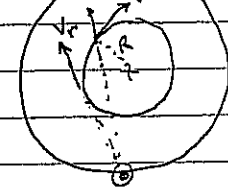
Oort constants A & B

local measures of Galactic kinematics

A is a measure of the local SHEAR (differential rotation)

$$A \equiv -\frac{1}{2}\left[R\frac{d\Omega}{dR}\right]_{R_0} = \frac{1}{2}\left(\frac{V}{R} - \frac{dV}{dR}\right)_{R_0}$$

This arises because the terminal velocity along the line of sight is related to position by



$$V_r = R_0 \sin l \left(\frac{V}{R} - \frac{V_0}{R_0}\right)$$

(line of sight radial velocity

For stars near to us ($d \ll R_0$),

$R - R_0$ is small. Doing a bit of trigonometry

$$V_r \approx d \sin(2l)\left[-\frac{R}{2}\frac{dV}{dR}\right]_{R_0} = A\, d \sin(2l)$$

Handy for nifty reason:

If you know A, you can get a kinematic estimate of d

Similarly, you can estimate A if you know d to a bunch of stars

Making a further trigonometric trick (expanding around $l \approx 90°$) we get:

$$V_r \approx 2AR_0(1 - \sin l)$$ only valid near $l \approx 90°$

so by measuring V_r in the direction we're headed, we get AR_0. This combination is well measured

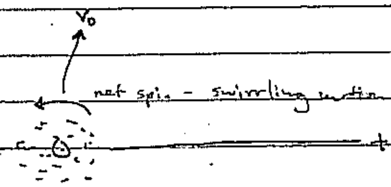
$$AR_0 \approx 115 \text{ km s}^{-1}$$

even if A & R_0 are separately uncertain

VORTICITY or CIRCULATION

Oort constant B

measure of vorticity



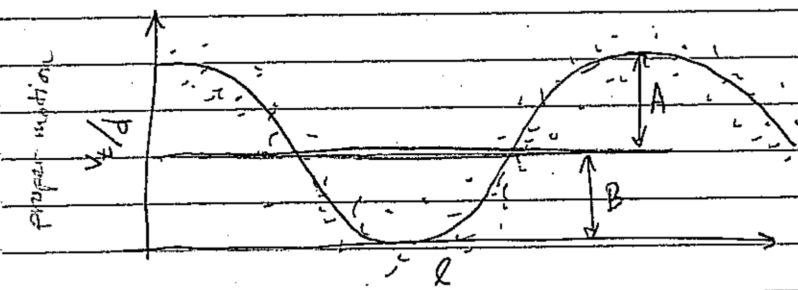
$$B = -\frac{1}{2}\left(\frac{V}{R} + \frac{dV}{dR}\right)_{R_0}$$

whereas A only depends on radial velocities v_r

need tangential velocities v_t (proper motions) for B

$$B = \frac{v_t}{d} - A\cos(2l)$$

basically the "excess" circulation motion not already attributable to shear



GALACTIC LONGITUDE

Note that these #s are not independent:

$\Omega_0 = \frac{V_0}{R_0} = A - B$ angular speed of LSR

$\left.\frac{dV}{dR}\right|_{R_0} = -(A+B)$ local slope of rotation curve

$A < |B|$ V(r) rising

$A > |B|$ V(r) declining

$A = |B|$ flat V(r)

BM ch. 10

Olling & Dehnen (2003) ApJ, 599, 275

In non-axisymmetric potentials (e.g. barred spirals like the Milky Way) there can be a net radial flow around some arbitrary point as well as net circulation

Chandrasekhar. 1942). At each position x in the Galaxy, there is a unique streaming velocity $\boldsymbol{v} = (U, V)$ (here we ignore the possibility of orbit crossing). The streaming velocity at some point x in the Galaxy with respect to an observer at the Sun may be expanded in a Taylor series (with local Cartesian coordinates: $\hat{e}_x$ and $\hat{e}_y$ pointing in directions $l = 0°$ and $l = 90°$),

$$\boldsymbol{v} = -\boldsymbol{v}_0 + \boldsymbol{H} \cdot \boldsymbol{x} + \mathcal{O}(x^2) \, , \tag{1}$$

where $\boldsymbol{v}_0$ is the velocity of the Sun with respect to the local streaming and

$$\boldsymbol{H} = \begin{pmatrix} \partial v_x/\partial x & \partial v_x/\partial y \\ \partial v_y/\partial x & \partial v_y/\partial \end{pmatrix}_{x=0} \equiv \begin{pmatrix} K+C & A-B \\ A+B & K-C \end{pmatrix} . \tag{2}$$

