

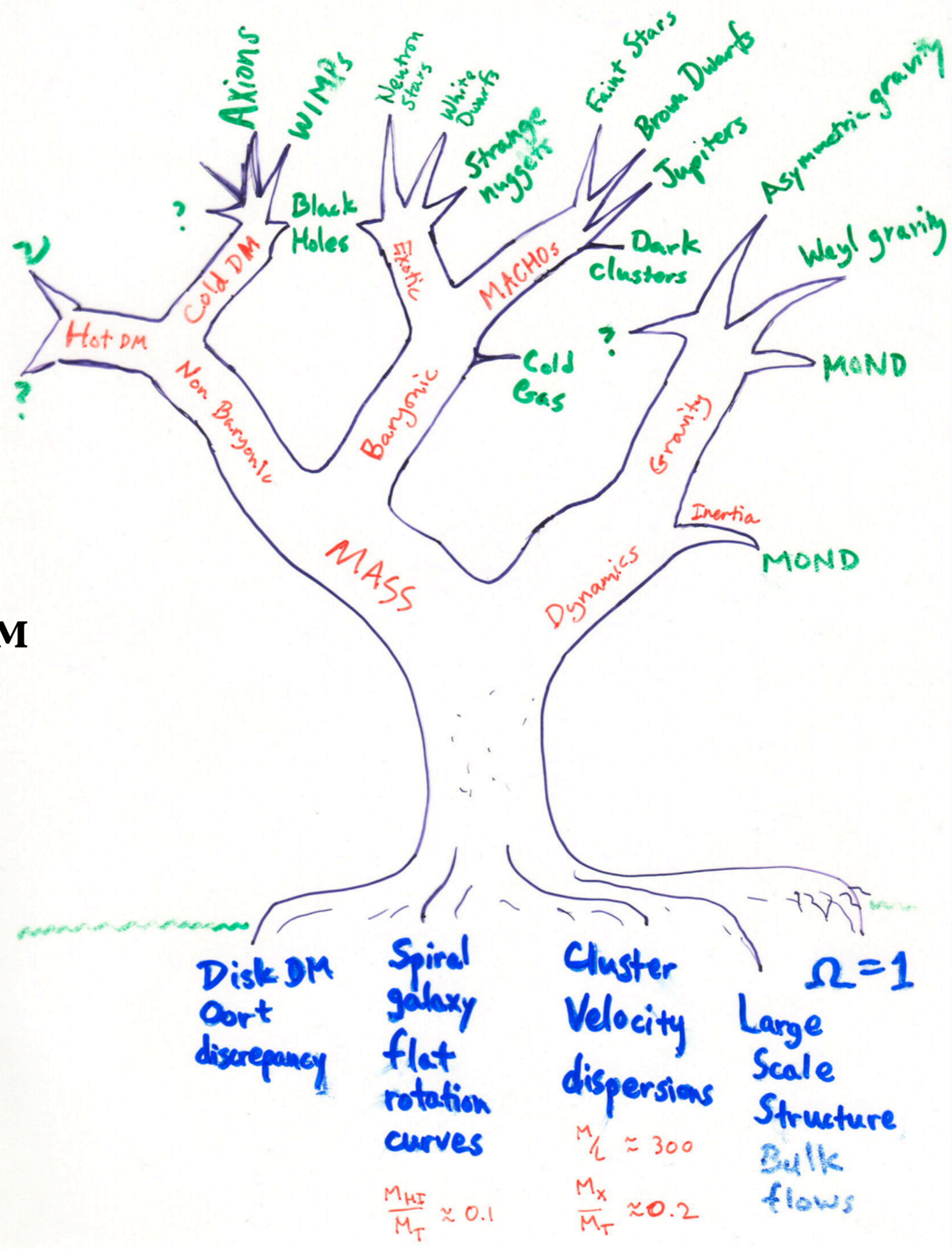
DARK MATTER

ASTR 333/433

TODAY

COSMIC MISSING MASS PROBLEM
MEASURES OF Ω_m

Homework 3 Due



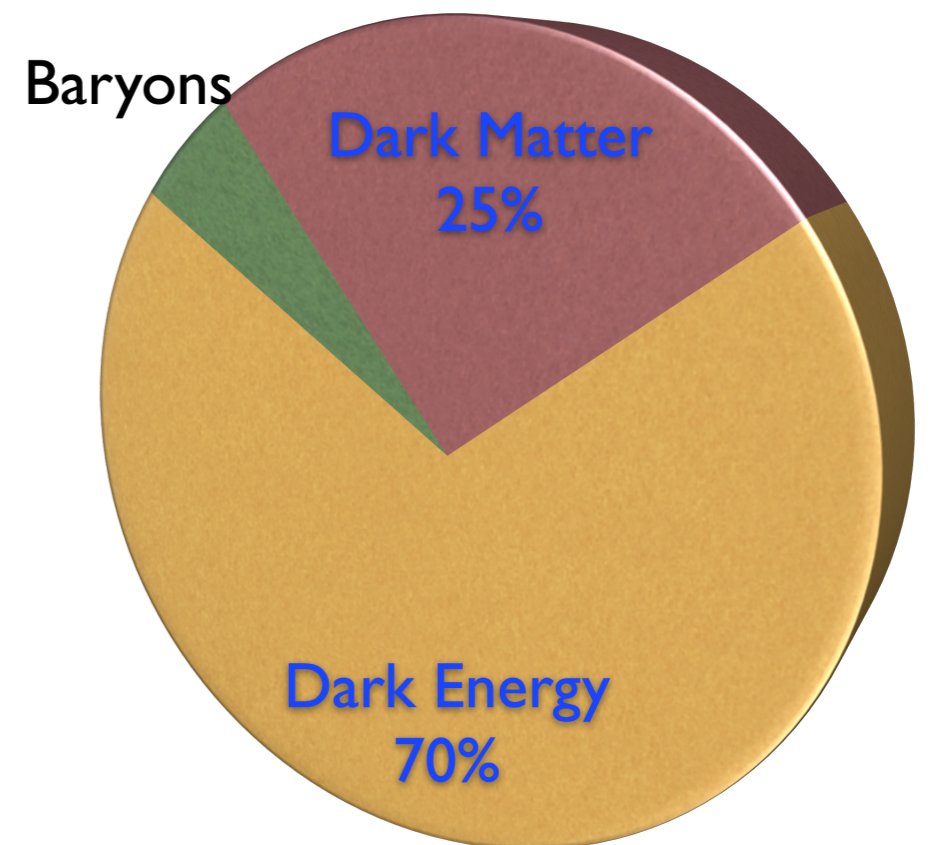
Empirical Pillars of the Hot Big Bang

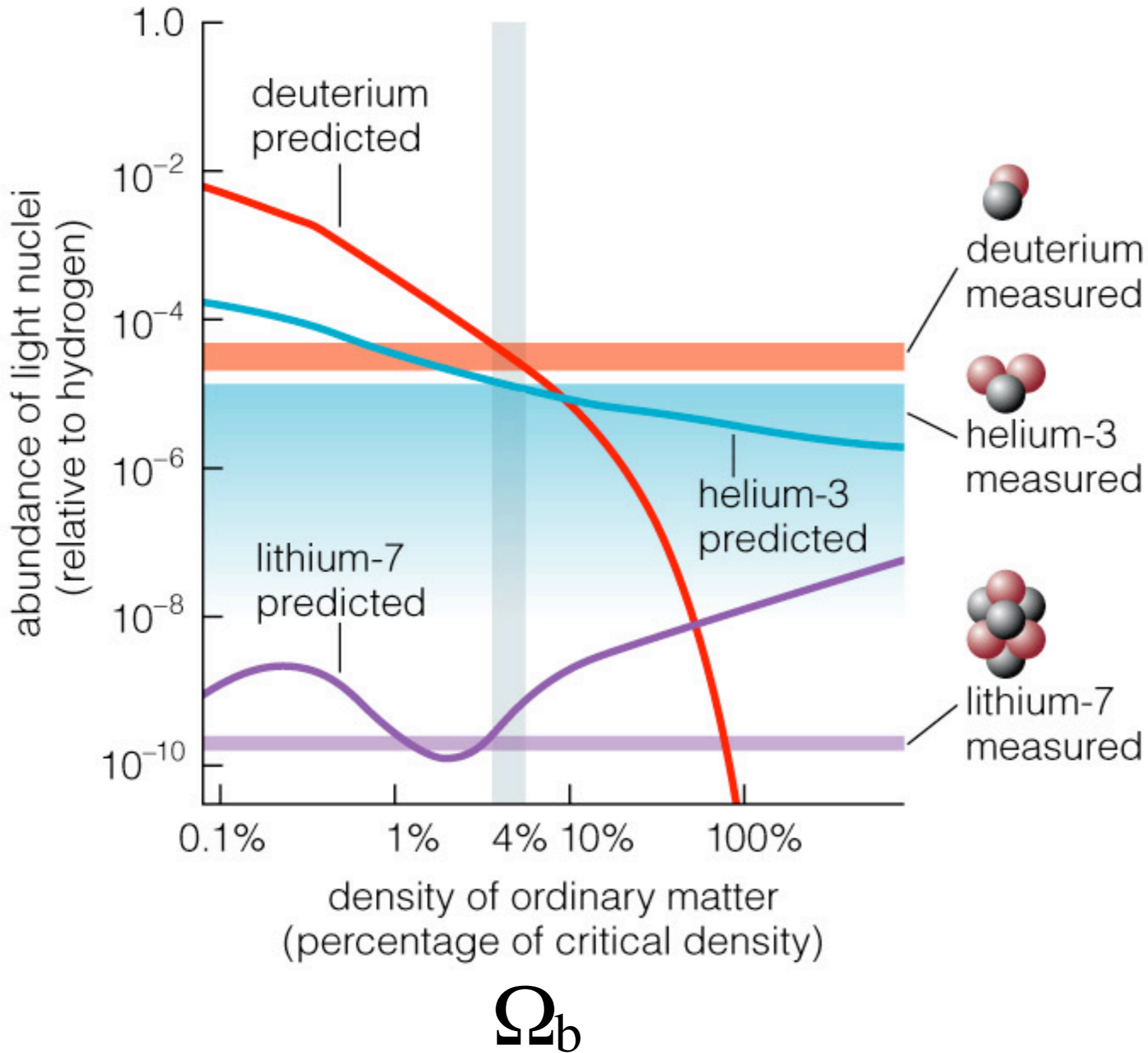
1. Hubble Expansion
2. Big Bang Nucleosynthesis Ω_b
3. Cosmic Microwave Background

