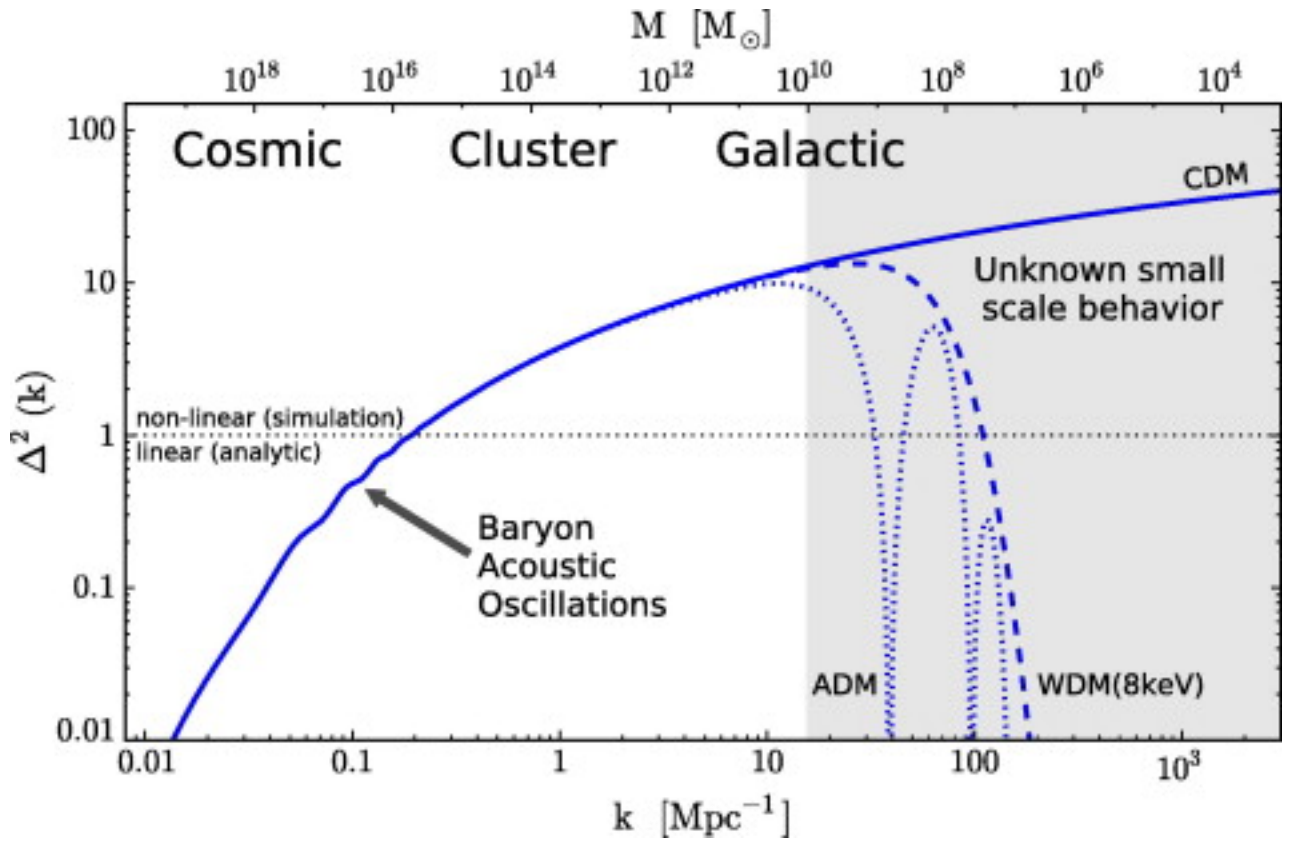
(e.g., diurnal and annual temperature variation)

Real space

Fourier space



So a smooth power spectrum has contributions from all frequencies, but also picks out which are more common.



- # Power spectrum of galaxies

$$\delta \equiv \frac{\delta\rho}{\rho}$$

The power spectrum is commonly used to quantify large scale structure.
It is the related to the 2 point correlation function via Fourier transform.

2 point correlation function: $\xi(r) = \langle \delta(\vec{x}) \cdot \delta(\vec{x} + \vec{r}) \rangle$

The 2 point correlation function is the probability of finding one galaxy near another in excess over a random distribution.

