Disk Stability

Ostriker & Peebles (1973)

- very early, rather primitive numerical simulations
- showed that thin, cold, rotating disks are unstable to the growth of $m=2$ modes (the bar instability).
- this problem remains unresolved (arXiv: 1601.03406).

O & P solution was to embed spiral disks in a deep potential well – a dark matter halo.

Split kinetic energy into rotational T and pressure Π components

so

$$K = T + \tfrac{1}{2}\Pi \qquad (T \text{ all in plane of disk. } \Pi \text{ not})$$

$$K = \tfrac{1}{2}|W|$$ Virial equilibrium condition
kinetic energy = half the potential energy

O & P found stability was achieved when

$$t \lesssim 0.14 \qquad t \equiv \frac{T}{|W|}$$ the rotational component of the KE normalized to the P.E.

This is crude & almost certainly wrong in detail, but was important early evidence for Dark Matter

e.g.: the Milky Way has $V_c \approx 220 \text{ km s}^{-1}$ $\quad T \sim V_c^2$
$\sigma \approx 20 \text{ km s}^{-1}$ $\quad \Pi \sim \sigma^2$

working these numbers, $t \approx 0.45$ [0.5 is the maximum]

Such a cold disk should be wildly unstable, yet spiral galaxies persist over a Hubble time

Dark Matter halos provide stability (maybe) provided $\Pi \gg T$: they must be dynamically hot, quasi-spherical, and dominate the mass

Other stability criteria

Global stability against non-axis-symmetric perturbations

$$X_m = \frac{\kappa^2 R}{2\pi m G\Sigma}$$

Goldreich & Tremaine '78, '79
Toomre 1981

m is mode number of perturbation – $m = 2$ for a bar or grand design spiral

κ is the epicyclic frequency
Σ the surface density

The higher the surface density Σ, the less stable the disk

The precise stability criterion is determined numerically and depends on the potential

$X > 3$ required for stability for a flat rotation curve

$X > 1$ suffices for a rising rotation curve

Lowest modes m suppressed first (Athanassoula et al 1987)

So a very high surface density disk is unstable
a merely high " " " can have $m=2$ modes (bars, grand design spirals)
a low " " " will have $m=2$ suppressed.

It is tempting to conclude the grand design spirals are nearly maximal – high enough surface density to drive $m=2$ modes
Perhaps flocculent spirals are lower Σ so $m=2$ suppressed

Bars & spiral arms are the natural consequence of disk self-gravity. If the halo becomes too dominant, all modes get suppressed.

The amplitudes of modes are usually expressed as Fourier components

$$A_m = \frac{1}{N} \sum_{j=1}^{N} e^{i[m\theta_j + p \ln(r_j)]}$$

A is the amplitude of mode m for N stars

m = 2 gives a bar for $p = 0$
" grand design spiral for $p > 0$

p is the "pitch angle" of a "logarithmic spiral"

LOCAL disk stability criterion "Toomre Q" (Toomre 1964)

$$Q = \frac{\sigma_r \kappa}{3.36 G \Sigma}$$

σ_r = radial velocity dispersion
Σ = local disk surface density
κ = epicyclic frequency

3.36 Constant chosen so that stability occurs for $Q \gtrsim 1$

Numerically, $Q \approx 1.4$ in solar neighborhood
Instability $Q \lesssim 1$ sometimes thought to be a criterion for star formation.

EPICYCLIC FREQUENCY

$$\kappa^2 = \frac{\partial^2 \Phi}{\partial R^2} = R \frac{\partial \Omega^2}{\partial R} + 4\Omega^2$$

where $\Omega = \frac{V}{R}$ is the orbital frequency.

κ is the frequency with which a star oscillates (in the plane) about the "guiding center" of a closed (circular) orbit.

Mass components

Stars in galaxies

- Spiral & Irregular galaxies — Exponential disks

$$\Sigma(R) = \Sigma_0 e^{-R/R_d}$$

is a decent first approximation to the azimuthally averaged face-on surface brightness.

Can measure light profile, assume a mass-to-light ratio, and solve the Poisson equation to obtain the gravitational potential of the observed stars.

- Elliptical galaxies 3D ellipsoids seen in projection
can be oblate, prolate, or triaxial
$a=b>c$; $a>b=c$; $a>b>c$
(all can be a fcn of radius ; position angle can twist)

Classical "$r^{1/4}$" or de Vaucouleurs profile

$$\Sigma(r) = \Sigma_e \, e^{-7.67\left[\left(\frac{r}{R_e}\right)^{1/4} - 1\right]}$$

R_e = effective radius (contains $\frac{1}{2}$ light)

$\Sigma_e = \Sigma(R_e)$

Sersic profile: generalized hybrid

$$\Sigma(r) = \Sigma_e \, 10^{-b_n\left[\left(\frac{r}{R_e}\right)^{\frac{1}{n}} - 1\right]}$$

adds shape parameter n

$n = 1 \rightarrow$ exponential disk
$n = 4 \rightarrow$ de Vaucoulers profile

Galaxies with both bulge & disk can be fit as separate components (e.g. exponential disk + "classical" $r^{1/4}$ bulge) or as a single Sersic profile (with n managing the transition from bulge to disk).