$$\Omega_m = \Omega_b + \Omega_{DM}$$

Auxiliary Hypotheses

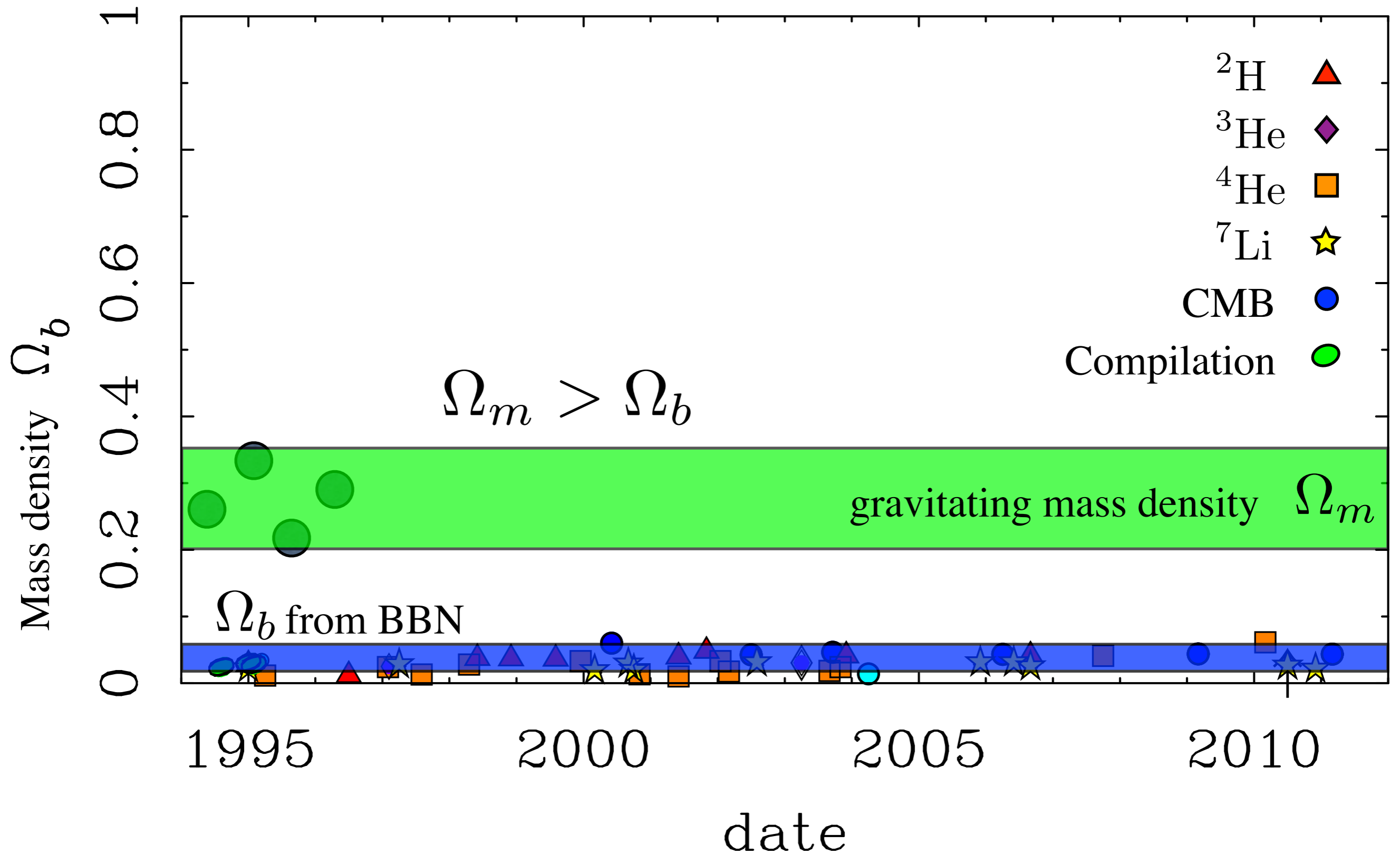
- Dark matter Ω_{DM}
- Dark Energy Ω_Λ





BBN gets the abundances of deuterium, helium, and lithium right if the mass density is about 4 to 5% of the critical density.

There's more mass than BBN allows in baryons



Measurements of the gravitating mass density

- Cluster M/L
 - measure M/L of a cluster, combine with measured luminosity density of universe.
- Weak lensing
 - measure shear over large scales
- Peculiar Velocity Field
 - measure deviations from Hubble flow
- Power spectrum of galaxies
- CMB fits

Measurements of the gravitating mass density

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- measure M/L of a cluster, combine with measured luminosity density of universe.

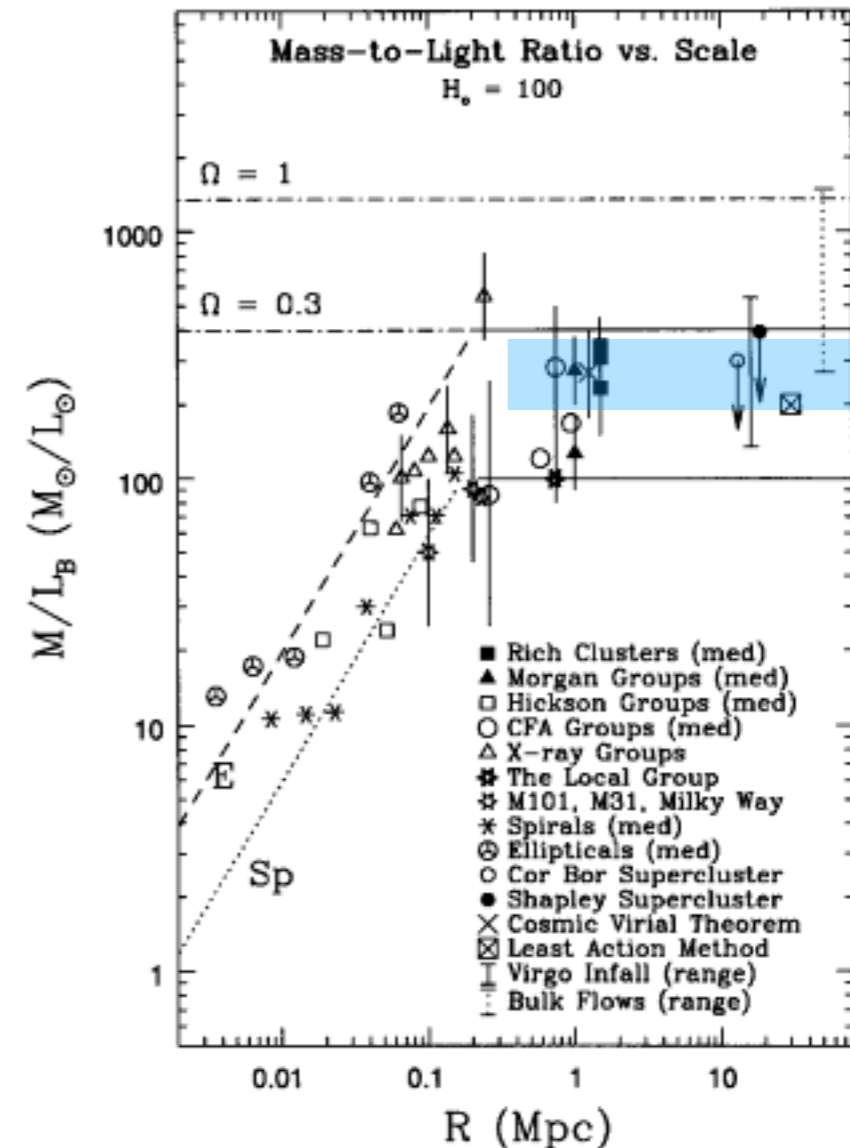
- j from integrating the luminosity function of galaxies:

$$\rho_m = \left(\frac{M}{L}\right) j$$

- Also, cluster baryon fractions:

$$f_b = \frac{M_b}{M_{tot}} \quad \circ \longrightarrow \quad \Omega_m = \frac{\Omega_b}{f_b}$$

- both assume clusters are representative of the whole.

