Cosmic Overdensities

It is conventional in cosmology to refer to structures by the density contrast they represent with respect to the critical density of the universe. The mass enclosed within a radius encompassing the over-density Δ is

$$M_\Delta = \frac{4\pi\Delta}{3}\rho_{crit}R_\Delta^3. \tag{1}$$

With $\rho_{crit} = 3H_0^2/8\pi G$, this becomes

$$M_\Delta = \frac{\Delta}{2G}H_0^2 R_\Delta^3. \tag{2}$$

By the same token, the circular velocity of a tracer particle at R_Δ is $V_\Delta^2 = GM_\Delta/R_\Delta$. Consequently,

$$M_\Delta = (\Delta/2)^{-1/2}(GH_0)^{-1}\, V_\Delta^3. \tag{3}$$

In ΛCDM, the density contrast $\Delta \approx 100$ marks the virial extent of a dark matter halo. For $H_0 = 72\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$,

$$M_{vir} = (4.6 \times 10^5\ \mathrm{M_\odot\,km^{-3}\,s^3})V_{vir}^3. \tag{4}$$

This includes all mass, dark and baryonic, that reside within the radius R_{vir}. This 'virial' radius is entirely notional; we do not really know what is going on that far out from any given galaxy. For historical reasons, it is conventional to reference the notional halo mass to an overdensity of 200, in which case

$$M_{200} = (3.3 \times 10^5\ \mathrm{M_\odot\,km^{-3}\,s^3})V_{200}^3. \tag{5}$$

This differs from equation 4 by the factor of $\sqrt{2}$ for the difference between $\Delta = 100$ and 200 seen in equation 3. This is frequently cited as the basis of the Tully-Fisher relation, though bear in mind that this refers to the total mass enclosed by R_{200} and the circular velocity of a test particle at that radius, which are not observed.