Power spectrum: $P(k) = \langle |\delta_k|^2 \rangle$ where $k = \dfrac{2\pi}{\lambda}$

where k is the wavenumber corresponding to the scale λ

Fourier transform:

$$\xi(\vec{r}) = \frac{V}{(2\pi)^3} \int |\delta_k|^2 e^{-i\vec{k}\cdot\vec{r}} d^3k$$

averaged over volume V

$|\delta_k|^2$ ↑ $P(k)$

the choice of "window function"
- how the volume is defined -
is a technical detail that matters

• Power spectrum of galaxies

$$\delta \equiv \frac{\delta\rho}{\rho} \qquad k = \frac{2\pi}{\lambda}$$

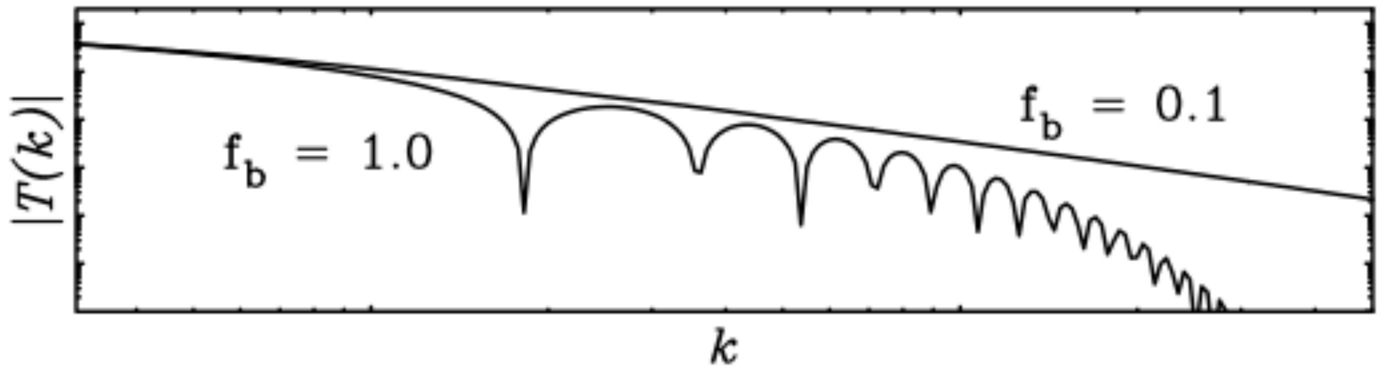
Power law power spectrum: $$P(k) = \langle |\delta_k|^2 \rangle \propto k^n$$

where n = 1 is scale free, with the same power on all scales.
This is observed to be nearly the case on large scales that have not yet collapsed. It is modulated on small scales by structure formation.