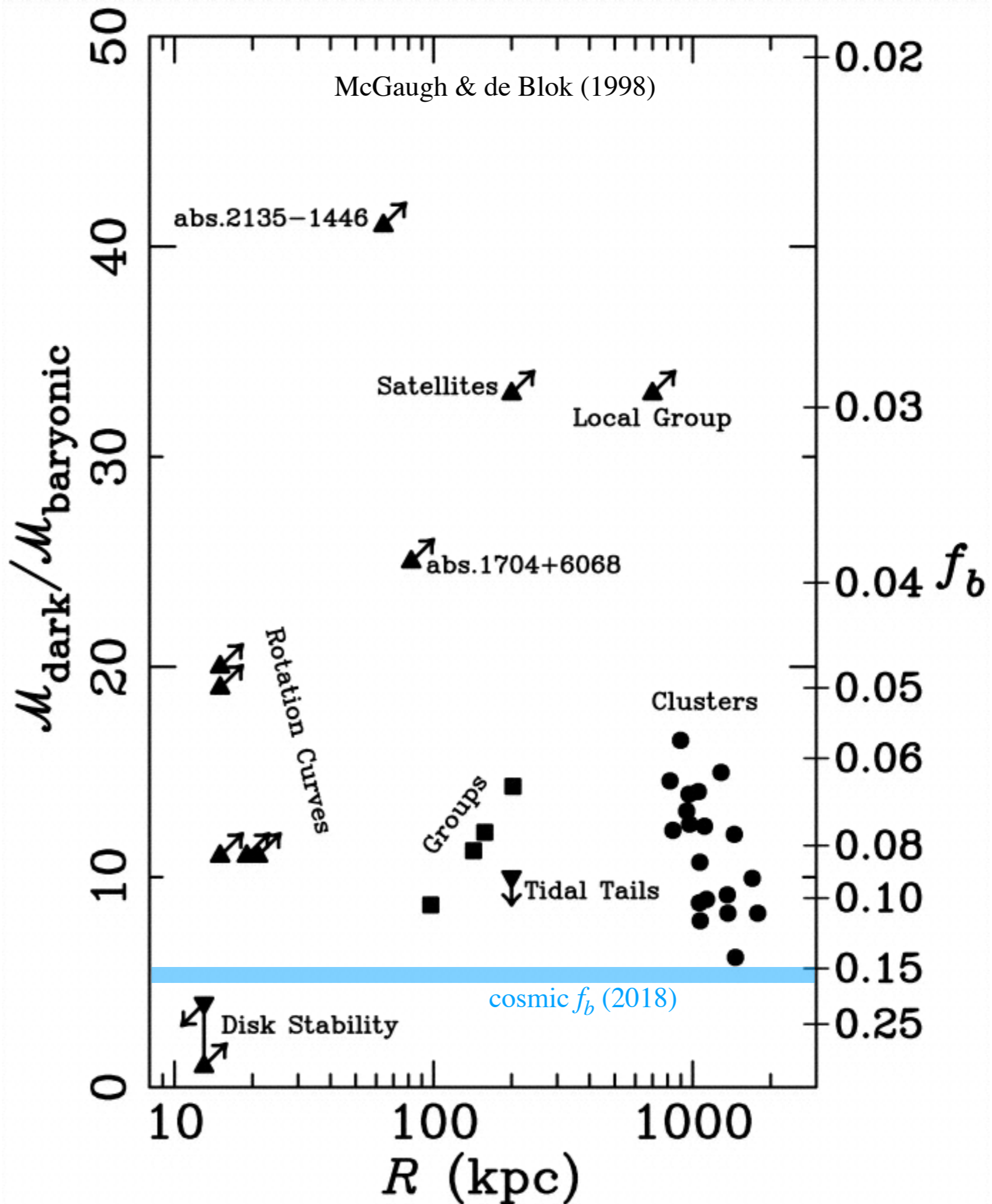


$$\Omega_m \approx \frac{1}{4}$$

FIG. 2.—Composite mass-to-light ratio of different systems—galaxies, groups, clusters, and superclusters—as a function of scale. The best-fit $M/L_B \propto R$ lines for spirals and ellipticals (from Fig. 1) are shown. We present median values at different scales for the large samples of galaxies, groups and clusters, as well as specific values for some individual galaxies, X-ray groups, and superclusters. Typical 1σ uncertainties and 1σ scatter around median values are shown. Also presented, for comparison, are the M/L_B (or equivalently Ω) determinations from the cosmic virial theorem, the least action method, and the *range* of various reported results from the Virgo-centric infall and large-scale bulk flows (assuming mass traces light). The M/L_B expected for $\Omega = 1$ and $\Omega = 0.3$ are indicated.

– cluster baryon fractions

$$f_b = \frac{M_b}{M_{tot}} \quad \circ \longrightarrow \quad \Omega_m = \frac{\Omega_b}{f_b}$$



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Measurements of the gravitating mass density

- Weak lensing
 - measure shear over large scales

Dark Energy Survey
arxiv:2002.11124

$$\Omega_m \approx 0.18 \pm 0.04$$

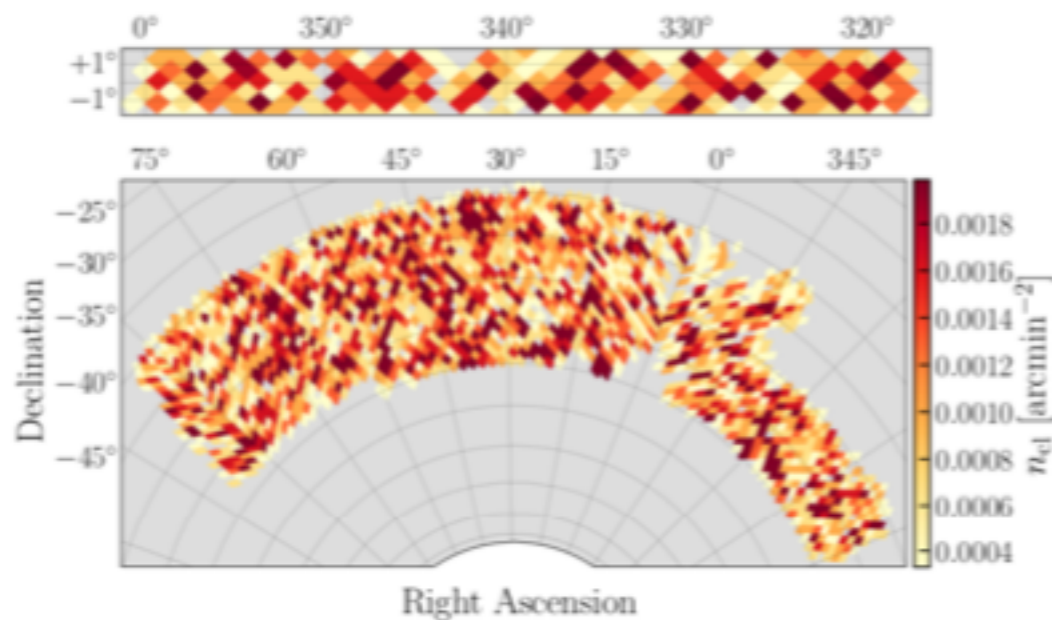
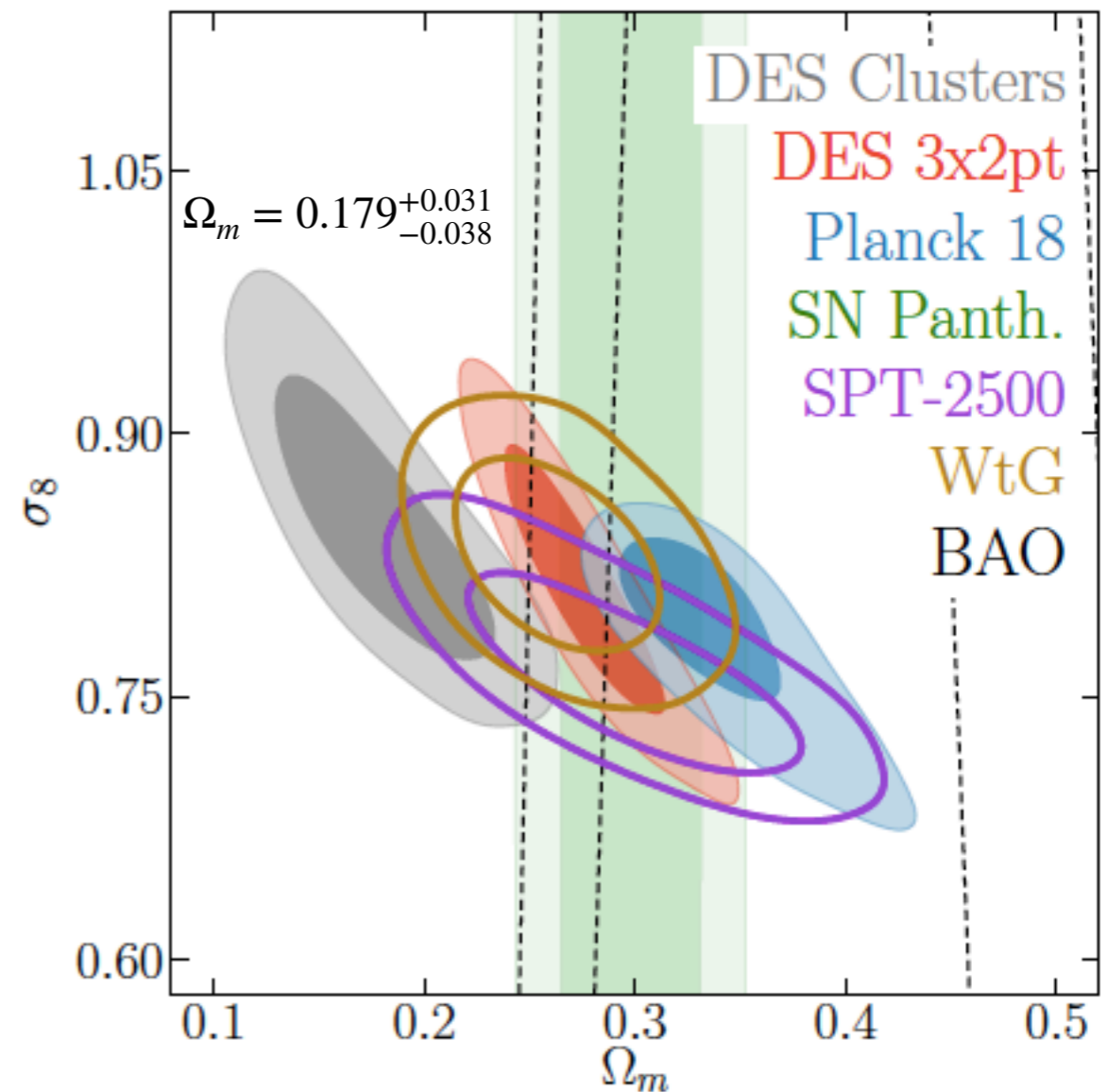


FIG. 1. The DES Y1 redMaPPer cluster density over the two non-contiguous regions of the Y1 footprint: the Stripe 82 region (116 deg²; *upper panel*) and the SPT region (1321 deg²; *lower panel*).



