Final Review

Orbital anisotropy

general mass estimator:

$$M(r) = \frac{r\,\sigma_r^2}{G}\left(\gamma_* + \gamma_\sigma - 2\beta\right)$$

$$\gamma_* = -\frac{d\ln n_*}{d\ln r}$$ logarithmic slope of tracer density profile $n_*(r)$

$$\gamma_\sigma = -\frac{d\ln \sigma_r^2}{d\ln r}$$ logarithmic slope of [3D, not line-of-sight] radial velocity dispersion profile $\sigma^2(r)$

$$\beta = 1 - \frac{\sigma_t^2}{\sigma_r^2}$$ anisotropy parameter ratio of tangential vs radial motion Measures kinetic energy in rotation vs. that in pressure support.

β can be a fcn of radius

Isotropy ($\beta=0$) implicitly assumed in most virial analyses

Luminosity function

Schechter fcn: $$\Phi(L) = \Phi^* e^{-L/L^*}\left(\frac{L}{L^*}\right)^{\alpha}$$

w/ characteristic density Φ^*, luminosity L^*, & faint end slope α

Integrated luminosity density $j = \Phi^* L^* \Gamma(\alpha+2)$

ΛCDM predicted mass fcn has steep slope $\alpha \approx -1.9$
Observed luminosity " " shallow " $\alpha \approx -1.2$

This is the origin of the mis-named missing satellite problem — lots of low mass halos are predicted (not just as satellites) while relatively few low mass galaxies are observed

Gravitational Lensing

Flavors: Strong lensing - multiple lensed images of single source
Weak lensing - mild distortion of source
Micro lensing - temporary brightening due to unresolved strong lensing of source by object that passes in front of it

Threshhold for strong lensing:

$$\Sigma > \Sigma_{crit} \simeq \frac{c^2}{4\pi G} \frac{D_S}{D_L D_{LS}}$$

Bend angle

$$\alpha_d = \frac{4GM}{bc^2} = \frac{D_S}{D_{LS}} (\theta_I - \theta_S)$$

true angle between

apparent angle between source & lens

Mass estimator ~~for weak lensing~~
e.g., within an arc in a cluster of galaxies

$$M(<\theta_0) = \left(1.1 \times 10^{14} M_\odot\right)\left(\frac{\theta_0}{30''}\right)^2\left(\frac{D_L}{D_S}\right)\left(\frac{D_{LS}}{1\,\mathrm{Gpc}}\right)$$

Microlensing optical depth

$$\tau = 2\pi \frac{\sigma_v^2}{c^2} \frac{D_L D_{LS}}{r D_S}$$

Not enough lensing events are observed towards the LMC to account for the dark matter. On the flip side, the lensing optical depth towards the bulge is rather high. Lensing objects are just ordinary stars if the disk is maximal.

12
12
24
36
48
60
72
84
96
108
120
132
144

Cosmology

Friedmann eqn

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \frac{1}{3}\Lambda c^2$$

$H = \frac{\dot{a}}{a}$ — Hubble expansion; mass density; geometry; cosmological constant (aka dark energy)

Critical density

$$\rho_{crit} = \frac{3H^2}{8\pi G} \qquad \Omega_m = \frac{\rho_m}{\rho_{crit}}$$

$\Omega_m > 1$ eventually halts expansion & recollapses
$\Omega_m \leq 1$ expands forever
$\Lambda > 0$ eventually causes expansion to accelerate

As $t \to 0$ $(z \to \infty)$ all solutions tend to $\Omega_m \to 1$

Cosmology needs non-baryonic dark matter because

1. $\Omega_m > \Omega_b$ from Big Bang Nucleosynthesis
2. Large Scale Structure formation
   - need growth factor of $\sim 10^5$ from CMB to now
   gravity + baryons will only do ~ 1000 since $\delta \sim a = 1090$

In addition to the cosmological missing mass, we also have 2 distinct missing baryon problem:

1. Sum of known baryons $< \Omega_b$

2. Halo-by-halo missing baryon problem

$$\frac{M_b}{M_{tot}} < f_{b,\,cosmic}$$ in bound objects like galaxies and groups of galaxies

$$\Omega_m = \frac{\rho_m}{\rho_{crit}}$$

Measurements of the cosmic mass density

- Combine cluster M/L with luminosity density j of the universe obtained by integrating the galaxy luminosity function

$$\rho_m = j\left(\frac{M}{L}\right) \qquad \Omega_m \approx 0.3$$

- Weak Lensing
  Measure shear caused by lensing over large scales
  $\Omega_m \approx 0.18$

- Peculiar Velocity Field / Redshift distortions
  Measure deviations from the Hubble flow $(\delta V/V)$ caused by mass over densities $(\delta\rho/\rho)$

$$\frac{\delta V}{V} \approx -\frac{1}{3}\Omega_m^{0.6}\frac{\delta\rho}{\rho} \qquad \Omega_m \approx 0.25$$

- Power Spectrum $P(k)$ of galaxies at $z \approx 0$ $\qquad P(k) = \delta_k^2$
  Shape and amplitude of the large scale structure traced by galaxies depends on $\Gamma = \Omega_m h \approx 0.2$ where $h = H_0/100$. In detail Γ also depends weakly on the baryon density Ω_b. $P(k) \sim k^n$ with $n \approx 1$

- Acoustic Power spectrum $C_\ell = \Delta T^2$ of Cosmic Microwave Background at $z = 1090$. Depends sensitively on all cosmic parameters, but basically

$$\Omega_m h^2 \approx 0.14 \qquad \Omega_m = 0.32$$

of which baryons are

$$\Omega_b h^2 = 0.022 \qquad \Omega_{cdm} h^2 = 0.12 \text{ to get } 3^{rd} \text{ peak right}$$

Really $\Omega_m h^3 \approx 0.096$ better approximates the degeneracy

WIMPs   Weakly Interacting Massive Particle

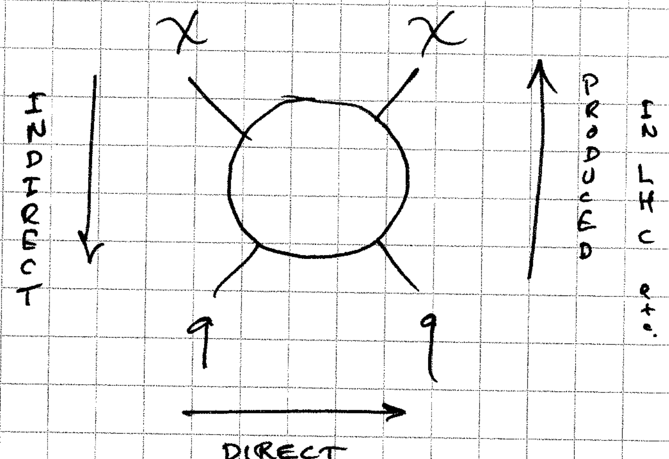
Leading candidate for the non-baryonic dark matter

Usually presumed to be the lightest stable particle in the hypothetical super symmetric sector (SUSY) (e.g., the neutralino)

In SUSY, every normal (standard model) particle has a SUSY partner particle. Minimal SUSY, with the fewest new parameters, called MSSM. (Minimal Super-Symmetric Model)

The WIMP miracle: the weak force interaction scale ($m_\chi \approx 100$ GeV) is about right to leave a relic density that's about right to be the dark matter ($\Omega_\chi \sim 0.1 \pm$ a few dex)

WIMP detection



Direct detection: WIMPs scatter off nuclei in underground labs

Indirect detection: WIMPs are their own anti-particle; can annihilate into standard model particles, creating a source of cosmic rays & γ-rays.

WIMP production: Particle colliders that achieve energies $> M_\chi c^2$ could create WIMPs which might be recognized as a deficit in detected debris mass-energy.