One way to think of it is the rms variation at each scale λ

$$M \sim \lambda^3 \qquad \delta_{\rm rms} \propto M^{-(n+3)/6}$$

cold dark matter smooths baryon acoustic oscillations

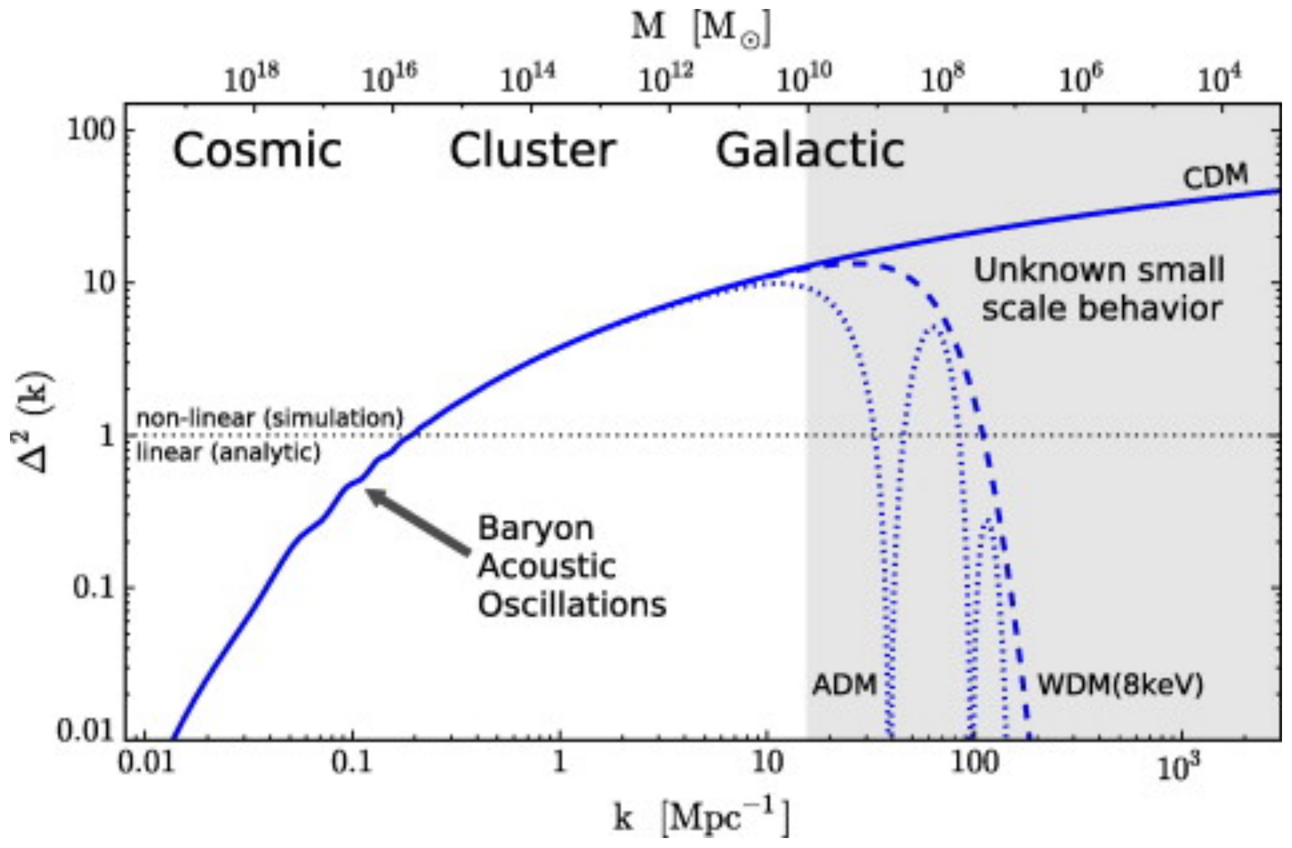


There is more rms variance on small scales, so more power there.
[On very large scales, the universe is homogeneous, so no variance.]

By convention, the normalization is set on a scale of 8 Mpc, where

$$\frac{\delta N_{gal}}{N_{gal}} = 1$$ with corresponding mass variance σ_8

Planck measures $\sigma_8 = 0.811 \pm 0.006$



From an accident report in the *Boston Driver's Handbook*:
"The guy was all over the road. I had to swerve several times before I hit him."

The power spectrum of SCDM missed badly:
too much power on small scales;
too little power on large scales.

<u>SCDM</u> ("Standard" CDM)

$$\Omega_m = 1$$
$$H_0 = 50$$

$\Omega_m h = 0.5$ expected

$\Omega_m h \approx 0.2$ observed

Schramm (1993) also expressed concern about the existence of quasars at $z \approx 4$ (!)