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Virgo-centric infall

The Virgo cluster is the largest nearby over-density. Its gravity distorts the Hubble flow. We fall towards it so it appears to recede less than it should by an amount that depends on its mass

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TONRY AND DAVIS (1981)

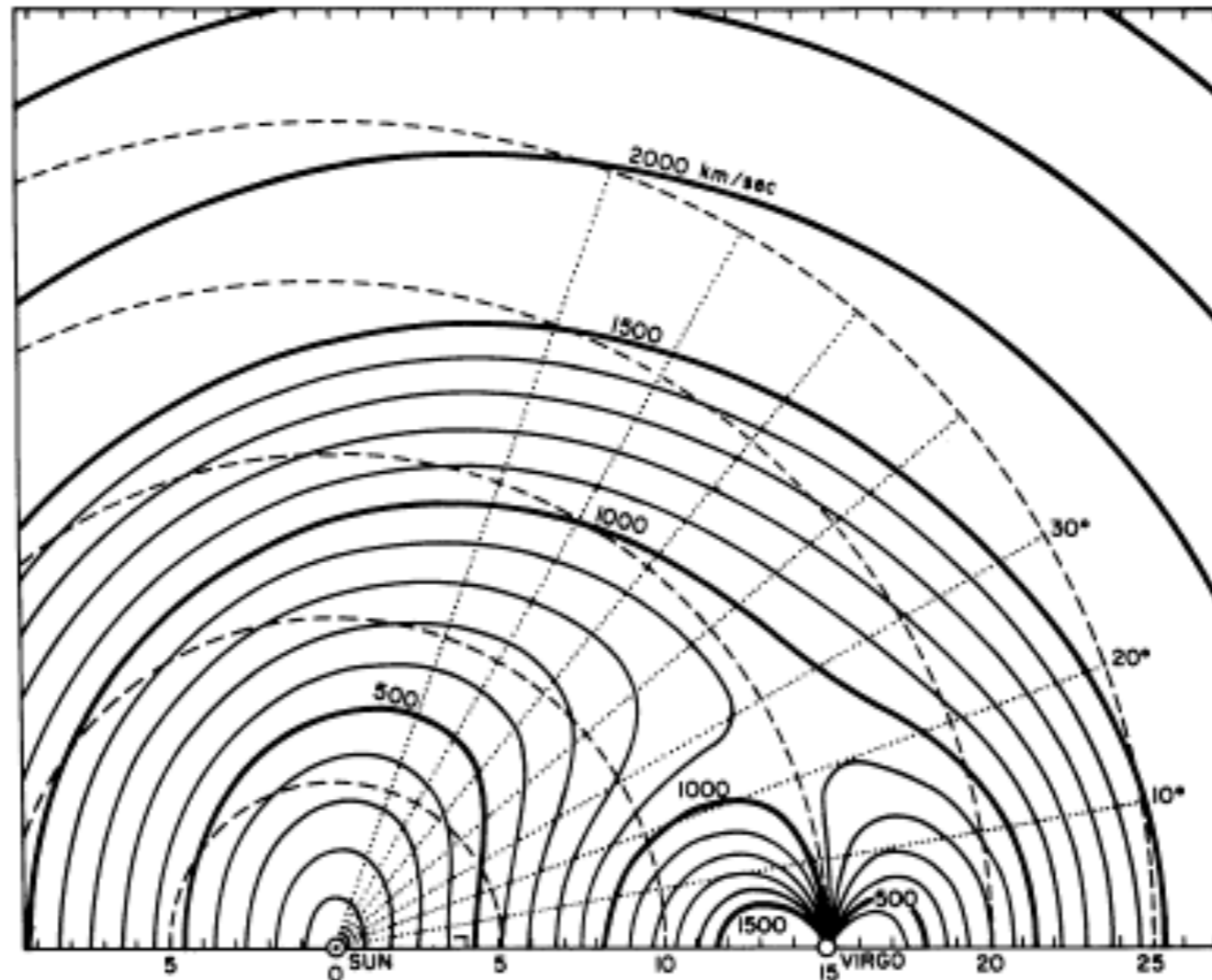


FIG. 1.—On a two-dimensional grid with the Earth and the Virgo cluster on the x axis, redshift contours are plotted for a Hubble flow perturbed by a Virgo-centric flow. An infall velocity of 400 km s^{-1} at our position is assumed. A pure Hubble flow would be concentric circles.

Measurements of the gravitating mass density

- Peculiar Velocity Field

- measure deviations from Hubble flow

in linear regime $\frac{\delta\rho}{\rho} \ll 1$

$$\frac{\delta v}{v} \approx \frac{d \ln H}{d \ln \rho} \frac{\delta \rho}{\rho} \approx - \frac{1}{3} \frac{\Omega_m^{0.6}}{b} \frac{\delta \rho_g}{\rho_g}$$

peculiar velocity

distortion in Hubble flow induced by

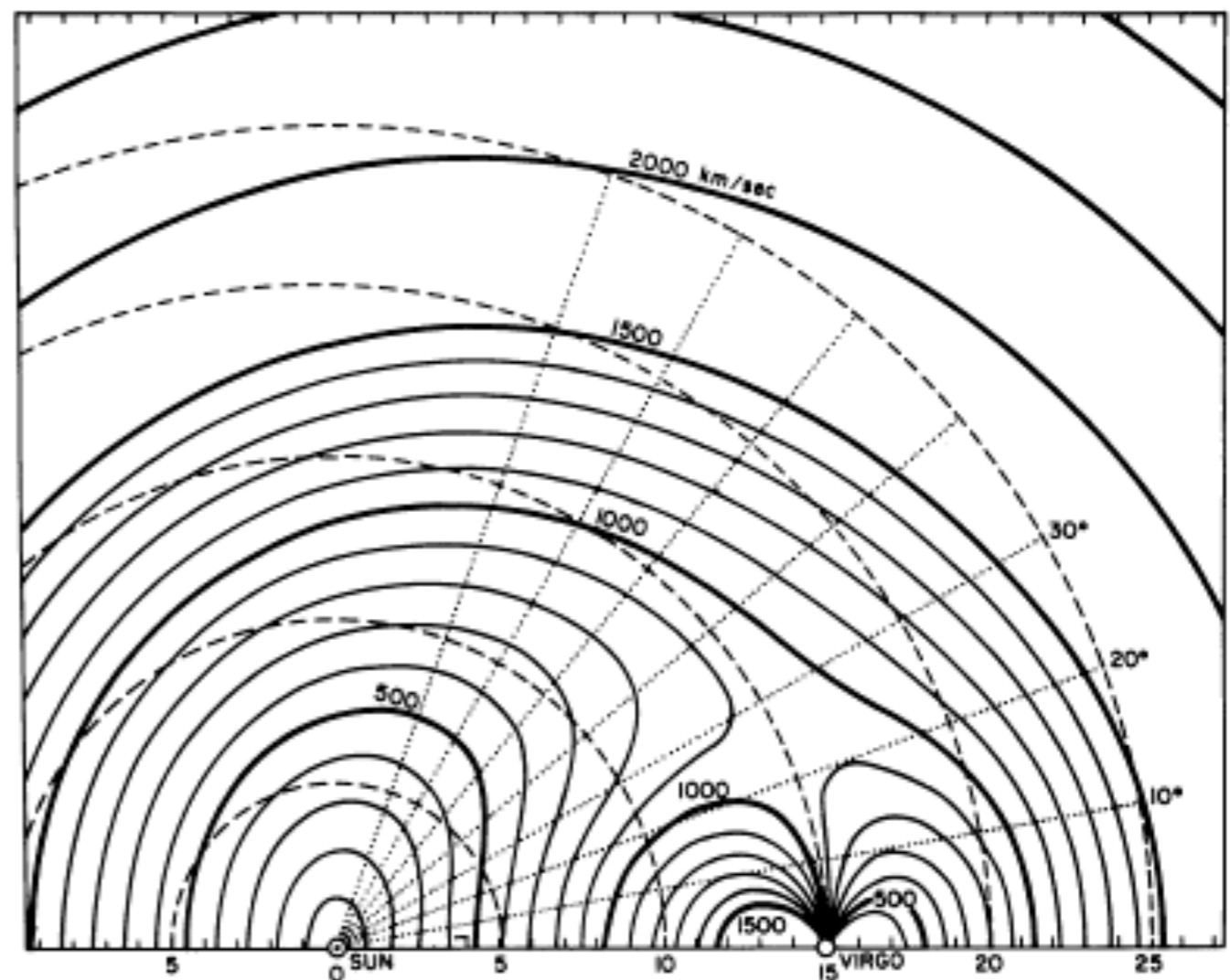
mass over-density

bias

BIAS b relates galaxy over-densities to mass over-densities

$$\Omega_m = 0.25 \pm 0.05$$

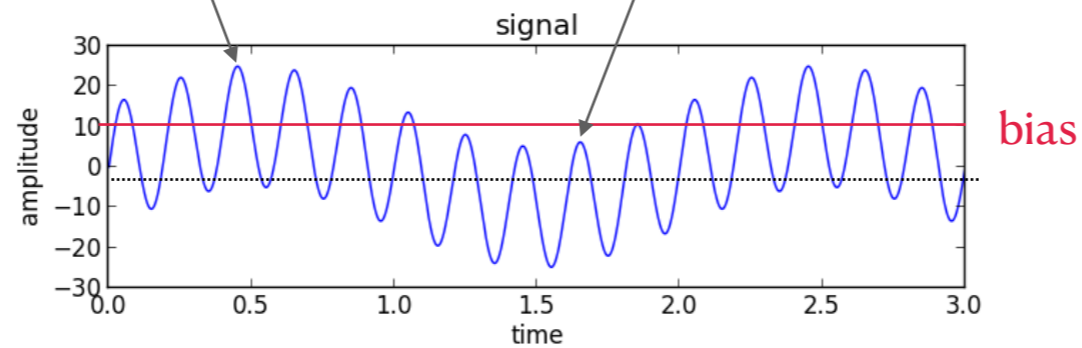
TONRY AND DAVIS



BIAS b
relates
galaxy over-densities
to mass over-densities

Peaks above line make a galaxy

Peaks below line do not



$$\frac{\delta\rho_g}{\rho_g} = b \frac{\delta\rho_m}{\rho_m}$$

$$\sigma_8 = \frac{1}{b} \quad \text{in a sphere of radius 8 Mpc}$$

Davis et al. (1980) found

$$\Omega_m = 0.4 \pm 0.1$$

with modern distances this becomes

$$\Omega_m = 0.25 \pm 0.05$$

basically unchanged for nearly 40 years

Lines are lines of constant Ω_m

ESTIMATES OF v_p

Velocity	Source
380 ± 75	Smoot and Lubin 1979
480 ± 75	Aaronson <i>et al.</i> 1980
350 ± 50	de Vaucouleurs and Bollinger 1979
$290 \pm 30^*$	Yahil 1980
190 ± 130	Schechter 1968

