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ASTR 333/433 — Dark Matter — Midterm Review — Fall 2021

OBSERVATIONAL EVIDENCE FOR MASS DISCREPANCIES

There are many lines of evidence for mass discrepancies in the universe. Historically, some of the more important include the

- Oort discrepancy in the solar neighborhood (Oort)
- Stability of galactic disks (e.g., Peebles & Ostriker)
- Flat rotation curves of spiral galaxies (Rubin, Bosma)
- Large velocity dispersions of stars in dwarf spheroidals (Aaronson; many others)
- Clusters of Galaxies
  - Large velocity dispersions of cluster member galaxies (Zwicky)
  - Gravitational lensing
  - Hydrostatic equilibrium of hot intracluster gas (as seen in X-rays and the SZ effect)
- Large Scale Structure (needs a boost to get it to grow: Peebles)
- Cosmic gravitating mass density exceeds baryon density from Big Band Nucleosynthesis: $\Omega_m > \Omega_b$

Together, the last two imply the need for some form of non-baryonic, dynamically cold dark matter that does not interact via electromagnetism (so zero interaction cross-section with photons). This implies the existence of a new kind of particle outside those known in the Standard Model of Particle Physics.

DARK MATTER CANDIDATES

- Baryonic
  - Conventional
    - Brown dwarfs, Jupiters, very faint stars
    - Very cold molecular gas (in thermal equilibrium with cosmic background radiation)
    - Warm-hot gas (diffuse gas that has a temperature near to 10^5 K is very hard to detect)
  - Exotic
    - White dwarfs, neutron stars, black holes (stellar remnants)
    - Strange nuggets (including 'macros'), primordial black holes
- Non-baryonic
  - Hot Dark Matter
    - Massive neutrinos
  - Cold Dark Matter
    - WIMPs (Weakly Interacting Mass Particles) and extensions
    - Primordial black holes, strange nuggets (act as non-baryonic CDM if made before BBN)
  - Other
    - Warm dark matter (WDM), Self-interacting dark matter SIDM, Superfluid dark matter, Axions, etc.

The existence of dark matter is an inference based on the assumption that gravity is normal; the observed discrepancies could also indicate a change in the force law.

VIRIAL THEOREM

Can be derived be requiring that the moment of inertia tensor be stationary for an object in equilibrium. Boils down to

$$2\langle \mathbf{K} \rangle + \langle \mathbf{W} \rangle = 0$$

where $\mathbf{K}$ is the kinetic energy and $\mathbf{W}$ is the gravitational potential energy. For N particles of mass m such that $M = Nm$,

$$M = 2\frac{\sigma^2}{G} R_{mathrmhms}$$

where σ is the velocity dispersion representing the kinetic energy and R_{hms} is the harmonic radius

$$\frac{1}{R_{\mathrm{hms}}} = \sum \frac{1}{r_{i,j}}$$

which appears because the gravitational potential depends on the separation between each pair of masses i & j. It is commonly approximated as

$$R_{\mathrm{hms}} \approx 1.25 R_e$$

where R_e is the radius containing half the total light. This is widely applicable in many systems (King), but not perfect in all. It also assumes that the distribution of mass traces the distribution of light. This is not obviously a good assumption if most of the mass is dark: the half-mass radius could be rather different from the half-light radius. For completeness, it is good to know that for a spherical system, the half mass radius in 3D is related to the projected half mass radius as

$$R_{1/2} = \frac{4}{3} R_e.$$

For anisotropic orbits in spherical systems, the mass estimator becomes

$$M(r) = \frac{r\sigma_r^2}{G} \left(-\frac{\partial \ln n_*}{\partial \ln r} - \frac{\partial \ln \sigma_r^2}{\partial \ln r} - 2\beta \right)$$

where σ_r is the radial velocity dispersion, n_* is the distribution of tracers, and β is the anisotropy parameter:

$$\beta = 1 - \frac{\sigma_t^2}{\sigma_r^2}$$

and σ_t is the tangential velocity dispersion. Note that for
circular orbits, $\sigma_r = 0$; $\beta \to -\infty$;
isotropic orbits, $\sigma_r = \sigma_t$; $\beta = 0$;
radial orbits, $\sigma_t = 0$; $\beta = 1$.
Note also that the anisotropy parameter quantifies how the kinetic energy is partitioned between radial and tangential motion.

DISK STABILITY

Ostriker & Peebles (1973) showed that a cold, thin, rotating disk of point masses (stars) was unstable to the unchecked growth of the bar instability. A bare disk operating under Newtonian gravity self-shreds after only a few dynamical times. This contradicts the observation that dynamically cold spiral galaxies exist and are ubiquitous throughout the universe. Observations to high redshift show that this situation is sustained over at least half a Hubble time. Their solution was to impose an external potential; i.e., that of an invisible dark matter halo. Numerically, stability is achieved for

$$\frac{T}{|W|} \lesssim 0.14$$

where T is the kinetic energy in rotation. The total kinetic energy $K = T + \Pi/2$ where Π is the kinetic energy in random motions. For spiral galaxies, $T \gg \Pi$, so $K \approx T$ and according to the virial relation we should have $T/W \approx 1/2$. To lower this to the requisite 0.14, we increase the gravitational potential energy W by adding an invisible component in the form of a static dark matter halo.

Athanassoula et al. (1987) showed that the reverse was also true. Some disk instability is needed to drive the observed bars and spiral structure. If the potential energy is increased with out limit, disks become too stable and never form the observed structures. This leads to a lower limit on disk mass.

The modern situation is much more complicated, and remains incompletely understood. Nevertheless, there is a clear need to stabilize disks, but also a tension between that and the need to not over-stabilize disks to the point that bars and spiral arms never form.

OORT DISCREPANCY

The restoring force K_z for vertical motions to a plane-parallel disk is

$$K_z = -\frac{\partial \Phi}{\partial z} = \frac{1}{n(z)} \frac{\partial (n\sigma_z^2)}{\partial z}$$

where $n(z)$ is the vertical profile of the tracer population, often taken to be exponential: $n(z) = n_0 e^{-z/z_0}$. Locally, this boils down to a simple relation between the vertical velocity dispersion σ_z, the surface density Σ, and the scale height z_0:

$$\sigma_z^2 = 2\pi G \Sigma z_0.$$

More generally,

$$K_z(R) = 2\pi G\Sigma(R) + 2z(A^2 - B^2) = 2\pi G\Sigma(R) - 2z\left(\frac{V}{R}\right)\left(\frac{dV}{dR}\right)$$

where A and B are the geenralized Oort parameters, and are themselved a fnction of radius. Note that in addition to the term directly attributable to the surface density $\Sigma(R)$, there is also a term for the shape of the gravitational potential in the radial direction that manifests as the $A^2 - B^2$ term. This latter term will cause a discrepancy between the dynamical surface density and the baryonic surface density even if there is no extra dark matter in the disk just because there is a quasi-spherical halo that causes the gradient in the rotation curve to deviate from the Keplerian case. This causes a lot of confusion in the literature, with seemingly contradictory claims that often boil down to differing assumptions about this term. (Of course, for a flat rotation curve, $dV/dR = 0$).

See also the more extensive hand-written notes.