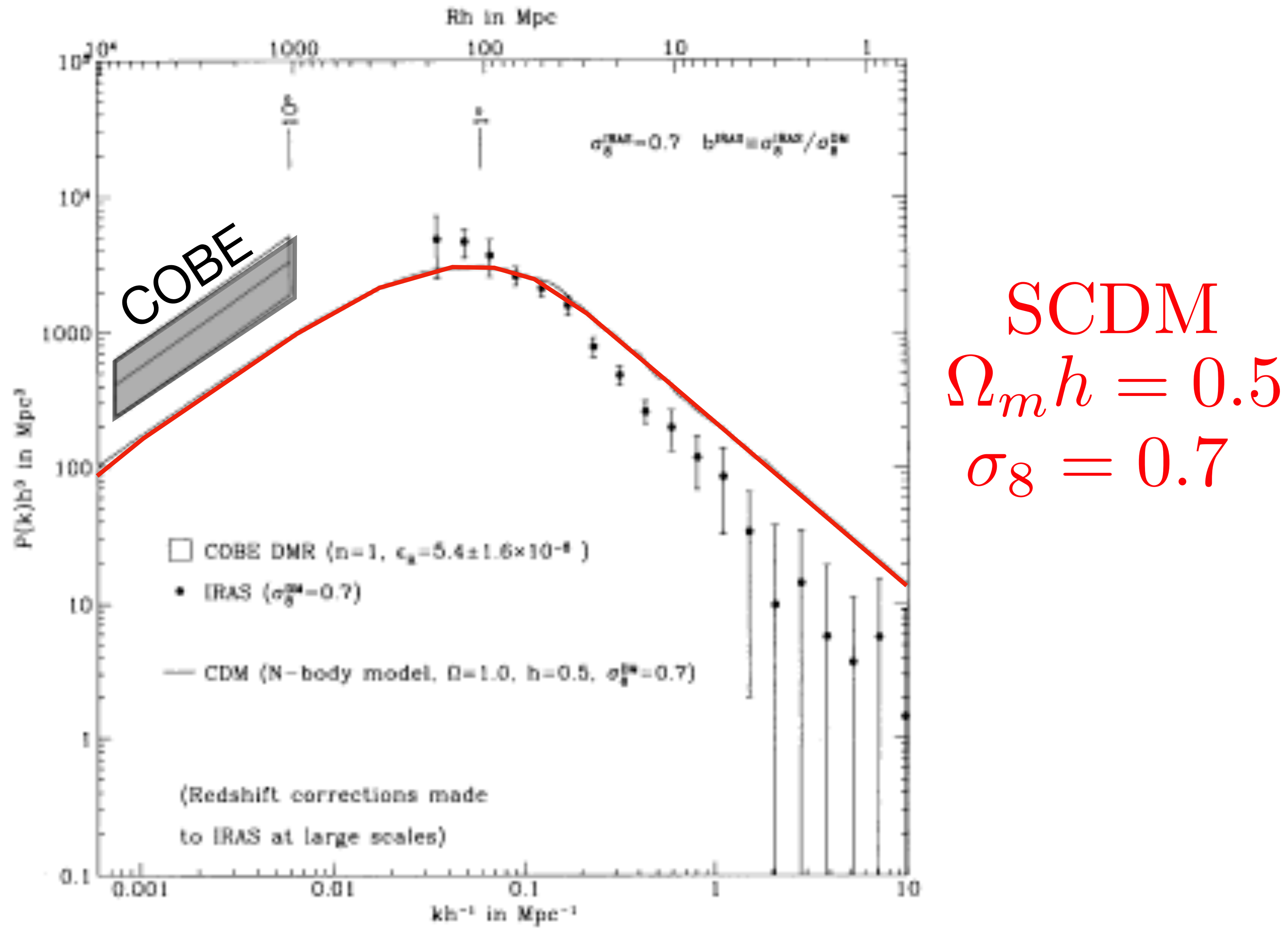