* Calculated with respect to the centroid at the local group as defined by Yahil *et al.* 1977.

$$\frac{\delta\rho}{\rho}$$

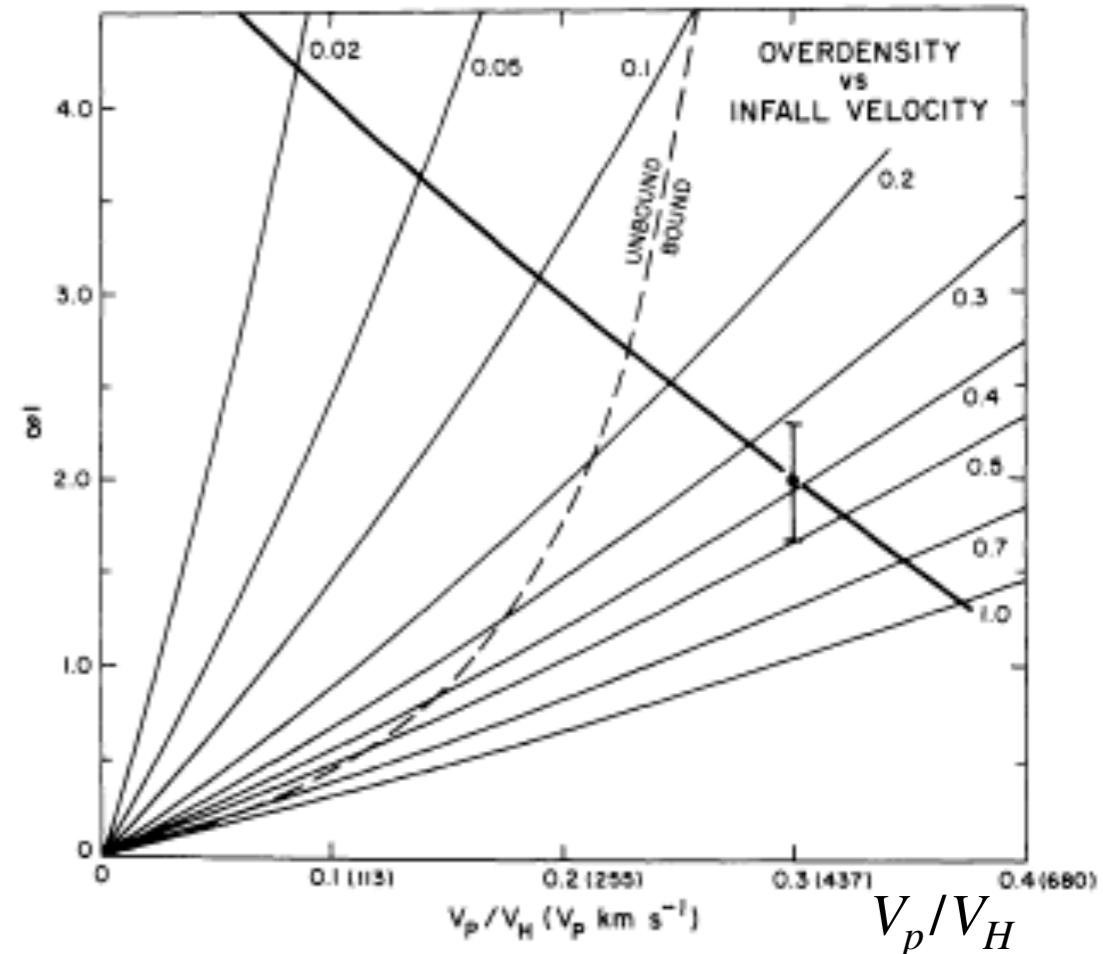


FIG. 1.—The mean overdensity of Virgo vs. v_p/v_H for various values of Ω . The x-axis is also labeled with v_p , using a recessional velocity to Virgo of 1020 km s^{-1} . The measured overdensity is prescribed by the heavy line, and is marked at the favored position as given by the anisotropy of the Hubble flow and microwave background radiation. The error bar is an estimate of the 90% confidence limit of our determination of $\bar{\delta}$. Models to the right of the dotted line are bound to Virgo.

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- Power spectrum of galaxies

$z = 0$

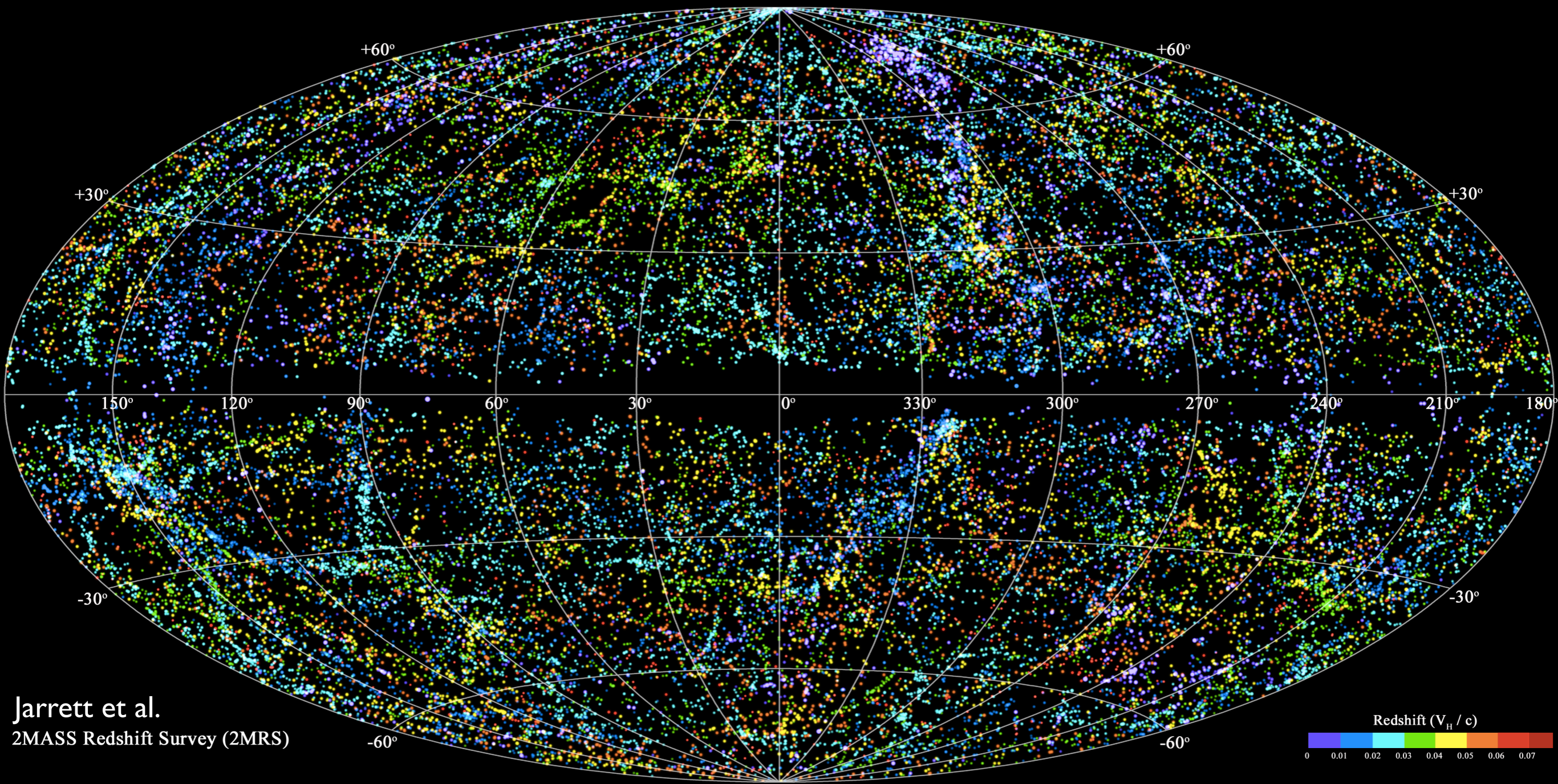
- CMB fits

$z = 1090$

Measurements of the gravitating mass density

- Power spectrum of galaxies

The power spectrum provides a statistical description of the clustering of galaxies, which are not randomly distributed in space



- Power spectrum of galaxies

$$\delta \equiv \frac{\delta\rho}{\rho}$$

The power spectrum is commonly used to quantify large scale structure. It is related to the 2 point correlation function via Fourier transform.

2 point correlation function: $\xi(\vec{r}) = \langle \delta(\vec{x})\delta(\vec{x} + \vec{r}) \rangle$


The 2 point correlation function is the probability of finding one galaxy near another in excess over a random distribution.

