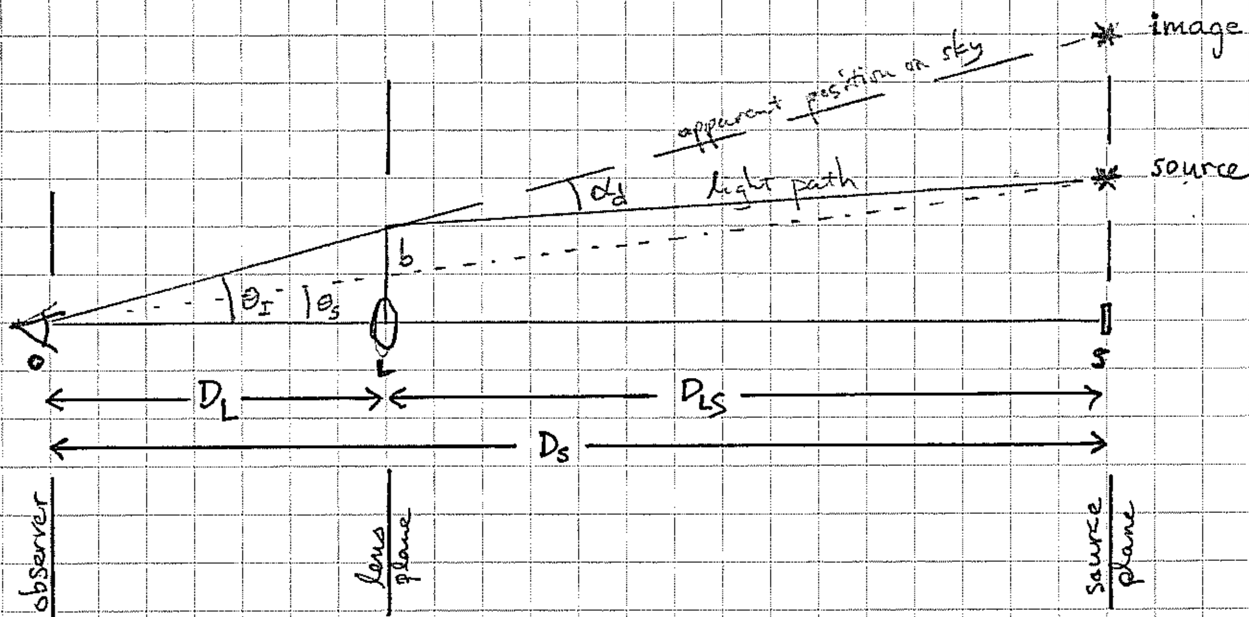
Gravitational Lensing

a relativistic phenomenon: GR required.

See Mo, vdB, & White § 6.6

Peacock ch. 4



famous factor of 2 in excess of Newton

Bend angle $\quad \alpha_d = \frac{2}{c^2}\int \underbrace{a_\perp \, dl}_{} = \frac{4GM}{c^2 b}$ for circular $M(<b)$

line integral of gravitational acceleration; impact parameter

Provides good constraint on mass enclosed by lens $M(<b)$

Thin lens approximation: sudden deflection

region over which light rays bent small compared to the distance to the source. Usually a good approx. in astro!

BUT: only holds if there is only a single lens along the l.o.s.!

$$\alpha_d = \frac{D_S}{D_{LS}}\left(\theta_I - \theta_S\right)$$

D_S, D_{LS} observed (redshift); θ_I observed; θ_S follows from geometry.

Generic flavors of gravitational lensing

Strong lensing - multiple images
Weak lensing - mild distortion of single image
microlensing - unresolved brightening from strong lensing

more specifically

weak lensing is a mild distortion that preserves a 1:1 relation between lens plane & image plane.

strong lensing generates multiple lenses with # depending on lens structure $\Sigma_L(r, \theta)$ and source location

strong & weak lensing separated by a critical surface density

$$\Sigma_{crit} = \frac{c^2}{4\pi G} \frac{D_S}{D_L D_{LS}}$$

Gravitational lensing conserves surface brightness. Results in a net magnification of source brightness as the lens bends light into our path that otherwise would have missed us.

above Σ_{crit}, get multiple images (strong lensing)

For cosmological source & lens at $z_s = 2$ & $z_L = 0.5$
$\Sigma_{crit} \approx 0.8 \text{ g cm}^{-2} \approx 3800 \ M_\odot \text{ pc}^{-2}$

surface densities in excess of $3800 \ M_\odot \text{ pc}^{-2}$ typically only occur near the centers of dense systems like giant elliptical galaxies.

Get multiple images, sometimes Einstein rings, useful for measuring H_0 through quasar variability (geometric path length different for each image; measure time delay, use speed of light to get distance)

BUT subject to mass sheet degeneracy (constant, uniform Σ causes offset in H_0)

Strong lensing always happens in bright places - no help with dark matter.

Mass inside giant arc:

$$M(<\theta_0) = \pi \Sigma_{crit} = \left(1.1\times10^{14} M_\odot\right)\left(\frac{\theta_0}{30''}\right)^2\left(\frac{D_L}{D_S}\right)\left(\frac{D_{LS}}{1 \text{ Gpc}}\right)$$

applied to clusters of galaxies showing arcs, gives
$M/L \approx 200 \ M_\odot/L_\odot$

Weak lensing: statistical detection of net arc-like features around lens

examples: Bullet cluster (far from cluster)
groups of galaxies
L^* galaxies – many individuals stacked

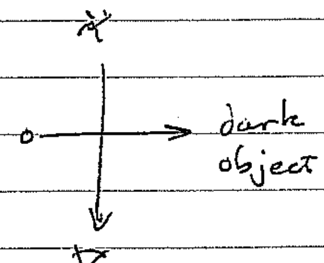
convergence: $K = \frac{\Sigma}{\Sigma_c}$ (< 1 for weak lensing - Enclosed κ)

gravitational shear:

$\gamma = \gamma_1 + i\gamma_2$ basically just residual shapes of galaxies

$$\nabla K = \begin{pmatrix} \gamma_{1,1} + \gamma_{2,2} \\ \gamma_{2,1} - \gamma_{1,2} \end{pmatrix}$$ get K from shapes

Microlensing



Magnification of background source caused by unresolved Einstein ring as lens moves across image

results in a net amplification A (can be large!)

# of expected microlensing events

$$N(>A) = \frac{2\tau}{(A^2-1) + A\sqrt{A^2-1}}$$

where the lensing optical depth is

$$\tau = \frac{4\pi G}{c^2} \int \frac{D_L D_{LS}}{D_S} \rho \, dl$$

for thin screen of lenses, $\tau \rightarrow \kappa = \Sigma/\Sigma_{crit}$

for MACHOs in the halo,

$$\tau = \frac{2\pi \sigma_v^2}{c^2} \frac{D_L D_{LS}}{r D_S}$$

to LMC $\tau = 5\times10^{-7}$ if all halo in MACHOs

$\tau < 1\times10^{-7}$ observed