Cold dark matter

CDM leading candidate (popularity-wise)

Need entirely new particle outside the
Standard Model of Particle Physics

Motivations
- $\Omega_m > \Omega_b$ something with mass that ain't baryons
- LSS growth factor 10^5 since $z \approx 1000$ (CMB)
  baryons only give 10^2 or so

CDM candidates

WIMP Weakly Interacting Massive Particle

presumed to be the lightest stable super symmetric partner particle
i.e., the neutralino in minimal supersymmetry (MSSM)

$m_\chi \sim 100$ GeV ($\approx$ a heavy nucleus, like Xenon)
$\sigma_\chi \sim$ weak nuclear scale

"WIMP miracle" – cross-section about right to leave the right relic abundance (Ω_m) to be the dark matter
IF $\sigma_\chi \sim$ weak force scale

other possibilities

WIMPZILLAS $m_\chi > $ TeV
Axions known to exist, but not necessarily in enough numbers
Light Dark Matter ($m_\chi \lesssim 10$ GeV)

Etc. (e.g. Q-balls)

Other Ideas for Dark Matter

- Warm Dark Matter

WDM $m \sim 1$ – a few keV

speed "just right" — not relativistic like ν
but not zero like CDM

most commonly invoked to suppress structure formation on "small" scales (like ν) without killing it on large scales

MIGHT help with small scale problems :
- missing satellites
- cusp/core
- too big to fail

- Self-Interacting Dark Matter

SIDM doesn't interact with baryons (like CDM) but has enhanced cross-section for self-interaction

Requires new force of nature that is only active in the dark sector.

"dark photons" transmit the interaction between SIDM particles

- Other — can always make up other DM candidates

Modified gravity — change force law
instead of invoking unseen mass

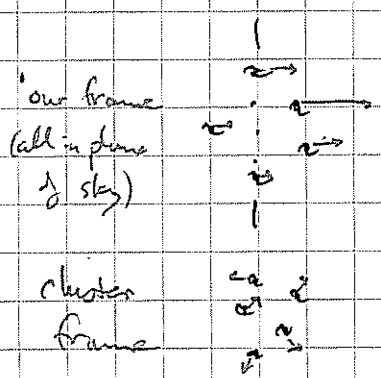
Early indications of dark matter

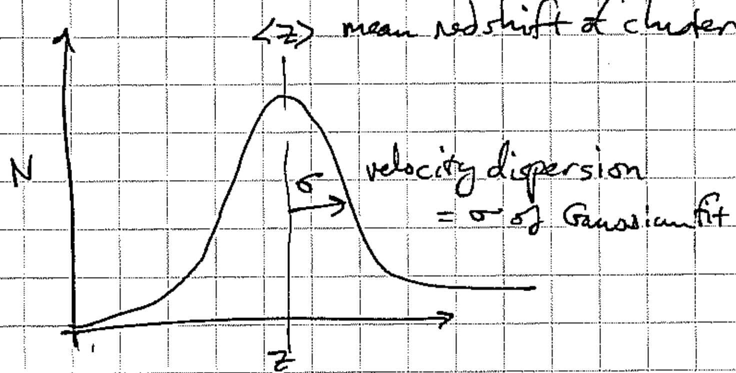
Oort (1932) discrepancy : z-motions of local stars

Zwicky (1930's) : velocity dispersions of clusters

Peebles & Ostriker (1973) : stability of galactic disks against the bar instability

Velocity dispersions of Clusters (Zwicky)





In general, σ need not be a single number as the real distribution need not be Gaussian, so one has a velocity dispersion tensor

$$\sigma_{ij}^2 = \overline{v_i v_j} - \overline{v_i}\,\overline{v_j}$$

In practice we only have line-of-sight velocities for cluster galaxies, so assume this 1D measured quantity is representative of the 3D space motion by <u>assuming</u> isotropy and <u>virial equilibrium</u>.

Virial Theorem

Following Bothun §. 4.1.1

Moment of Inertia of system of N particles

$$I = \sum_{i=1}^{N} m_i r_i^2 = \sum_{i=1}^{N} m_i (x_i^2 + y_i^2 + z_i^2)$$

time derivative of moment of inertia

$$\frac{dI}{dt} = \dot{I} = \sum_{i=1}^{N} m_i (2x_i\dot{x}_i + 2y_i\dot{y}_i + 2z_i\dot{z}_i)$$

$$\frac{1}{2}\frac{d^2I}{dt^2} = \frac{1}{2}\ddot{I} = \underbrace{\sum_{i=1}^{N} m_i(\dot{x}^2 + \dot{y}^2 + \dot{z}^2)}_{mv^2 = \text{kinetic energy}, 2T} + \underbrace{\sum_{i=1}^{N} m_i(x_i\ddot{x}_i + y_i\ddot{y}_i + z_i\ddot{z}_i)}_{\vec{r}\cdot(m\vec{a}) \;\; \text{potential energy}, W}$$

in general $\boxed{\frac{1}{2}\ddot{I} = 2K + W}$

After a few dynamical times, a self-gravitating system will obtain VIRIAL EQUILIBRIUM (become "virialized") in which the time-averaged moment of inertia is steady:

$\langle \dot{I} \rangle \to$ constant so $\langle \ddot{I} \rangle \to 0$ [Note - this is violated during mergers, reinstated afterwards.] time-varying gravitational potential can rearrange orbits

$$\boxed{\frac{1}{2}\langle \ddot{I} \rangle = 0 = 2\langle K \rangle + \langle W \rangle}$$ VIRIAL THM

For self gravitating system of particles,

$$W = -\frac{G}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} \frac{m_i m_j}{r_{ij}}$$

IFF all m_i the same (or nearly so), then can replace sums →

$$\sum_i^N \sum_j^N m_i m_j = N(N-1) m_i \rightarrow M^2 \quad \text{for large } N$$

(where M is the total mass)

so $$W = -\frac{GM^2}{2R_{hms}}$$ where R_{hms} is the harmonic mean separation

$$\frac{1}{R_{hms}} \equiv \frac{1}{N} \sum_{i=1}^{N} \frac{1}{r_i}$$

OK, so now use this: $2\langle T\rangle + \langle W\rangle = 0 \rightarrow \langle T\rangle = -\frac{1}{2}\langle W\rangle$

$K > 0$
$W < 0$

$$K = \frac{1}{2} N v_i^2 m_i$$
$$\langle K\rangle = \frac{1}{2} M \langle v_i\rangle^2 \qquad \sigma_v = \langle v_i^2\rangle^{1/2}$$

σ_v rms velocity dispersion

$$2\langle K\rangle + \langle W\rangle = 0 = 2 \cdot \frac{1}{2} M \sigma_v^2 - \frac{1}{2} \frac{GM^2}{R_{hms}}$$

W negative!

$$M \sigma_v^2 = \frac{1}{2} \frac{GM^2}{R_{hms}}$$

$$\boxed{M = \frac{2\sigma_v^2 R_{hms}}{G}}$$ VIRIAL MASS ESTIMATOR

need to measure σ_v, R_{hms} to get mass

In practice, $R_{hms} \approx 1.25\, R_{eff}$ is a good approximation

R_{eff} = half light radius → hence common assumption "light traces mass"

i.e., $R_{eff}(\text{light}) = R_{eff}(\text{mass})$

This is a more serious issue than $R_{hms} \approx 1.25 R_{eff}$ approximation

Applied to clusters to get M
→ M/L → M/L × j → Ω_m

σ_v 3D velocity dispersion – take care in relating this to L.O.S measured σ_{LOS} – – $\sigma_{LOS} = \sigma_v$ ONLY IF system isotropic

$$\sigma_{LOS}^2 = \sum_j (\sigma_j / \sin\theta_j)^2$$

Example application of the Virial thm to a typical rich cluster of galaxies (similar to Coma)

$$M = \frac{2\sigma^2 R_{hms}}{G} \qquad R_{hms} \approx 1.25\,R_e \qquad R_e \approx 1\ \mathrm{Mpc}$$

$\sigma \approx 1000\ \mathrm{km\,s^{-1}}$

$G = 4.3\times10^{-6}\ \mathrm{kpc\ km^2 s^{-2} M_\odot^{-1}}$

$$M = \frac{2\,(\overset{\mathrm{km/s}}{10^3})^2\,(\overset{\mathrm{kpc}}{1250})}{4.3\times10^{-6}}$$

for Coma

$M \approx 6\times10^{14}\ M_\odot \qquad L_B \approx 8\times10^{12}\ L_\odot$

$\frac{M}{L} = 75\ \frac{M_\odot}{L_\odot} \qquad$ (≈ 200 MWs in B-band)

Things to worry about:

- clusters never "virialized" (in dynamical equilibrium) — always show substructure
- interlopers — non-cluster members might contaminate σ.
- X-ray gas: stars not representative of baryons,
  so $M/M_b \sim 7$ even though $M/L \sim 70$
  (i.e., gas outweighs stars by factor of ~ 10)
  ... actually (at least) TWO missing mass ~~problems~~ types in clusters,
  one of which turned out to be the X-ray gas