Relic density of particles determined by when they freeze out

number density x cross-section = expansion rate

Freeze out condition: $n\sigma \approx H$

HOT (relativistic)
e.g., neutrino

$T_\nu \gg m_\nu$ so number still around just depends on the photon density

$$\Omega_\nu h^2 = \frac{\sum m_\nu}{93 \text{ eV}}$$

current limits

$$0.06 \leq \sum m_\nu \leq 0.12$$

neutrino oscillations — structure formation

COLD (non-relativistic)
e.g., WIMP

$T_X \ll m_X$ particle-antiparticle pairs have time to annihilate, so

$$n \sim (m_X T)^{3/2} e^{-\frac{m_X}{T}}$$

$$\frac{\Omega_X}{0.2} \approx \frac{x_{fo}}{20}\left(\frac{10^{-8}\text{ GeV}^{-2}}{\sigma}\right)$$

$20 \lesssim x_{fo} < 50$
annoying quantum factor

$$\sigma \sim \frac{g^4}{m_X^2}$$

where **g** is the coupling strength
(e.g., the weak nuclear force)

Lee-Weinberg limit: $m_X > 2$ GeV to not over-produce cosmic mass density

Modified Gravity

Many have tried to modify gravity to eliminate the need for dark matter

Paul Atriedes: Many have tried and failed?

Rev. Mother: Many have tried and died.

One can generically exclude length-scale-based modifications, as the discrepancy does not appear at a particular size. It does, however, appear at a characteristic acceleration, as hypothesized by MOND:

Basic tenets of MOND:

1. New scale with dimensions of acceleration, $a_0 = 1.2 \times 10^{-10}\ m\,s^{-2}$
2. Normal Newtonian dynamics hold for $a \gg a_0$
3. Dynamics are scale invariant for $a \ll a_0$

   such that $(\vec{r}, t) \rightarrow \lambda(\vec{r}, t)$

The effective force law for low accelerations $a \ll a_0$ tends towards the asymptotic limit $a \rightarrow \sqrt{g_N a_0}$ where g_N is the normal Newtonian acceleration.

The Newtonian and MOND regimes are bridged by a smooth interpolation function $\mu(x)$

$$g_N = \mu(x)\, a \qquad \text{where} \quad x = \frac{a}{a_0}$$

subject to $\mu \rightarrow 1$ for $x \gg 1$

$\mu \rightarrow x$ for $x \ll 1$

e.g., $\mu = \dfrac{x}{1+x}$

This is the original algebraic MOND formulation

One could also write

$$a = \nu(y)\, g_N \qquad \text{where} \quad y = \frac{g_N}{a_0}$$

subject to $\nu \rightarrow 1$ for $g_N \gg a_0$ $(y \gg 1)$

$\nu \rightarrow y^{-1/2}$ for $g_N \ll a_0$ $(y \ll 1)$

e.g., $\nu(y) = \dfrac{1}{1 - e^{-\sqrt{y}}}$

This is QUMOND

More general theories approximately reduce to one of these

MOND can be interpreted as a modification of either

Gravity $\quad F = \frac{GMm}{r^2}$

or

Inertia $\quad F = ma \quad$ i.e., $M_{grav} \neq m_{inertia}$!

The original idea was the latter, but it requires a non-local theory to ~~be~~ conserve momentum and energy.
A modification that conserves both is AQUAL

$$\vec{\nabla}\left(\mu\left(\frac{|\nabla\Phi|}{a_0}\right)\vec{\nabla}\Phi\right) = 4\pi G\rho$$

This is a modification of gravity, but only Newtonian, not GR.
Finding a satisfactory theory ~~that~~ is generally covariant remains an outstanding theoretical problem.
Nevertheless, the basic tenets suffice to predict

- The BTFR
- the shape of rotation curves
- the Central Density Relation

etc. : e.g., Renzo's rule is implicit

MOND breaks the strong equivalence principle
as it depends on the external field g_{ex} as well as
the internal field g_{in}

Regime	Mass Estimator	
$g_{in} > a_0$	$M = \frac{RV^2}{G}$	(Newtonian regime)
$a_0 > g_{in} > g_{ex}$	$M = \frac{1}{a_0 G} V^4$	~~Deep~~ (Deep MOND regime)
$a_0 > g_{ex} > g_{in}$	$M = \frac{a_0}{g_{ex}} \frac{RV^2}{G}$	(Quasi-Newtonian regime) $G_{eff} \rightarrow \left(\frac{g_{ex}}{a_0}\right) G$

If $g_{ex} > a_0 > g_{in}$, you're still in the Newtonian regime

This is called the External Field Effect (EFE)