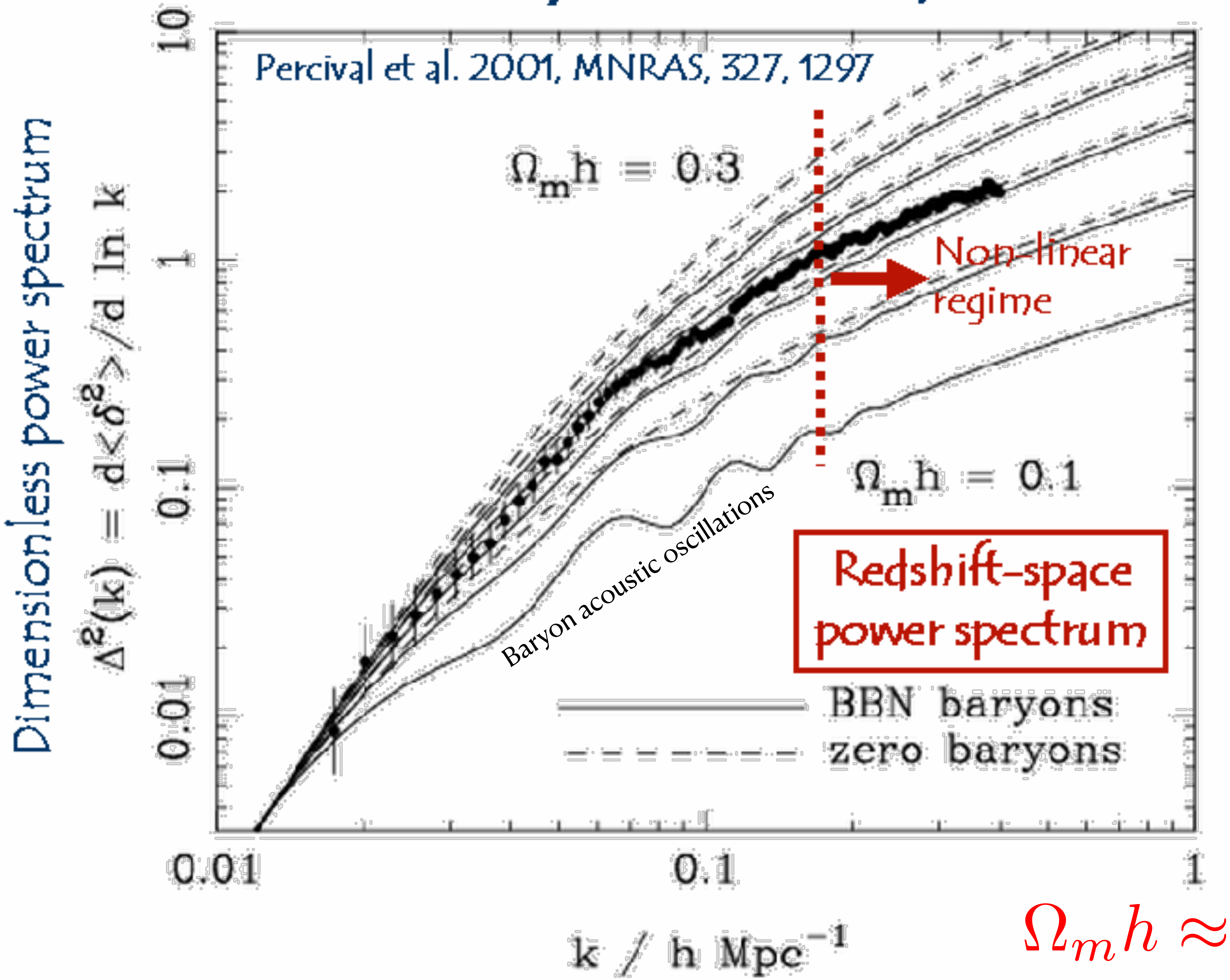


