MIDTERM REVIEW

Observational Evidence for mass discrepancies

- Oort discrepancy in solar neighborhood
- flat rotation curves of spiral galaxies
- clusters of galaxies
  - velocity dispersions
  - hydrostatic equilibrium of X-ray gas
  - gravitational lensing of background galaxies
- large scale structure
- $\Omega_m > \Omega_b$

Early indications included

Oort (1932) - factor of ~2 discrepancy in solar neighborhood

Zwicky (1933) - factor of ~100 discrepancy in clusters

Ostriker & Peebles (1973) - factor of ~10 discrepancy in bar instability in disks

WIMPS favored because
- $\Omega_m > \Omega_b$
- LSS formation

Dark Matter Candidates

Baryonic

e.g. Brown dwarfs
black holes
cold H_2
Jupiters
old white dwarfs
etc.

Non-Baryonic

Hot
ν

other
Warm DM
SIDM
pretty much anything you wanna make up.

Cold (CDM)
WIMPs
Black holes
Strange nuggets
Axions
other particle DM candidates

Dark matter is the usual inference; the discrepancy might also indicate a change in dynamical laws

Virial Theorem

Can be derived from stationary moment of inertial tensor

boils down to $2\langle K\rangle + \langle W\rangle = 0$

Kinetic E   Potential Energy

for N particles of equal mass m such that $M = Nm$,

$$M = \frac{2\sigma^2 R_{hms}}{G}$$

where the harmonic radius R_{hms} is usually approximated as $R_{hms} \gtrsim 1.25 R_e$

Vertical Force (Oort: restoring force to disk)

$$K_z = -\frac{\partial \Phi}{\partial z} = \frac{1}{\nu}\frac{\partial(\nu\sigma^2)}{\partial z}$$

where $\nu(z)$ is the vertical profile of tracer population

locally this boils down to

$$\sigma_z^2 = 2\pi G \Sigma z_0$$

e.g. $\nu(z) = \nu_0 e^{-z/z_0}$

Disk Stability

LOCAL: Toomre Q: $Q = \frac{\sigma_R \kappa}{3.36 G\Sigma}$ locally stable if $Q \gtrsim 1$

GLOBAL: $X_m = \frac{\kappa^2 R}{2\pi m G\Sigma}$ higher surface densities less stable

Ostriker & Peebles: $t \lesssim 0.14$ where $t = \frac{T}{|W|}$

with

$K = T + \frac{1}{2}\Pi$. T = rotational kinetic energy

Π = kinetic energy in random motions

Exponential Disks

2D face-on surface brightness profile

$$\Sigma(R) = \Sigma_0 e^{-R/R_d}$$

Σ_0 = central surface density
R_d = scale length

This integrates to a total luminosity $L = 2\pi \Sigma_0 R_d^2$

The enclosed luminosity is a simple fn of ~~the~~ scale length

$$L(<x) = 2\pi \Sigma_0 R_d^2 \left[1 - (1+x)e^{-x}\right] \quad \text{where} \quad x = \frac{R}{R_d}$$

in 3D one can have a "double exponential" model

$$\rho = \rho_0 e^{-R/R_d} e^{-z/z_0}$$

For general light profiles we can fit the

Sersic profile

$$\Sigma(R) = \Sigma_e e^{-b_n\left[(R/R_e)^{\frac{1}{n}} - 1\right]}$$

which reduces to the exponential form for $n = 1$
and is equivalent to the de Vaucoulers profile for $n = 4$

Potential - Density Pairs

Poisson Eqn $\quad \nabla^2 \Phi = 4\pi G \rho$

has an analytic solution for a handful of Φ-ρ pairs.

