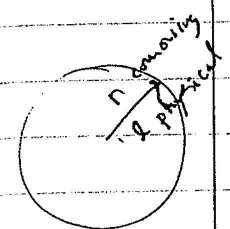

Note error in Bothun's derivation (see web page)

Newtonian cosmology

homogeneous, isotropic universe of pressureless matter
↳ $v \ll c$

Apply Gauss's thm to any spherical region (homogeneity)

$$M = \frac{4\pi}{3}\rho l^3 \qquad \text{mass conservation: } \rho \sim l^{-3} \sim R^{-3}$$

Gravity: $\cancel{\ddot{l} = -\frac{4\pi G}{3}\rho l = -\frac{GM}{l^2}}$

$$\ddot{l} = -\frac{GM}{l^2} = -\frac{4\pi G}{3}\rho l$$

note: $l = Rr$, r unchanging in time: $\underline{l(t) = r\,R(t)}$

so

$$\ddot{l} = (\ddot{rR}) = \cancel{r}\ddot{R} = -\frac{4\pi G}{3}\rho \cancel{r} R$$

comoving coordinate drops out
Need only expansion term $R(t)$

$$\ddot{R} = -\frac{4\pi G}{3}\rho R = -\frac{4\pi G}{3}\frac{(\rho R^3)}{R^2}$$

trick: multiply by $\dot{r}$

$$\dot{R}\ddot{R} = -\frac{4\pi G}{3}\frac{(\rho R^3)}{R^2}\dot{R} = -\frac{4\pi G}{3}(\rho R^3)\frac{\dot{R}}{R^2}$$

note: $\frac{d}{dt}(\dot{R}^2) = 2\dot{R}\ddot{R} \qquad \frac{d}{dt}\left(\frac{1}{R}\right) = -\frac{\dot{R}}{R^2}$

so

$$\frac{1}{2}\frac{d}{dt}(\dot{R}^2) = -\frac{4\pi G}{3}(\rho R^3)\frac{d}{dt}\left(\frac{1}{R}\right).$$

hence

$$\boxed{\dot{R}^2 - \frac{8\pi G}{3}\rho R^2 = 2\varepsilon}$$

ε an energy-like integration constant ~~[illegible]~~

NOTE: NO STATIC SOL'N WITH $\dot{R} = 0$

$$\frac{1}{H_0^2}\left(\frac{\dot a}{a}\right)^2 = \Omega_{m_0}(1+z)^3 + \Omega_{r_0}(1+z)^4 + \Omega_{k_0}(1+z)^2 + \Omega_{\Lambda_0}$$

Radiation domination

$$\left(\frac{\dot a}{a}\right)^2 = H_0^2\Omega_{r_0} a^{-4}$$

$$\frac{\dot a}{a} = H_0\Omega_{r_0}^{1/2} a^{-2}$$

$$\dot a = \frac{da}{dt} = H_0\Omega_{r_0}^{1/2} a^{-1}$$

$$\int a\,da = \int H_0\Omega_{r_0}^{1/2}\,dt$$

$$\frac{1}{2}a^2 = H_0\Omega_{r_0}^{1/2} t$$

$$a = (2H_0)^{1/2}\Omega_{r_0}^{1/4} t^{1/2} \quad : \quad a \sim t^{1/2}$$

$$H = \cancel{H_0\Omega_{r_0}^{1/2}}\cdot 2^{-1}\cdot\frac{1}{\cancel{H_0\Omega_{r_0}^{1/2}}}\,t^{-1}$$

$$H = \frac{1}{2t} \qquad \varepsilon = \alpha T^4$$

$$\Omega_r = \frac{8\pi G}{3H^2}\frac{\alpha T^4}{c^2}$$

$$a^2 = 2H_0\Omega_{r_0}^{1/2} t = \frac{2H_0}{H_0}\underbrace{\sqrt{\frac{8\pi G\alpha}{3}}}\frac{T^2}{c}t$$

$$T^2 t = 1.9\times10^{20}\,K^2 s$$

Matter domination

$$\left(\frac{\dot a}{a}\right)^2 = H_0^2\Omega_{m_0} a^{-3}$$

$$\frac{\dot a}{a} = H_0\Omega_{m_0}^{1/2} a^{-3/2}$$

$$\frac{da}{dt} = \dot a = H_0\Omega_{m_0}^{1/2} a^{-1/2}$$

$$a^{1/2}da = H_0\Omega_{m_0}^{1/2}\,dt$$

$$\frac{2}{3}a^{3/2} = H_0\Omega_{m_0}^{1/2} t$$

$$a = \left(\frac{3}{2}H_0\Omega_{m_0}^{1/2}\right)^{2/3} t^{2/3} \qquad a \sim t^{2/3}$$

$$H = \frac{\dot a}{a} = \cancel{H_0\Omega_{m_0}^{1/2}}\,\frac{3}{2}\,\frac{t^{-1}}{\cancel{H_0\Omega_{m_0}^{1/2}}}$$

$$H = \frac{2}{3t}$$

curvature

$$\left(\frac{\dot{a}}{a}\right)^2 = H_0^2 \Omega_{k_0} a^{-2}$$

$$\frac{\dot{a}}{a} = H_0 \Omega_{k_0}^{1/2} a^{-1}$$

$$da = H_0 \Omega_{k_0}^{1/2} dt$$

$$\underline{a = H_0 \Omega_{k_0}^{1/2} t} \qquad \text{linear expansion} \quad \underline{a \sim t}$$

Λ domination

$$\left(\frac{\dot{a}}{a}\right)^2 = H_0^2 \Omega_{\Lambda_0} = \cancel{H_0^2} \frac{c^2}{3\cancel{H_0^2}} \Lambda = \frac{c^2}{3} \Lambda$$

$$\frac{\dot{a}}{a} = \frac{c}{\sqrt{3}} \Lambda^{1/2}$$

$$\frac{da}{a} = \frac{c}{\sqrt{3}} \Lambda^{1/2} dt$$

$$\ln a = c\sqrt{\frac{\Lambda}{3}}\, t$$

$$a = e^{c\sqrt{\frac{\Lambda}{3}}\, t} \quad \underline{a \sim e^t} \,! \qquad \text{exponential expansion}$$
