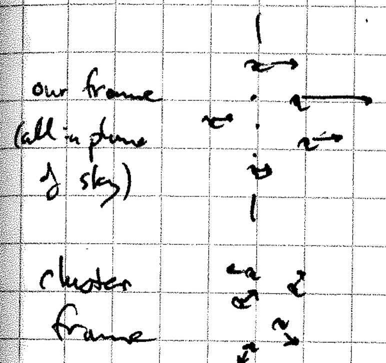
Early indications of dark matter

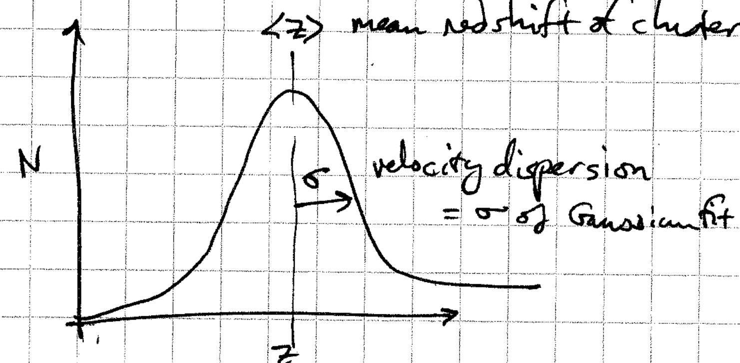
Oort (1932) discrepancy : z-motions of local stars

Zwicky (1930's) : velocity dispersions of clusters

Peebles & Ostriker (1973) : stability of galactic disk against the bar instability

Velocity dispersions of Clusters (Zwicky)





In general, σ need not be a single number as the real distribution need not be Gaussian, so one has a velocity dispersion tensor

$$\sigma_{ij}^2 = \overline{v_i v_j} - \overline{v_i}\,\overline{v_j}$$

In practice we only have line-of-sight velocities for cluster galaxies, so assume this 1D measured quantity is representative of the 3D space motion by <u>assuming</u> isotropy and <u>virial equilibrium</u>.

Virial Theorem

Following Bothun §. 4.1.1

Moment of Inertia of system of N particles

$$I = \sum_{i=1}^{N} m_i r_i^2 = \sum_{i=1}^{N} m_i (x_i^2 + y_i^2 + z_i^2)$$

time derivative of moment of inertia

$$\frac{dI}{dt} = \dot{I} = \sum_{i=1}^{N} m_i (2x_i\dot{x}_i + 2y_i\dot{y}_i + 2z_i\dot{z}_i)$$

$$\frac{1}{2}\frac{d^2I}{dt^2} = \frac{1}{2}\ddot{I} = \underbrace{\sum_{i=1}^{N} m_i(\dot{x}^2 + \dot{y}^2 + \dot{z}^2)}_{mv^2 = \text{kinetic energy}, 2T} + \underbrace{\sum_{i=1}^{N} m_i(x_i\ddot{x}_i + y_i\ddot{y}_i + z_i\ddot{z}_i)}_{\vec{r}\cdot(m\vec{a}) \;\; \text{potential energy}, W}$$

in general

$$\boxed{\frac{1}{2}\ddot{I} = 2K + W}$$

After a few dynamical times, a self-gravitating system will obtain VIRIAL EQUILIBRIUM (become "virialized") in which the time-averaged moment of inertia is steady:

$\langle \dot{I} \rangle \to$ constant so $\langle \ddot{I} \rangle \to 0$ [Note - this is violated during mergers, reinstated afterwards] ~~time-varying gravitational~~ potential can rearrange orbits

$$\boxed{\frac{1}{2}\langle \ddot{I} \rangle = 0 = 2\langle K \rangle + \langle W \rangle}$$ VIRIAL THM

For self gravitating system of particles,

$$W = -\frac{G}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{m_i m_j}{r_{ij}}$$

IFF all m_i the same (or nearly so), then can replace sums →

$$\sum_i^N \sum_j^N m_i m_j = N(N-1) m_i \to M^2$$ for large N

(where **M** is the total mass)

so $$W = -\frac{GM^2}{2R_{hms}}$$ where R_{hms} is the harmonic mean separation

$$\frac{1}{R_{hms}} \equiv \frac{1}{N} \sum_{i=1}^{N} \frac{1}{r_i}$$

OK, so now use this: $2\langle T\rangle + \langle W\rangle = 0 \to \langle T\rangle = -\frac{1}{2}\langle W\rangle$

$$\boxed{\begin{matrix} K > 0 \\ W < 0 \end{matrix}}$$

$$K = \frac{1}{2} N v_i^2 m_i$$

$$\langle K\rangle = \frac{1}{2} M \langle v_i\rangle^2 \qquad \sigma_v = \langle v_i^2\rangle^{1/2}$$

σ_v rms velocity dispersion

$$2\langle K\rangle + \langle W\rangle = 0 = 2\,\frac{1}{2} M \sigma_v^2 - \frac{1}{2}\frac{GM^2}{R_{hms}}$$

W negative!

$$M\sigma_v^2 = \frac{1}{2}\frac{GM^2}{R_{hms}}$$

$$\boxed{M = \frac{2\sigma_v^2 R_{hms}}{G}}$$ VIRIAL MASS ESTIMATOR

σ_v 3D velocity dispersion — take care in relating this to L.O.S measured σ_{LOS} — $\sigma_{LOS} = \sigma_v$ ONLY IF system isotropic

need to measure σ_v, R_{hms} to get mass

In practice, $R_{hms} \approx 1.25\, R_{eff}$ is a good approximation

R_{eff} = half light radius → hence common assumption "light traces mass" i.e., $R_{eff}(\text{light}) = R_{eff}(\text{mass})$

This is a more serious issue than $R_{hms} \approx 1.25 R_{eff}$ approximation

$$\sigma_{LOS}^2 = \sum_j (\sigma_j / \sin i_j)^2$$

Applied to clusters to get M

→ M/L → M/L × j → Ω_m

Example application of the Virial thm to a typical rich cluster of galaxies (similar to Coma)

$$M = \frac{2\sigma^2 R_{hms}}{G}$$

$R_{hms} \approx 1.25 R_e$ $\quad R_e \approx 1$ Mpc

$\sigma \approx 1000$ km s^{-1}

$G = 4.3\times10^{-6}$ kpc km^2 s^{-2} $M_\odot^{-1}$

$$M = \frac{2\,(\underbrace{10^3}_{\text{km/s}})^2\,(\underbrace{1250}_{\text{kpc}})}{4.3\times10^{-6}}$$

$$M \approx 6\times10^{14}\ M_\odot$$

for Coma

$L_B \approx 8\times10^{12}\ L_\odot$

$$\frac{M}{L} = 75\ \frac{M_\odot}{L_\odot}$$

($\approx$ 200 MWs in B-band)

Things to worry about:

- clusters never "virialized" (in dynamical equilibrium) — always show substructure
- interlopers — non-cluster members might contaminate σ.
- X-ray gas: stars not representative of baryons,
  so $M/M_b \sim 7$ even though $M/L \sim 70$
  (i.e., gas outweighs stars by factor of ~ 10)
  ... actually (at least) TWO missing mass ~~problems~~ types in clusters,
  one of which turned out to be the x-ray gas