MOND - Modified gravity (or inertia)

All we really know is that there is a discrepancy:
ordinary gravity (Newton & Einstein) cannot explain
the data with only the known baryons (what we can see).

Obvious hypothesis: rather than invoke dark matter,
try tweaking the force law

Many attempts to modify gravity have been made.
Tried and failed - things go south quickly
if you write down the wrong force law.

e.g., ALL length-scale based modifications can be
EXCLUDED.
The mass scale does not appear at a
characteristic length scale.

Generically: predict the wrong slope for Tully-Fisher

e.g., suppose force law becomes $\frac{1}{r}$ at $r > R_f$

$$\frac{F}{m} = a = \frac{GM}{r^2} \longrightarrow \frac{GM}{rR_f} \quad \text{for } r > R_f$$

$$\frac{V^2}{r} = \frac{GM}{rR_f}$$

$$V^2 = \left(\frac{G}{R_f}\right) M$$ hence:

Rotation curves flat ✓ $V \sim$ const
Tully-Fisher slope 2 ✗ $M \sim V^2$

MOND: alter force law at acceleration scale, a_0

empirically, $a_0 \approx 10^{-10}\ m\,s^{-2}$

limits:

g_N = normal Newtonian expr.

$$a \rightarrow g_N \quad a \gg a_0 \qquad g_N = \frac{GM}{r^2} \text{ for point mass}$$

$$a \rightarrow \sqrt{g_N a_0} \quad a \ll a_0$$

$$(a^2 = g_N a_0)$$

Transition made smoothly by interpolation fcn $\mu(x)$

$$x = \frac{a}{a_0} \qquad \mu \rightarrow 1 \text{ for } x \gg 1$$

$$\mu \rightarrow x \text{ for } x \ll 1$$

so

$$\mu(x)\, a = g_N$$

in practice $\quad \mu(x) = \dfrac{x}{1+x}$

or $\dfrac{x}{\sqrt{1+x^2}}$

or $1 - e^{-x}$

OR $\mu \leftrightarrow \nu^{-1}$

$$a = \nu(y)\, g_N$$

Can alternatively write $\quad a = \nu g_N \quad$ i.e. $V_c^2 = \nu(y) \cdot V_b^2$

with $\quad \nu(y) = \left[1 - e^{-\sqrt{y}}\right]^{-1}$

$y \rightarrow 1 \quad a \gg a_0$

$y \rightarrow y^{-1/2}$

MOND can be interpereted as either a modification of

GRAVITY $\quad F = \dfrac{GMm}{r^2} \quad$ (modification of Newton NOT GR)

OR

INERTIA $\quad F = ma \quad$ maybe at low accelerations

inertial $m \neq$ gravitational charge

In deep MOND limit

$$a = \sqrt{g_N a_0}$$

$$\frac{v^2}{r} = \sqrt{\frac{GM}{r^2} a_0}$$

$$V^4 = a_0 GM \longrightarrow \text{TF, No } \Sigma \text{ or } R_d \text{ residuals}$$

χ geometry term $\chi \approx 0.8$ for disks

For pressure supported systems

$$\sigma^4 = \frac{4 a_0 GM}{81}$$

geometry term for spheres

In EFE regime, $a_{in} < a_{ex} < a_0$

e.g. dSph   e.g., MW

$$\mu(x) V^2 = \frac{GM}{r} \quad \text{where} \quad x \approx g_{ex}/a_0$$

(really, vector sum of all $\vec{g}$)

$=1$ in Newton, otherwise the same. Looks like G is bigger

$$G_{eff} = \frac{G}{\mu} \approx \frac{G a_0}{g_{ex}}$$

Original form doesn't conserve momentum or energy

Fixed by modified Poisson eqn

$$\nabla \cdot \left[\mu\left(\frac{|\nabla \Phi|}{a_0}\right) \nabla \Phi \right] = 4\pi G \rho$$

(conformally invariant in 3D. Regular Poisson only in 2D)

In MOND regime, dynamics are invariant under transformations $(t, \vec{x}) \to \lambda(t, \vec{x})$