Halo models

A brief guide to common dark matter halo models

- Pseudo-isothermal $\rho \sim \text{const}$ $r \to 0$
  $\rho \sim r^{-2}$ $r \to \infty$

  Works well for fitting rotation curves empirically motivated

- NFW (Navarro - Frenk - White 1997)

  emerges from $\rho \sim r^{-1}$ $r \to 0$
  computer simulations $\rho \sim r^{-3}$ $r \to \infty$
  of structure formation
  for self-gravitating but otherwise non-interacting
  dark matter particles in an expanding universe.
  Provides a poor description of real rotation curves

- Burkert

  The Burkert profile is an attempt to reconcile the best features of p-ISO & NFW halos:

  $\rho \sim \text{const}$ $r \to 0$
  $\rho \sim r^{-3}$ $r \to \infty$

- Einasto profile

  The ~~E~~ Einasto profile adds a 3^{rd} parameter to better fit numerical simulation results. Observationally it is indistinguishable from the NFW halo.

Halo models

Many models have been proposed for dark matter halos

A brief guide to some of the more common that one may run across in the literature

- the Pseudo-Isothermal halo

WORKS WELL FOR FITTING OBSERVED ROTATION CURVES

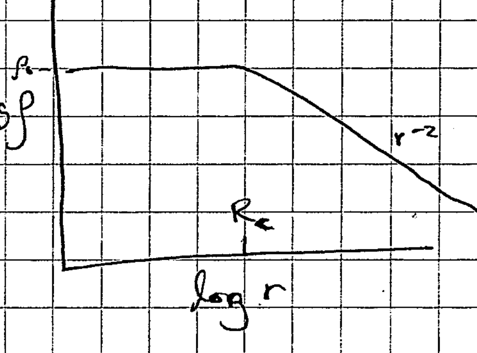
This was the form most commonly assumed after the discovery of flat rotation curves. In order to obtain $V(R) \sim$ constant requires $\rho \sim r^{-2}$. This occurs in an isothermal sphere in which the velocity dispersion of stars can be described in analogy to a perfect gas with $\sigma^2 = \frac{kT}{m}$ (see BT 4.3).

This behavior persists to $r = 0$, contrary to the data, hence the "pseudo" part, which introduces a constant density core so that

$$\rho \sim \text{constant} \quad r \to 0$$
$$\rho \sim r^{-2} \quad r \to \infty$$

$$\rho(r) = \frac{\rho_0}{1 + (r/R_c)}$$

where ρ_0 = core density, R_c = core radius



The rotation curve is

$$V_{p\text{-}iso}(r) = V_\infty \sqrt{1 - \left(\frac{R_c}{r}\right) \tan^{-1}\left(\frac{r}{R_c}\right)}$$

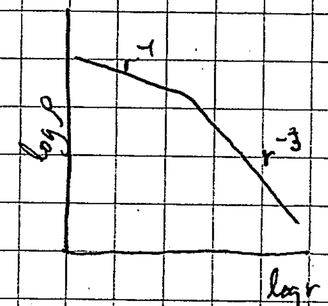
with $V_\infty = \sqrt{4\pi G \rho_0 R_c^2}$

with the obvious temptation to associate $V_f = V_\infty$

— NFW halos

Following the formalism used by Jerry Sellwood in his notes on NFW ~~halos~~ linked on the course web page under review literature,

$$\rho(r) = \frac{\rho_c r_s^3}{r(r+r_s)^2}$$



This leads to the rotation curve

$$V(R) = V_{200}\sqrt{\frac{\ln(1+cx) - \frac{cx}{1+cx}}{x\left[\ln(1+c) - \frac{c}{1+c}\right]}}$$

where $x = \frac{r}{R_{200}}$; $c = \frac{R_{200}}{r_s}$ ("concentration")

R_{200} is the radius that encloses a density $200\times$ the critical density of the universe

$$\rho_{crit} = \frac{3H_0^2}{8\pi G} \qquad \Delta \equiv \frac{\rho}{\rho_{crit}}$$

$\Delta \approx 200$ is roughly the "virial" overdensity, i.e., material within this overdensity has had time to settle.
[Strictly speaking $\Delta = 186$ for $\Omega_m = 1$.
The virial overdensity is more like $\Delta \approx 100$ in ΛCDM, but we persist in referencing everything to $\Delta = 200$]

This is all notional.

