Scaling relations

. Tully - Fisher

self gravitating disks :

$$V^2 = \frac{GM}{r} \qquad M \text{ disk dominated}$$

$$V^4 = \frac{G^2M^2}{r^2} \qquad \Sigma = \frac{M}{r^2} = \text{surface density enclosed by } r$$

$$V^4 = G^2 M \Sigma \qquad M = \Upsilon L$$

or observationally $\qquad I = \Upsilon \Sigma \qquad \Upsilon = \text{dynamical } M/L$

$\hookrightarrow$ surface brightness

$$V^4 = G^2 \Upsilon^2 L I$$

i.e. $L \propto V^4$ provided Υ & I constant.

It was once thought all galaxies had the same surface brightness (Freeman's Law).

This derivation predicts a shift in TF with surface brightness I. Such a shift is not observed. Consequently, there is a fine-tuning :

$$\Upsilon^2 I = \text{constant}$$

so $\quad I \sim \Upsilon^{-1/2}$

lower surface brightness galaxies must be progressively more DM dominated

TF, non-self-gravitating disk (DM everywhere dominant)

$$M_\Delta = \frac{4\pi}{3}\Delta \rho_{crit} R_\Delta^3 \qquad V_\Delta = \frac{GM_\Delta}{R_\Delta}$$ "virial" quantities

combining these with $\rho_{crit} = \frac{3H_0^2}{8\pi G}$ gives

$$M_\Delta = \left(\frac{\Delta}{2}\right)^{-1/2}\left(\frac{1}{GH_0}\right) V_\Delta^3$$

so $M_\Delta = C\, V_\Delta^3$

$$M_b = f_d f_b M_\Delta \qquad V_f = f_v V_\Delta$$

$$\frac{M_b}{f_d f_b} = C\left(\frac{V_f}{f_v}\right)^3$$

$f_b = 0.15$ universal baryon fraction $f_b = \frac{\Omega_b}{\Omega_m}$

$f_d = ?$ disk fraction < 1 only constraint

$f_v = f(v) \simeq$ unity $\longrightarrow$ 1-1.3 typically

Should increase w/ Σ & therefore with L

if f_d & f_v constant,

$M_b \sim V_f^3$ slope too shallow, $\underbrace{f_v(\Sigma): 3 \to 3.3 \pm 0.2}$

so infer $f_d \propto V_f$ (or f_v or both)

adiabatic compression

more fine-tuning, feedback

$\rightarrow$ observed baryon content of bound structures a strong function of mass: halo-by-halo missing baryon problem

Scaling relations

TF, Faber-Jackson : $L \sim \sigma^4$

Fundamental Plane

Virial Fundamental Plane: $M \sim \sigma^2 R \sim \Sigma R^2$

$\sigma^2 \sim \Sigma R$

observed FP "tilted" $R \sim \sigma^2 \Sigma^{-1}$

wrt virial version: $R \sim \sigma^{1.4} I^{-0.8}$

depends on who you ask

Baryon fractions:

universal cosmic baryon fraction $f_b = \frac{\Omega_b}{\Omega_m}$ - baryon density / - gravitating mass density

in individual galaxies, we have the "disk" baryon fraction:

$$f_d = \frac{M_b}{f_b M_{tot}} = \frac{M_* + M_g \text{ (observed)}}{f_b \text{ (total dynamical mass)}}$$

don't know what M_{tot} is; typically associate it with M_{200}

For rich clusters of galaxies (w/ $M_{tot} \approx 10^{15} M_\odot$)

it works out:

$$M_b \approx f_b M_{tot} \quad \text{so} \quad f_d \approx 1$$

If we look at smaller systems, f_d departs systematically from unity ($f_d < 1$) becoming ever smaller for smaller objects

$$f_d = \tanh\left(\frac{V_f}{900}\right)$$ works OK — maybe 700 km/s

Example feedback scheme (Dutton 2009 MNRAS, 396, 121)

Supernova drives outflows, which are assumed to move at the local escape velocity to maximize mass removal. ($< V_{esc}$ doesn't escape
$> V_{esc}$ moves less mass for same energy)

Energy driven wind model:

$$\Delta M_{eject}(R) = \frac{2\epsilon_{EFB}\,\eta_{SN}\,E_{SN}\,\Delta M_{*}(R)}{V_{esc}^2(R)}$$

ejected mass from radius R

mass of stars formed at radius R

$E_{SN} \simeq 10^{51}$ erg $= 5\times10^7\ \mathrm{km^2 s^{-2}} M_\odot$

$\eta_{SN} = 8.3\times10^{-3}$ # SN per $M_\odot$ of stars formed (this # for a Chabrier IMF)

ϵ_{EFB} = fraction of kinetic energy injected into wind

Usually a large # in simulations (0.25 - 1)
usually a small # observed (0.02 - 0.1)

Momentum driven wind model

$$\Delta M_{eject}(R) = \frac{\epsilon_{MFB}\,p_{SN}\,\eta_{SN}\,\Delta M_{*}(R)}{V_{esc}(R)}$$

$p_{SN} = 3\times10^4\ M_\odot\ \mathrm{km\,s^{-1}}$ is momentum produced by one SN

ϵ_{MFB} is again the coupling efficiency to the ISM

This formulation maximizes the impact of SN.