Power spectrum: $P(k) = \langle |\delta_k|^2 \rangle$ where $k = \frac{2\pi}{\lambda}$

where k is the wavenumber corresponding to the scale λ

Fourier transform:

$$\xi(\vec{r}) = \frac{V}{(2\pi)^3} \int |\delta_k|^2 e^{-i\vec{k}\cdot\vec{r}} d^3k \quad \text{averaged over volume } V$$


 $P(k)$

Large Scale Structure

Quantified with the **correlation function** $\xi(r)$ which is the Fourier transform of the **power spectrum** $P(k)$.

The correlation function is the excess probability of finding a galaxy near another galaxy over that in a random distribution.

$$\frac{dN}{N} = [1 + \xi(r)]dV \quad \xi(r) = \frac{V}{(2\pi)^3} \int P(k)e^{-\vec{k}\cdot\vec{r}} d^3k$$

$$P(k) \propto |\delta(k)|^2 \propto k^n$$

$$\xi(r) \propto r^{-(n+3)}$$

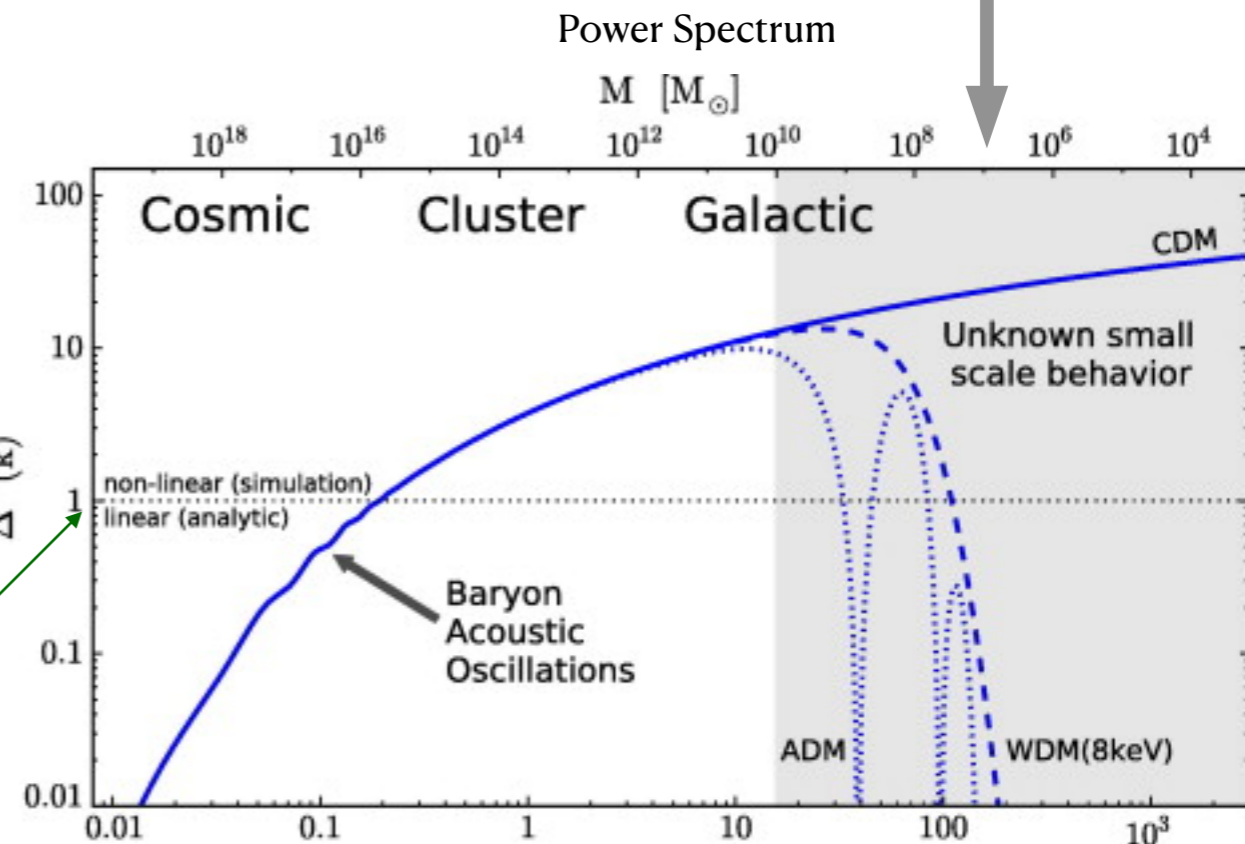
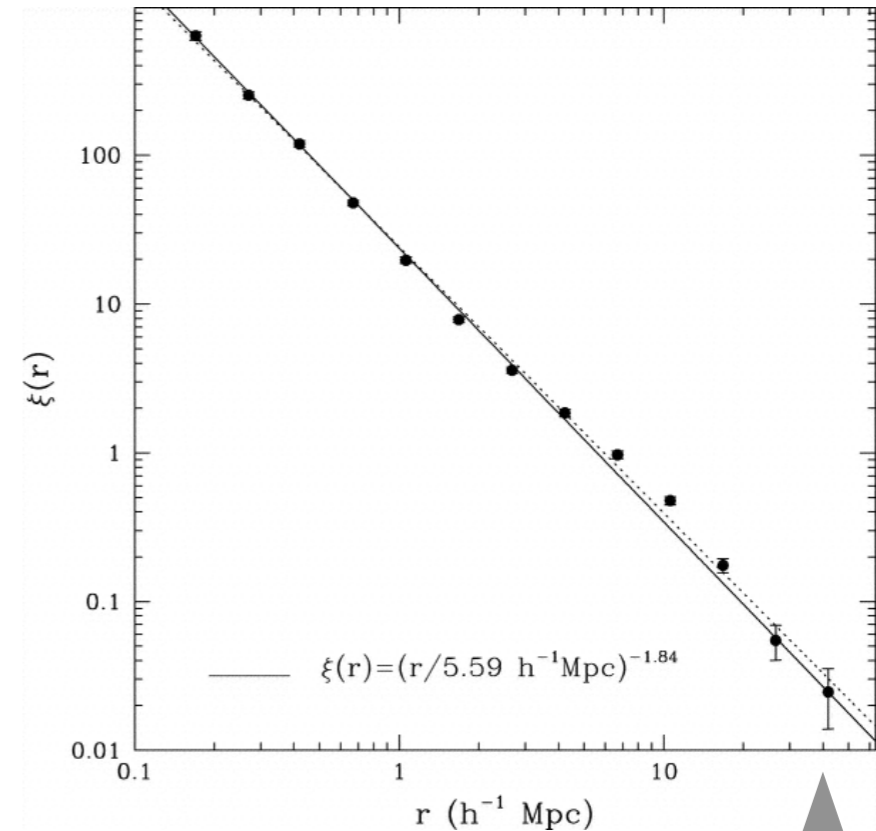
$$\delta \equiv \frac{\delta\rho}{\rho}$$

Harrison-Zeldovich spectrum has $n = 1$, which is a Gaussian random field. Inflation predicts $n \approx 1$, but different flavors of Inflationary theory predict slightly different values depending on the shape of the Inflationary potential (the Inflaton). Planck measures

