A few notional time scales

- crossing time $\quad t_c = \frac{2R}{V} \quad \frac{\text{diameter}}{v \text{ or } \sigma}$

typical time to go from one side of a system to the other

Question: will an O star live many crossing times?

- dynamical time $\quad t_d = \sqrt{\frac{3\pi}{16 G\rho}}$

typical orbital time in homogeneous sphere of density ρ

- relaxation time $\quad \frac{t_R}{t_c} = \frac{N}{48 f^2} \approx \frac{N}{6 \ln(N/2)}$

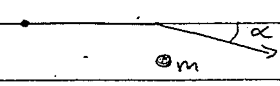
↳ N objects carrying a fraction f of the mass

typical time to forget initial conditions

for MW (smoother) w/ $N \approx 10^{11}$, $t_R \gg$ age of Universe

for GC (grainier) w/ $N \approx 10^6$, $t_R \approx 10$ Gyr ~ age of Universe

Relaxation is the cumulative result of many weak gravitational encounters that only nudge the vector $\quad \alpha = \frac{\Delta V_\perp}{V} = \frac{2Gm}{bV^2}$



Strong encounters (w/ ΔP.E. ~ K.E.) are rare in the solar neighborhood $\quad t_{strong} \sim 10^{15}$ yr $\gg$ U

Homework Hint

$$\Phi = v^2 = \frac{GM}{R} \quad \text{for a spherical mass distribution}$$

but $\nabla^2\Phi = 4\pi G\rho$

so you can't just set $M = \frac{4\pi}{3}\rho R^3$

to get $\rho(r)$ because $\rho(r) \neq \langle\rho\rangle_{<r}$

need to do the integral.

When solving the Poisson equation,
take care with the choices of geometry!

Cartesian:

$$\nabla^2\Phi = \frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2} + \frac{\partial^2\Phi}{\partial z^2}$$

Cylindrical:

$$\nabla^2\Phi = \frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial\Phi}{\partial R}\right) + \frac{1}{R^2}\frac{\partial^2\Phi}{\partial\theta^2} + \frac{\partial^2\Phi}{\partial z^2}$$

Spherical:

$$\nabla^2\Phi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\Phi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\,\frac{\partial\Phi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\Phi}{\partial\phi^2}$$

Nb: by convention, use R for cylinders & r for spheres