$\therefore$ It helps to know ∇^2 in the right coordinate system

Halo models

pseudo-isothermal   empirically motivated

$$\rho_{iso}(r) = \frac{\rho_0}{1 + (r/R_c)^2}$$

characterized by
core radius R_c
flat velocity V_∞

$$V(r) = V_\infty \sqrt{1 - \frac{R_c}{R} \tan^{-1}\left(\frac{R}{R_c}\right)}$$

$$V_\infty = \sqrt{4\pi G \rho_0 R_c^2}$$

NFW   derived from simulations

$$\rho_{NFW}(r) = \frac{4\rho_s}{(r/r_s)(1 + r/r_s)^2}$$

characterized by
c
$R_{200}/V_{200}/M_{200}$

$$V(r) = V_{200} \sqrt{\frac{\ln(1+cx) - \frac{cx}{1+cx}}{x\left[\ln(1+c) - \frac{c}{1+c}\right]}}$$

$$x = \frac{r}{R_{200}} \qquad c = \frac{R_{200}}{r_s}$$

$$V_{200} = h R_{200} \quad \text{where} \quad h = \frac{H_0}{100}$$

V_{200} in km/s, R_{200} in kpc

$$M_{200} = \frac{4\pi}{3}(200) R_{200}^3$$

mention Einasto
Burkert

Galactic Definitions

Orbital Frequency $$\Omega = \frac{V}{R}$$

Oort constant A $$A = \frac{1}{2}\left(\frac{V}{R} - \frac{dV}{dR}\right)_{R_0}$$

Oort constant B $$B = -\frac{1}{2}\left(\frac{V}{R} + \frac{dV}{dR}\right)_{R_0}$$

Note that $$\Omega = A - B$$

Epicyclic Frequency $$\kappa^2 = -4B\Omega = -4B(A-B)$$

For a circular orbit, $$V = \frac{2\pi R}{P_\Omega}$$

where P_Ω is the orbital period.

Therefore $$P_\Omega = \frac{2\pi}{\Omega}$$

Similarly, the epicyclic period $$P_\kappa = \frac{2\pi}{\kappa}$$

Laws of Galactic Rotation

1. Flat rotation curves : $V(R) \to$ constant for large R (V_f)

   Flatness persists indefinitely far out.

2. Sancisi's Law : For any feature in the light, there is a corresponding feature in the rotation curve, and vice-vers.

3. The Baryonic Tully-Fisher Relation

$$M_b = A V_f^4 \qquad M_b = M_* + M_g \qquad M_* = \Upsilon_* L$$

$$M_g = \eta (M_{HI} + M_{H_2})$$

$$A = 48.5 \pm 3.5 \; M_\odot \, (\mathrm{km\,s^{-1}})^{-4} \quad [2020]$$

(RNAAS, 4, 45)

4. The Central Density Relation

$$\Sigma_{dyn}(R=0) = \Sigma_\dagger f(\Sigma_*(R=0))$$

Lelli et al (2016)
Milgrom (2016)

5. The Radial Acceleration Relation

$$g_{obs} = F(g_{bar}) = \frac{g_{bar}}{1 - e^{-\sqrt{g_{bar}/g_\dagger}}}$$

$$g_\dagger = 1.20 \pm 0.02 \times 10^{-1\cdot} \; \mathrm{m\,s^{-2}}$$

modulo Υ_* systematics

The first four laws are implied by the fifth.
They are all related by the acceleration scale $g_\dagger$

$$g_\dagger = G\Sigma_\dagger \;; \qquad g_\dagger = \frac{\zeta V_f^4}{G M_b}$$

$\zeta \simeq 0.8$ for a finite thickness disk

Cosmological Framework

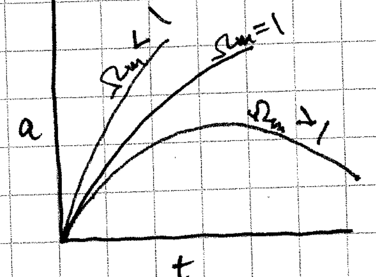
Dark matter halos are thought to form by gravitational collapse of over-dense regions in an otherwise expanding universe.

$a = \frac{1}{1+z}$

A little necessary context:

Friedmann eqn: $\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3}$

scale size — mass density — curvature — cosmological constant



a is the scale size of the universe — a dimensionless quantity that encodes the physical separation between comoving coordinate tracers (e.g., galaxies w/ zero peculiar motion)

a(t) is the expansion history of the universe following from the solution of Friedmann's eqn.