$$n = 0.965 \pm 0.004$$

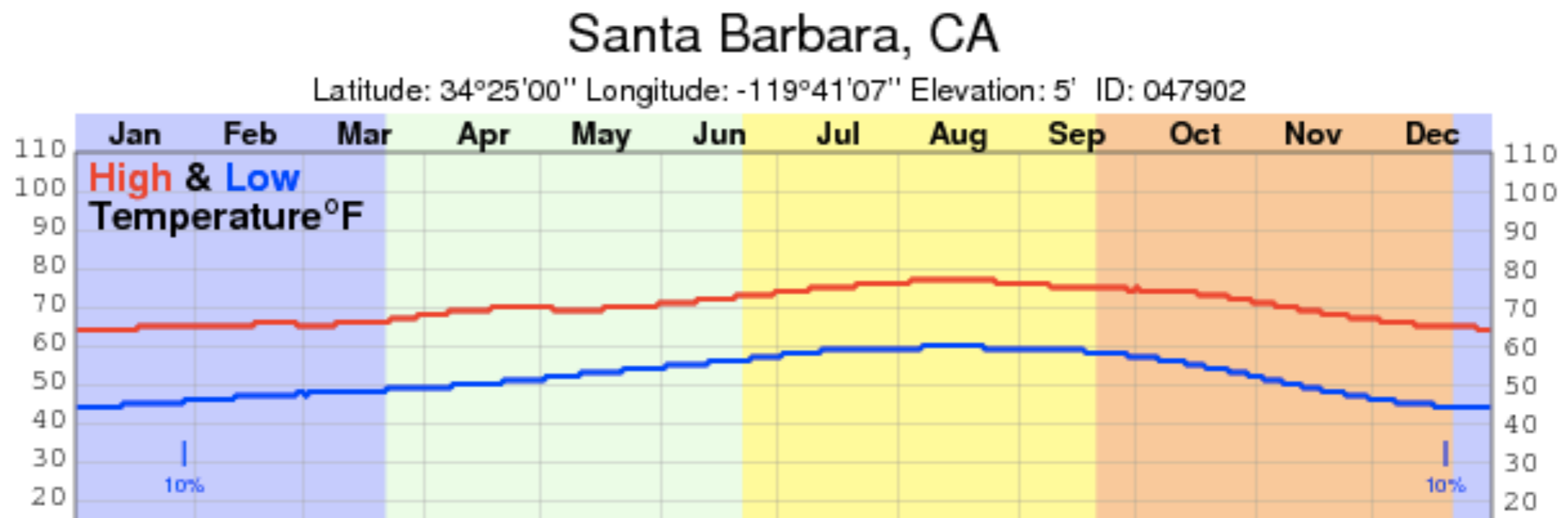
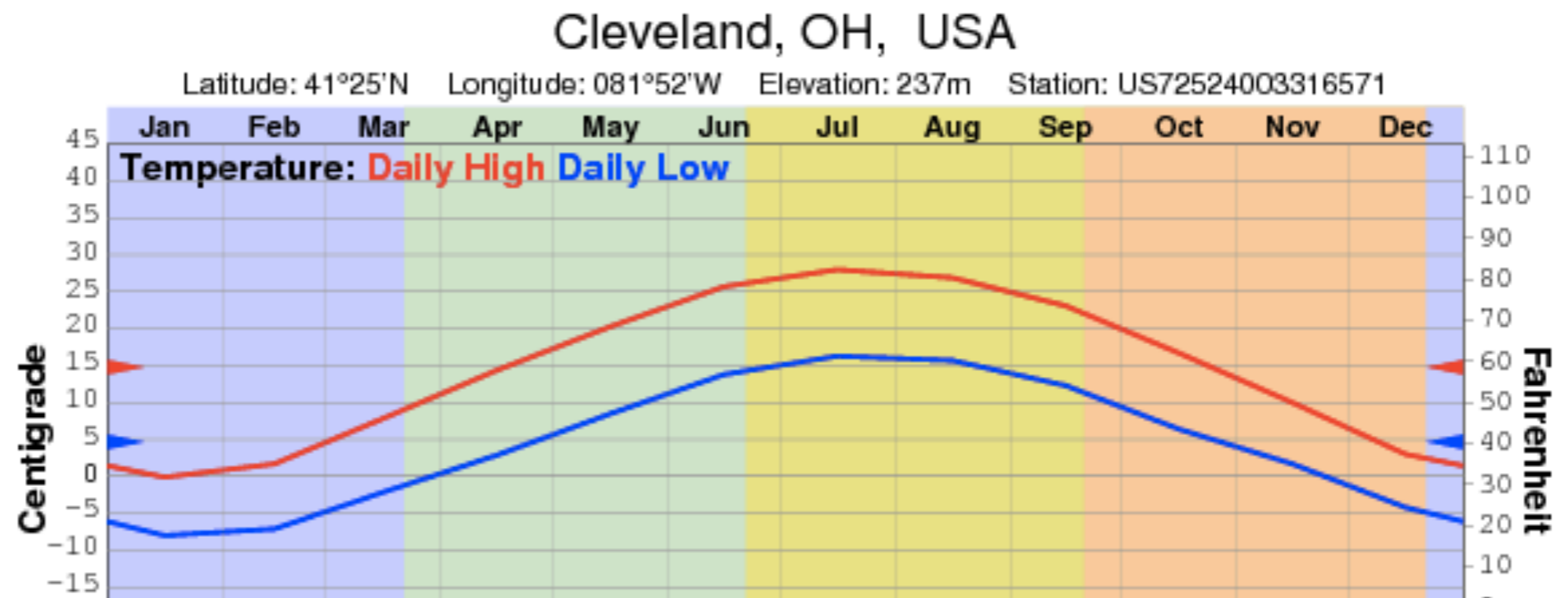
$\delta > 1$ marks the transition to the non-linear regime where perturbation theory no longer applies.

SDSS correlation function (Zehavi et al 2005)



Power Spectrum

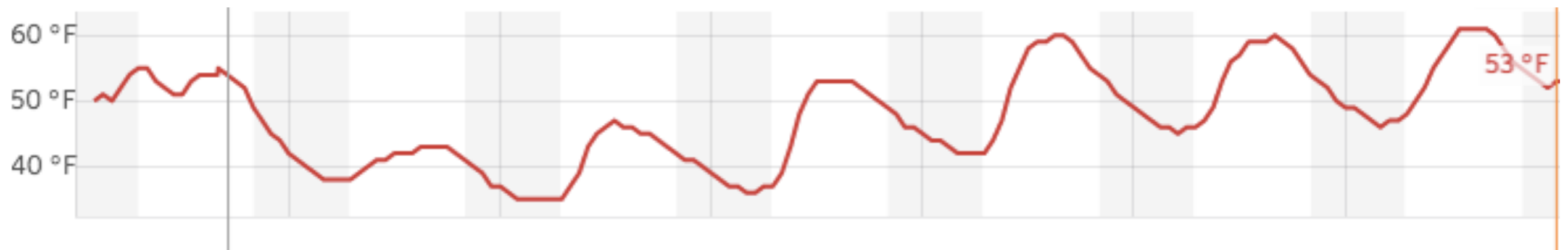
Example: weather in Cleveland and Santa Barbara
More power on long time scales in Cleveland (seasonal variation)



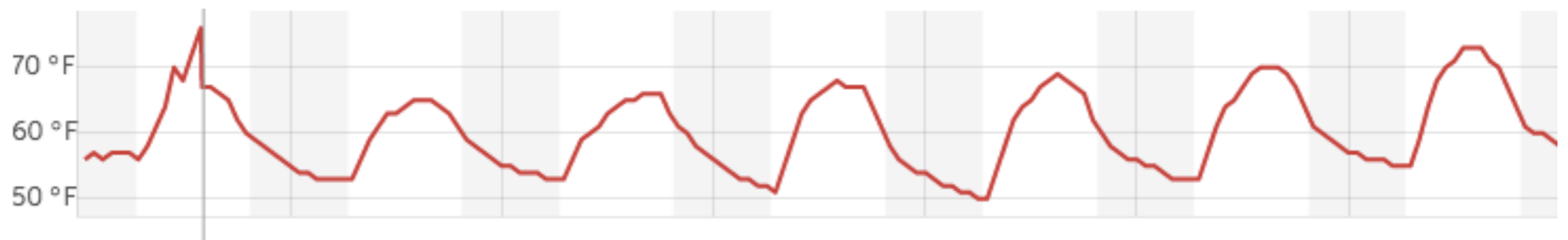
Power Spectrum

Example: weather in Cleveland and Santa Barbara
Similar power on short time scales in Santa Barbara (diurnal variation)

Cleveland forecast



Santa Barbara forecast



A power spectrum is a Fourier transform that quantifies the relative variability on different scales

- Power spectrum of galaxies

$$\delta \equiv \frac{\delta\rho}{\rho}$$

$$k = \frac{2\pi}{\lambda}$$

Power law power spectrum: $P(k) = \langle |\delta_k|^2 \rangle \propto k^n$

where $n = 1$ is scale free, with the same power on all scales.

This is observed to be nearly the case on large scales that have not yet collapsed. It is modulated on small scales by structure formation.

One way to think of it is the rms variation at each scale λ

$$M \sim \lambda^3 \qquad \delta_{\text{rms}} \propto M^{-(n+3)/6}$$

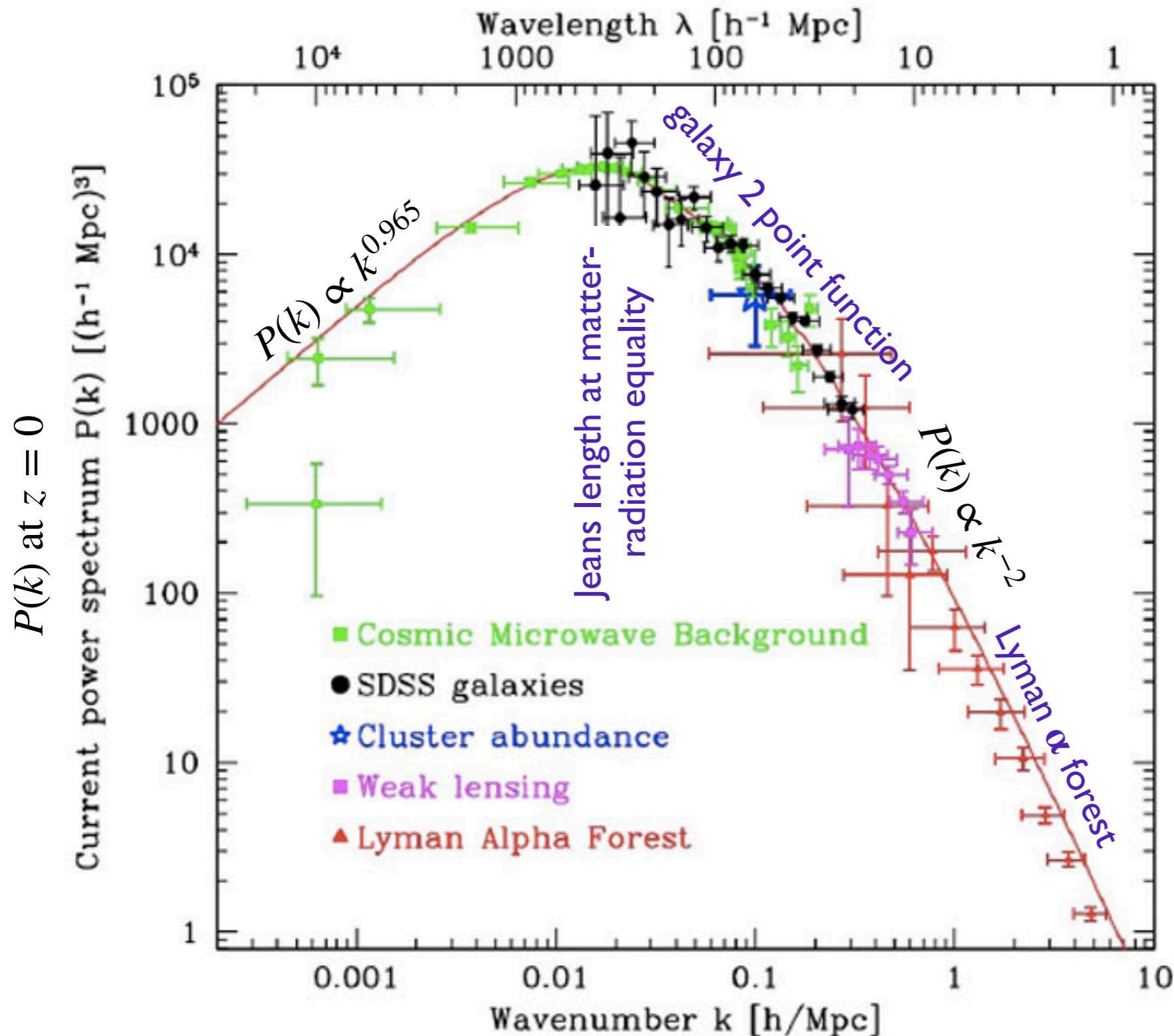
There is more rms variance on small scales, so more power there.

