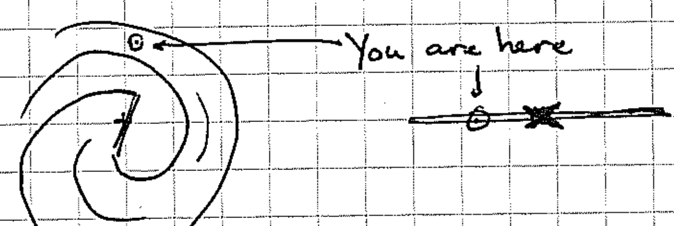
The Oort Discrepancy

The Milky Way, like most spiral galaxies,
is a thin, dynamically cold, rotating disk of stars & gas.

Thin: $R_d : z_0 = 8:1$ is typical, with considerable scatter
Dynamically Cold: $V/\sigma \gg 1$
For the Milky Way, $V \approx 200$ km s^{-1}
$\sigma \approx 20$ km s^{-1}

This is typical. Understanding the stability & persistence of thin spiral disks is a challenge, but the universe is littered with them.



edge-on

Face-on

To a decent first approximation, the orbit of a star in the Milky Way is approximately circular, ~~with~~ in the plane of the disk, while executing harmonic oscillation in the z-direction.

Oort (1932) considered the Z-motions of stars in the solar neighborhood, extending and applying earlier work by Kapteyn and Jeans.

Poisson equation

$$\nabla^2 \Phi = 4\pi G \rho$$

Φ = gravitational potential
ρ = 3D density ($M_\odot\, pc^{-3}$)

In cylindrical coordinates R, θ, z

$$\nabla^2 \Phi = \frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial \Phi}{\partial R}\right) + \underbrace{\frac{1}{R^2}\frac{\partial^2 \Phi}{\partial \theta^2}}_{\to 0} + \frac{\partial^2 \Phi}{\partial z^2}$$

Assume axis-symmetry, so $\frac{\partial \Phi}{\partial \theta} = 0$. Not really valid if spiral arms are strong

integrate once over z

$$\Sigma = \int_{-|z|}^{|z|} \rho \, dz$$

for distributions symmetric in z $[\rho(z) = \rho(-z)]$ this is equivalent to $\int_0^{|z|}$ times 2,
so $4\pi G\rho \to 2\pi G \Sigma |z|$

radial force $K_R = \frac{\partial \Phi}{\partial R} = \frac{V^2}{R}$

vertical force $K_z = \frac{\partial \Phi}{\partial z} = \int \frac{\partial^2 \Phi}{\partial z^2} dz$

so now we've eliminated one derivative to have

$$2\pi G \Sigma = |K_z| - |z| \frac{1}{R}\frac{\partial}{\partial R}(R K_R)$$

$R K_R = R \cdot \frac{V^2}{R} = V^2$

so

$$K_z = 2\pi G \Sigma + \frac{z}{R}\frac{\partial V^2}{\partial R}$$

restoring force to plane

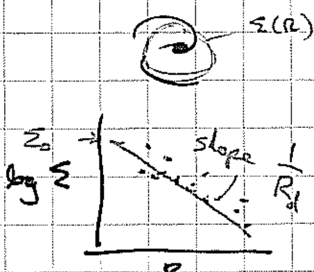
surface density of plane w/ $z < |z|$

radial force term amounts to square of rotation velocity

By convention, vertical force has been measured at $|z| = 1.1$ kpc, which contains most of the disk mass, but still has stars left to measure

Double exponential model

The azimuthally averaged light profile of spiral galaxies can tolerably be approximated by an "exponential disk"



$$\Sigma(R) = \Sigma_0 e^{-R/R_d}$$

Σ_0 central surface brightness
R_d disk scale length

This describes the 2D face-on distribution of star light.
The z-distribution can also be approximated as exponential, so
In 3D

$$\rho(R,z) = \rho_0 e^{-R/R_d} e^{-z/z_0}$$

We can use this to integrate the Poisson equation and replace K_z with the vertical velocity dispersion σ_z

The result is $\sigma_z^2 = K\pi G z_0 \Sigma$

where the constant K depends on the mass distribution.
Usually it is taken to be $K=2$ (~~uniform slab~~ isothermal)
or $K=1.5$ (pure exponential vertical dis)

In general, this applies to all components —
every population of stars (AV, KIII, etc)
as well as gas, and dark matter.
The constant K can be determined numerically for arbitrary $\Sigma(z)$, but for realistic cases $1.5 \leq K \leq 2$

Will return to this with the Jeans equations and the concept of the phase space. These give a different approach to individual tracer populations, as young stars are thinner & dynamically colder than older stellar populations

MEASUREMENTS

There have been many attempts to measure the local surface density of the disk since Oort:

e.g.,

Kuijken & Gilmore 1989

Holmberg & Flynn 2004

Bienaymé et al 2014 — to $z > 2$ kpc!

Bovy & Rix 2013 — K_{11} over large range in radii, not just locally!

numerically, the accounting goes (at the solar circle, $R = R_0$)

at R_0

$$\left.\begin{array}{l} \Sigma_* = 38\ M_\odot\,pc^{-2} \\ \Sigma_{gas} \simeq 14\ M_\odot\,pc^{-2} \end{array}\right\} \Sigma_b = 52\ M_\odot\,pc^{-2}$$

$$\Sigma_{dyn} \simeq 74\ M_\odot\,pc^{-2} > \Sigma_b$$

That $\Sigma_{dyn} > \Sigma_b = \Sigma_* + \Sigma_g$ is a mass discrepancy.

The dynamics wants more mass than is directly seen.

The difference — $22\ M_\odot\,pc^{-2}$ — is presumably the integrated portion of a quasi-spherical dark matter halo between $-1.1 \le z \le 1.1$ kpc.

For comparison, the local V-band surface brightness is $\sim 27\ L_\odot\,pc^{-2}$. So the mass-to-light ratio of the stars of the Milky Way is a reasonable

$\Upsilon_*^V = 1.4\ M_\odot/L_\odot$.