FIG. 10.—Solid curve is the real space power spectrum of the full nonlinear CDM *N*-body simulation (as in Fig. 3) normalized to the real space variance of *IRAS* galaxies ($\sigma_8 = 0.7$). The points are the *IRAS* redshift space $\tilde{P}(k)$ from Fig. 4, rescaled by eq. (17) with $\Omega = 1$ and $b = 1$; this is then, apart from the effects of the convolution in eq. (14), an approximation to the power spectrum of *IRAS* galaxies in *real* space on large scales if the *IRAS* galaxies are unbiased. The box indicates the power spectrum inferred from the *COBE* DMR measurements, assuming a $n = 1$ spectral index and $\epsilon_H = (5.4 \pm 1.6) \times 10^{-6}$ (Smoot et al. 1992; Wright et al. 1992). Note that when the CDM model is normalized to the *IRAS* variance, it produces excessive power on small scales while simultaneously failing to produce sufficient power on large scales to match the *COBE* results.

Fisher et al. (1993) ApJ, 402, 42

All this is solved by LCDM - provided that we are no longer concerned about the flatness/coincidence problem.

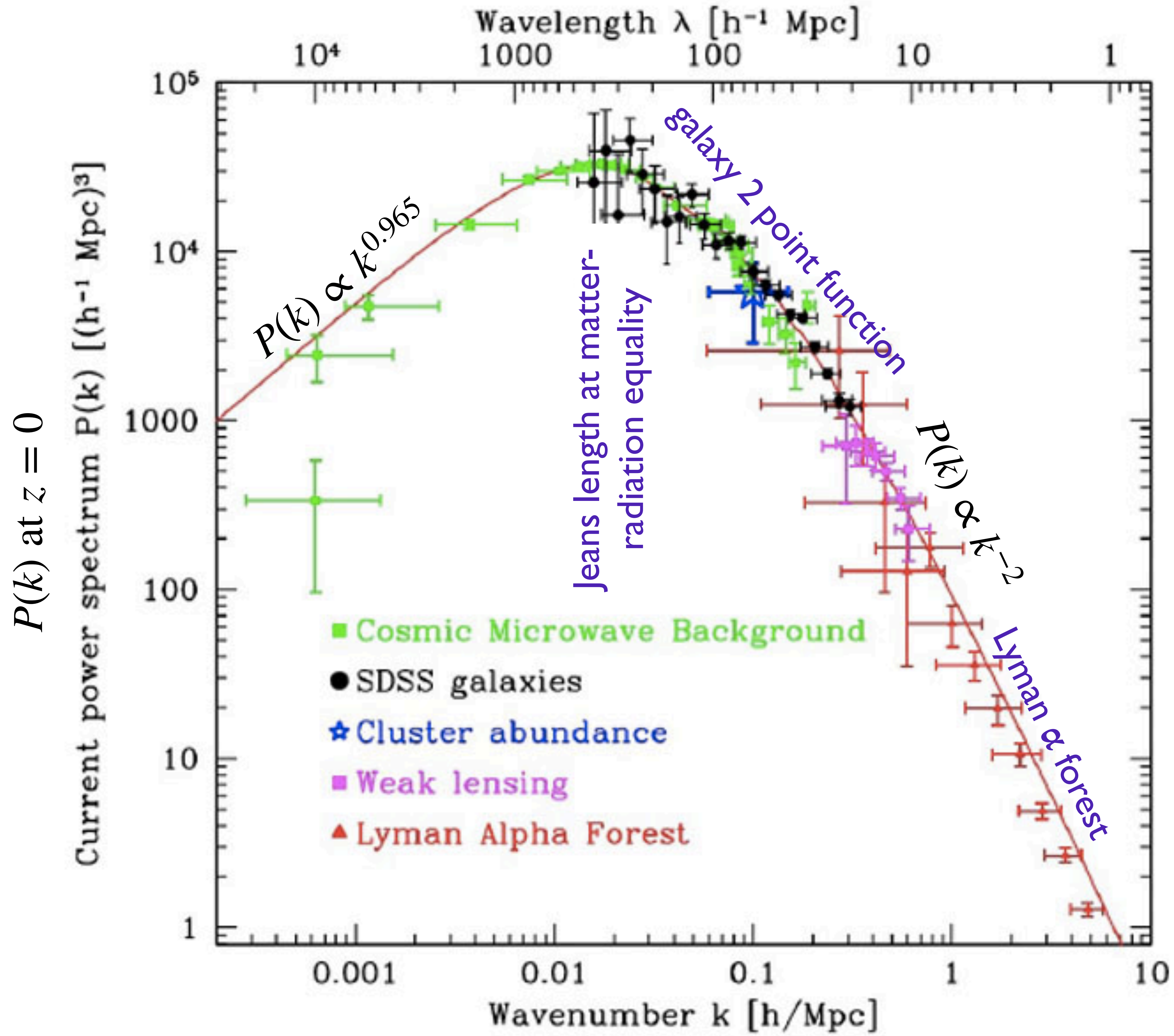
The Galaxy Power Spectrum



$\Omega_m h \approx 0.2$

Planck estimates: $n = 0.965 \pm 0.004$

$$\sigma_8 = 0.811 \pm 0.006$$



Jeans length at matter-radiation equality

$$\lambda_J = c_s \sqrt{\frac{\pi}{G\rho}}$$

sound speed of photon-baryon fluid

$$c_s^2 = \frac{\partial P}{\partial \rho} = \frac{1}{3}c^2$$

Eqn of state for photons,

$$P = \frac{1}{3}\rho c^2$$

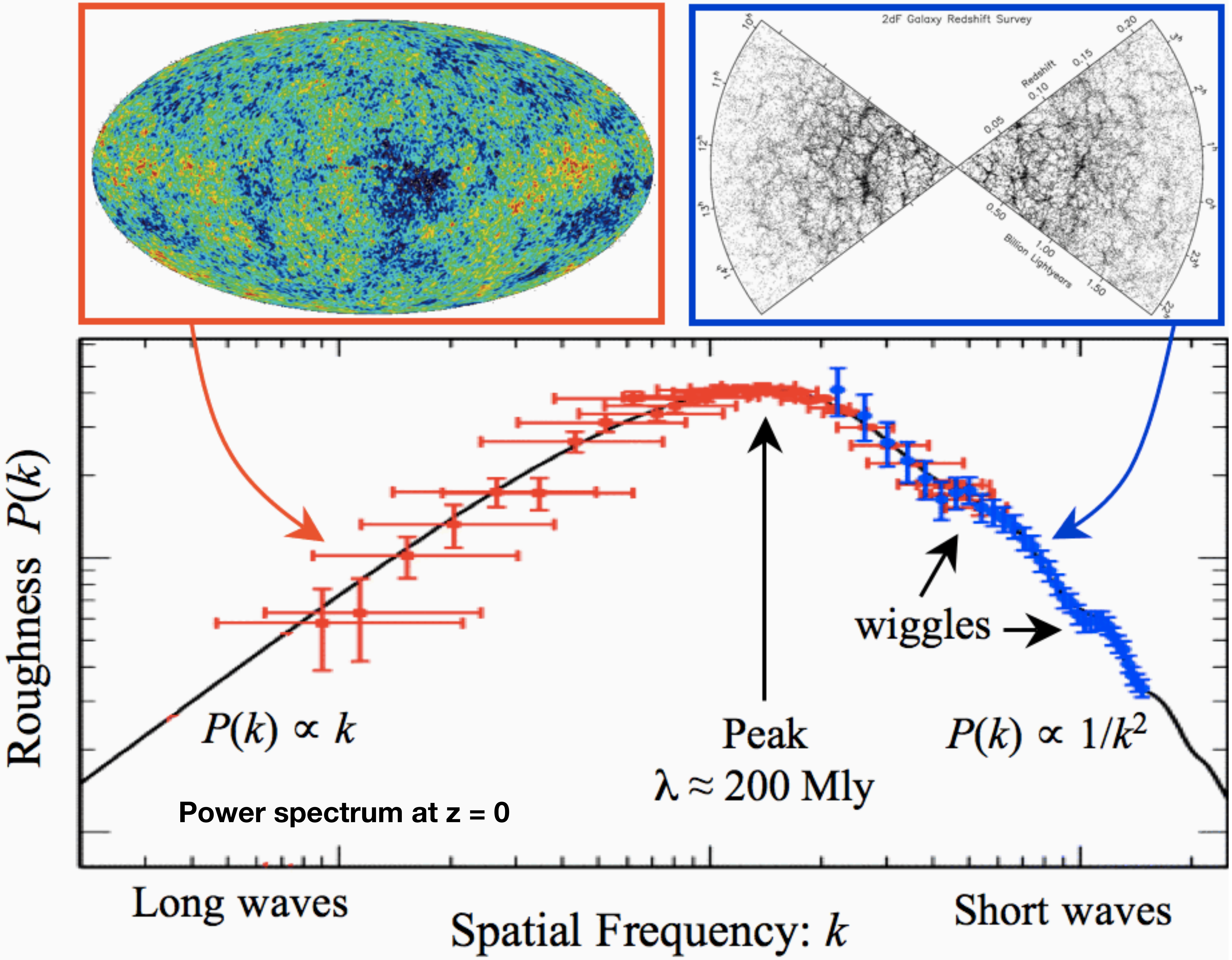
(P here is pressure)

imprints standard rod on surface of last scattering.

at smaller scales, things go non-linear from gravitational collapse, pressure, dissipation, feedback, etc. Described by a **Transfer function**

$$T(k) \equiv \frac{\delta_k(z=0)}{D(z)\,\delta_k(z)}$$

where D(z) is the linear growth factor - what it would have been without all these nasty non-linear effects.


2dF Galaxy Redshift Survey
Redshift
0.05
0.10
0.15
0.20
0.50
1.00
1.50
Billion Lightyears
Roughness $P(k)$
$P(k) \propto k$
Peak
$\lambda \approx 200$ Mly
wiggles
$P(k) \propto 1/k^2$
Power spectrum at z = 0
Long waves
Spatial Frequency: k
Short waves