[On very large scales, the universe is homogeneous, so no variance.]

By convention, the normalization is set on a scale of 8 Mpc, where

$$\frac{\delta N_{gal}}{N_{gal}} = 1 \quad \text{with corresponding mass variance} \quad \sigma_8$$

Planck estimates: $n = 0.965 \pm 0.004$
 $\sigma_8 = 0.811 \pm 0.006$



Jeans length at matter-radiation equality

$$\lambda_J = c_s \sqrt{\frac{\pi}{G\rho}}$$

sound speed

$$c_s^2 = \frac{\partial P}{\partial \rho} = \frac{1}{3}c^2$$

at smaller scales, things go non-linear from gravitational collapse, pressure, dissipation, feedback, etc. Described by a Transfer function

$$T(k) \equiv \frac{\delta_k(z=0)}{D(z)\delta_k(z)}$$

where $D(z)$ is the linear growth factor - what it would have been without all these nasty non-linear effects.

From an accident report in the *Boston Driver's Handbook*:
“The guy was all over the road. I had to swerve several times before I hit him.”

The power spectrum of SCDM missed badly:
 too much power on small scales;
 too little power on large scales.

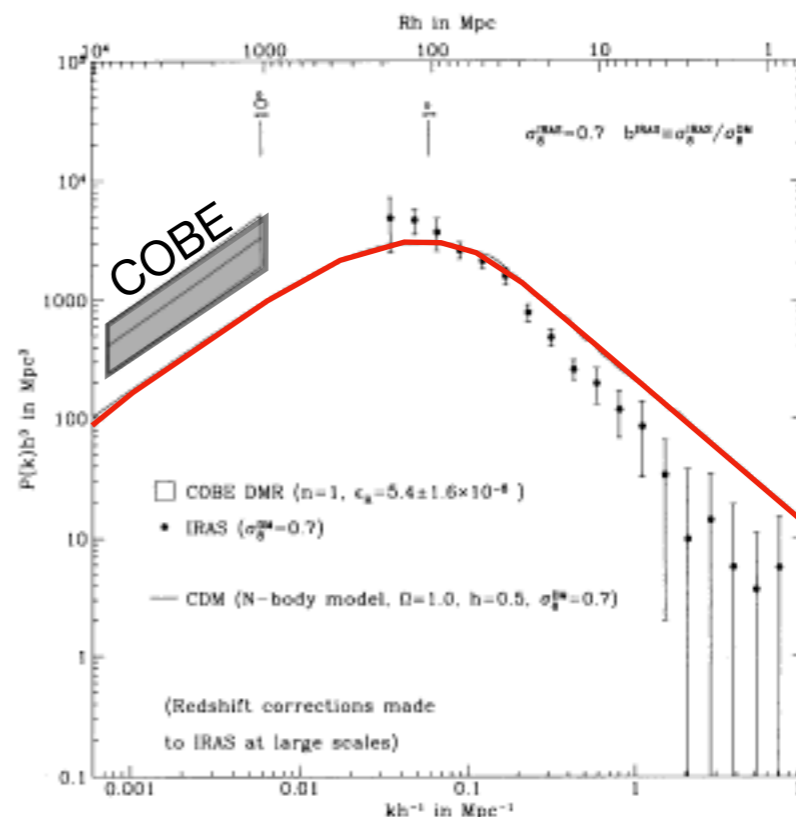
SCDM (“Standard” CDM)

$$\Omega_m = 1$$

$$H_0 = 50$$

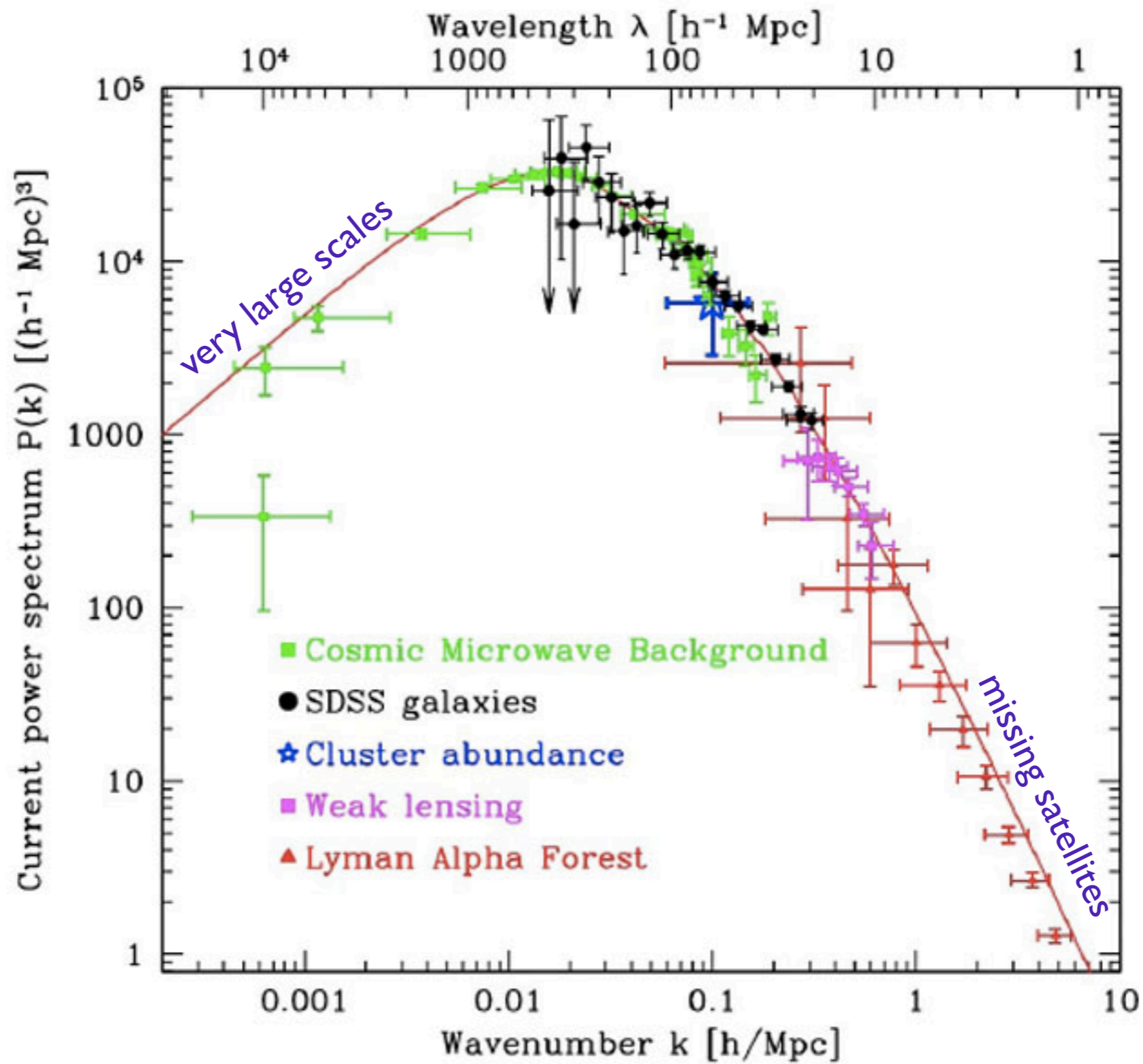
$$\Omega_m h = 0.5 \quad \text{expected}$$

$$\Omega_m h \approx 0.2 \quad \text{observed}$$



SCDM
 $\Omega_m h = 0.5$
 $\sigma_8 = 0.7$

FIG. 10.—Solid curve is the real space power spectrum of the full nonlinear CDM N -body simulation (as in Fig. 3) normalized to the real space variance of *IRAS* galaxies ($\sigma_8 = 0.7$). The points are the *IRAS* redshift space $\bar{P}(k)$ from Fig. 4, rescaled by eq. (17) with $\Omega = 1$ and $b = 1$; this is then, apart from the effects of the convolution in eq. (14), an approximation to the power spectrum of *IRAS* galaxies in *real* space on large scales if the *IRAS* galaxies are unbiased. The box indicates the power spectrum inferred from the *COBE* DMR measurements, assuming a $n = 1$ spectral index and $\epsilon_H = (5.4 \pm 1.6) \times 10^{-6}$ (Smoot et al. 1992; Wright et al. 1992). Note that when the CDM model is normalized to the *IRAS* variance, it produces excessive power on small scales while simultaneously failing to produce sufficient power on large scales to match the *COBE* results.



LCDM

$$\Omega_m = 0.3$$

$$H_0 = 70$$

$$\Omega_\Lambda = 0.7$$

$$\Omega_m h = 0.21$$

$$\sigma_8 = 0.83$$

$$n_s = 0.965$$

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- Cluster M/L $\Omega_m \approx 0.25$ Bahcall et al. (1995)
 - measure M/L of a cluster, combine with measured luminosity density of universe.
- Weak lensing $\Omega_m \approx 0.18 \pm 0.04$ Dark Energy Survey arxiv:2002.11124
 - measure shear over large scales
- Peculiar Velocity Field $\Omega_m = 0.25 \pm 0.05$ Tonry & Davis (1980)
 - measure deviations from Hubble flow
- Power spectrum of galaxies $\Omega_m h = 0.213 \pm 0.023$
 $\Omega_m = 0.3$ for $h = 0.71$
- CMB fits $\Omega_m = 0.315 \pm 0.007$ Planck Collaboration (2018)
also gives $h = 0.674 \pm 0.005$

Tegmark et al. (2004)