Note that the Hubble parameter $H = \frac{\dot{a}}{a}$ is the expansion rate.

H must vary with time; its current measured value is the misnamed Hubble "constant" $H_0 = \left(\frac{\dot{a}}{a}\right)_0$ measured now at $t = t_0$, the age of the U.

It is also convenient to define the density parameter $\Omega_m = \frac{\rho}{\rho_{crit}}$

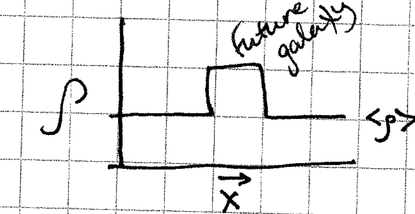
which is the ratio of the actual mass density to the critical density

$\rho_{crit} = \frac{3H^2}{8\pi G}$ that defines the over/under between eternal expansion and eventual recollapse.

Note that ρ_{crit} evolves with H, as do ρ & Ω_m.
Only if $\Omega_m = 1$ exactly does it remain 1 for eternity

The Friedmann eqn can be derived from the eqn. of motion of a point on the surface of a uniform expanding sphere, at least in the absence of Λ. It does not depend on scale. So a good first approximation to galaxy formation is to treat the volume that will collapse to form a galaxy as a locally overdense universe with $\Omega_m > 1$.

This is often called a top-hat overdensity



Note that $\vec{x}$ represents all 3 spatial dimensions. The over-dense region is spherical, so Friedmann's eqn applies.

Indeed, as $t \to 0$, $\Omega_m \to 1$, so the curvature and Λ terms may be ignored for the mean $\langle\rho\rangle$.

The solution for the uniform background universe (in which the top-hat is embedded) is

$$\frac{a}{a_0} = \left(\frac{3}{2} H_0 t\right)^{2/3}.$$

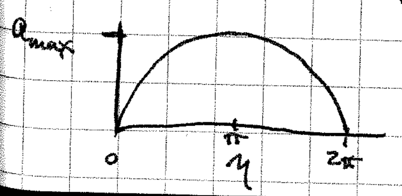
For the top hat, $\Omega_m > 1$, the condition necessary for it to collapse.

But only a little > 1: the initial condition is set by the fluctuations in the CMB when $t \approx 10^5$ yr and $\frac{\delta\rho}{\langle\rho\rangle} \approx 10^{-5}$ — galaxies are thought to arise from the gravitational growth and collapse of these initially tiny over-densities. (Under-densities become voids.)

For $\Omega_m > 1$ the solution is

$$\frac{a}{a_0} = \frac{1}{2} \frac{\Omega_{m,0}}{\Omega_{m,0} - 1} (1 - \cos\eta) \; ; \; H_0 t = \frac{1}{2} \frac{\Omega_{m,0}}{(\Omega_{m,0} - 1)^{3/2}} (\eta - \sin\eta)$$

η is the "development parameter" representing time to recollapse



η runs from 0 to 2π with maximum expansion at $\eta = \pi$

also used in timing argument

Hierarchical Galaxy Formation

1. Dark matter halos start to form early provided the dark matter is
   1. cold (slow moving)
   2. does not interact with photons

   They start small.
2. Baryons fall into the potential wells provided by nascent dark matter halos
   - this can't happen before $z \approx 200$ because the baryons are thermally coupled to the cosmic radiation field (the CMB) until that time ("decoupling")
3. Gas dissipates, condenses to centers of halos
   - compresses halos
   - forms disks
4. Stars form in disks
   - feedback from star formation may re-heat gas and stop further star formation
5. Halos grow by successive mergers
   - star formation truncated by merger (maybe sometimes with a burst)
   - disks destroyed; transformed into elliptical galaxy
6. Accretion continues
   - disks may re-form, resulting in disk + bulge structure
7. Merging subsides (around $z \approx 1$ in ΛCDM)
   - gradual, more gentle accretion of both dark matter and intergalactic gas may continue