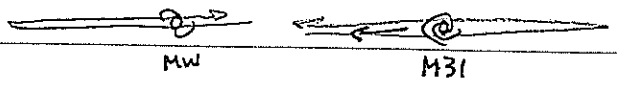
Start with slides covering the missing satellite problem and too big to fail

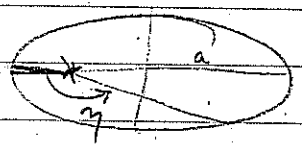
Timing argument in Local Group



M31 is approaching us at ~ 120 km/s Not in Hubble expansion!
So - what orbit is it on?
We know the universe is ~ 13 Gyr old,
So the MW-M31 pair has had time to expand then turn around & start to re-collapse, but haven't collided yet. Kahn & Woltjer (1959)

Position on radial orbit: $r = a(1 - e\cos\eta)$

$$t = \sqrt{\frac{a^3}{G(M+m)}}\,(\eta - e\sin\eta)$$ time since pericenter



e = eccentricity of orbit
a = semi-major axis
η = development parameter
$0 \le \eta \le 2\pi$

can differentiate the above expressions to obtain

$$\frac{dr}{dt} = \frac{dr/d\eta}{dt/d\eta} = \sqrt{\frac{G(m+M)}{a}}\,\frac{e\sin\eta}{1-e\cos\eta} = \frac{r}{t_0}\,\frac{e\sin\eta\,(\eta - e\sin\eta)}{(1-e\cos\eta)^2}$$

($e\sin\eta$)

Assume $r=0$ initially at $t=0$; t_0 = age of universe.
Use measured distance (r) and velocity (dr/dt) to estimate mass.
gives $m+M \approx 5\times10^{12}\,M_\odot$

Anisotropy parameter

In general, orbits of stars in pressure supported systems need not be isotropic, i.e. ($\sigma_{l.o.s.} \neq \sigma_{maj}$)

σ is different in different directions

σ_r, $\sigma_t = \sigma_\theta = \sigma_\phi$ radial & tangential

In general

$$M(r) = \frac{r\sigma_r^2}{G}\left(\gamma_* + \gamma_\sigma - 2\beta\right)$$

where $\gamma_* = -\frac{d\ln n_*}{d\ln r}$ logarithmic slope of stellar density profile (measurable)

$\gamma_\sigma = -\frac{d\ln \sigma_r^2}{d\ln r}$ logarithmic slope of $\sigma_r^2(r)$ [radial, not l.o.s]

$\beta = 1 - \frac{\sigma_t^2}{\sigma_r^2}$ anisotropy parameter

σ_t = " " " tangential (θ, ϕ) direction

σ_r = velocity dispersion in radial direction within body

extreme cases

circular orbits	$\sigma_r = 0$	$\beta = -\infty$
isotropy	$\sigma_r = \sigma_t$	$\beta = 0$ — implicitly assumed in most virial estimates; $\sigma_{los} = \sigma_r = \sigma_t$
radial	$\sigma_t = 0$	$\beta = 1$

more generally, $\beta < 0$ "tangential" bias

$\beta > 0$ "radial" bias

β can vary with r. This is the biggest systematic uncertainty in mass modeling elliptical galaxies