Note: the NFW density profile diverges, but only logarithmically in mass.

$$V_{esc}^2 = 2|\Phi|$$

The potential is finite, so an escape velocity can be defined (unlike for the pseudo-isothermal halo)

Einasto halo

A better fitting fcn for simulated halos than NFW at the expense of an extra parameter

Merritt et al (2006):

$$\rho(r) = \rho_e e^{-d_n\left[\left(\frac{r}{r_e}\right)^{1/n} - 1\right]}$$

If that looks familiar, it is because the Einasto profile is the 3D equivalent of the Sersic profile used for fittin projected surface densities

ρ_e = density at radius r_e that contains $\frac{1}{2}$ of the total mass

d_n is a hassle to obtain, but is well approximated by

$$d_n \simeq 3n - \frac{1}{3} + \frac{0.0079}{n} \quad \text{for} \quad n > \frac{1}{2}$$

Simulated halos have $4.6 < n < 8.2$

In the notation of Navarro et al. (2004), $\alpha = \frac{1}{n}$

who found the mid-point $\alpha \simeq 0.17$

The variable n is sometimes invoked to explain cores in real galaxies, but this is not correct. Both the change in slope and its location (at very small radii) fail to explain observations. It is too small an effect - really just a tweak to NFW

Einasto halos do have the nice property of a finite total mass

$$M(r) = 4\pi n r_e^3 \rho_e e^{d_n} d_n^{-3n} \gamma(3n, x)$$ where γ is the incomplete gamma fcn

integrating $M(r)$ to ∞ turns γ into Γ $\qquad x = d_n \left(\frac{r}{r_e}\right)^{1/n}$

$$M_{tot} = 4\pi n r_e^3 \rho_e e^{d_n} d_n^{-3n} \Gamma(3n)$$

Empirical DM Halo — McGaugh et al (2007), Walker et al (2009)

Just fit the portion of the data attributable to dark matter. Adjust M_*/L & choose that which minimizes scatter - in both BTFR and $V_{DM}(R)$, as it happens. Get

$$\log_{10} V(R) = 1.47^{+0.15}_{-0.19} + \frac{1}{2} \log R$$

V in km s^{-1}
R in kpc

slope $\frac{1}{2}$ fixed to NFW value (best fit 0.49)

this can also be expressed in terms of the enclosed mass

$$M_{DM}(R) = 200^{+200}_{-120} \left(\frac{R}{pc}\right)^2$$

M in $M_\odot$
R in pc this time

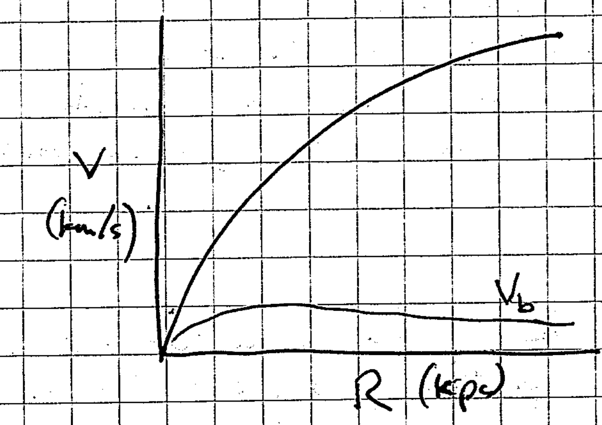
note: this is not the same as M_{total}, which increases linearly

or $$g_{DM} = 3^{+3}_{-2} \times 10^{-11} \text{ m s}^{-2}$$

What is the null hypothesis in ΛCDM?

CDM forms dark matter halos whose properties are specified by cosmic parameters: Ω_m, H_0, σ_8 - basically the density of matter at the time of halo formation. Baryons subsequently settle in the dark matter halos to form the observed galaxies.

When the dark matter dominates over the baryons (as in low surface brightness galaxies), the obvious null hypothesis is that the baryons just trace the potential of the dark matter:



$V \gg V_b$

The NFW halos predicted by CDM simulations are basically the same at the same mass, so our null hypothesis is that V(R) is [basically] the same for galaxies of the same mass.

This is NOT observed. There is considerable "diversity" in V(R) at fixed mass when R is measured in physical units (e.g., kpc).

Strangely, there is uniformity if R is measured in scale lengths (R_d). $V(R/R_d)$ is similar at fixed mass. The mass "knows" about the light.