The parameters A, B, C, and K are the Oort constants; they measure the local divergence (K), vorticity (B), and azimuthal (A) and radial (C) shear of the velocity field generated by closed orbits. The Oort constants may also be expressed in terms of cylindrical coordinates (R, φ) with the Sun at (R_0, 0) (cf. Chandrasekhar 1942),

V_ϕ = speed in circular direction

V_R = speed in radial direction

$$2A = \frac{v_\varphi}{R} - \frac{\partial v_\varphi}{\partial R} - \frac{1}{R}\frac{\partial v_R}{\partial \varphi} \, , \tag{3a}$$

$$2B = -\frac{v_\varphi}{R} - \frac{\partial v_\varphi}{\partial R} + \frac{1}{R}\frac{\partial v_R}{\partial \varphi} \, , \tag{3b}$$

$$2C = -\frac{v_R}{R} + \frac{\partial v_R}{\partial R} - \frac{1}{R}\frac{\partial v_\varphi}{\partial \varphi} \, , \tag{3c}$$

$$2K = \frac{v_R}{R} + \frac{\partial v_R}{\partial R} + \frac{1}{R}\frac{\partial v_\varphi}{\partial \varphi} \, , \tag{3d}$$

evaluated at the solar position (we use the convention that φ increases clockwise, i.e., in the direction of Galactic rotation). In the axisymmetric limit, $C = K = 0$,[4] and

Now there can also be

C radial shear

D or K local divergence

$$A_{\rm sym} = \frac{1}{2}\left(\frac{v_\varphi}{R} - \frac{\partial v_\varphi}{\partial R}\right)_{R=R_0} , \tag{4a}$$

$$B_{\rm sym} = \frac{1}{2}\left(-\frac{v_\varphi}{R} - \frac{\partial v_\varphi}{\partial R}\right)_{R=R_0} , \tag{4b}$$

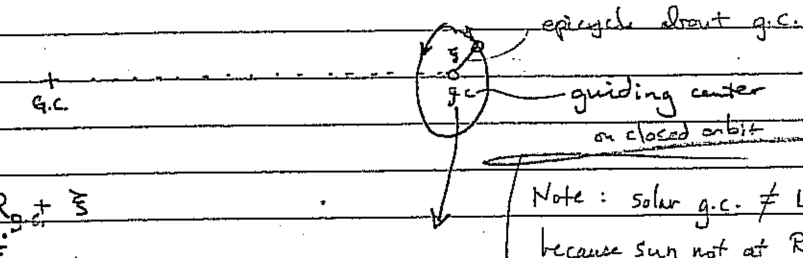
equivalent to the equations actually given by Oort, where R_0 is the Sun's distance to the Galactic center. In an axisymmetric galaxy, the circular (closed) orbits have velocity $v_\varphi^2 = R\,\partial\Phi/\partial R$, and measurements of the Oort constants provide a direct constraint on the Galactic potential Φ. For instance, a harmonic potential results in solid-body rotation, $A = 0$, and B equal to the (constant) rotation fre-

C & K probably a few km s^{-1} kpc^{-1}

Olling & Dehnen estimate $C \approx 10$ km s^{-1} kpc^{-1} — pretty big!
Kuijken & Tremaine (1994) had ~~estimated~~ $C \approx |K| < 1$ km s^{-1} kpc^{-1} — very small!

Epicycle approximation §3.3 of Sparke & Gallagher

Stars not on perfectly circular orbits

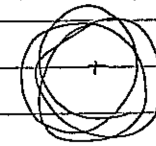


Note : solar g.c. $\neq$ LSR
because sun not at $R_{g.c.}$
except @ phase $\phi = \pm 90°$

$$R = R_{g.c.} + \xi$$
$$\ddot{R} = \ddot{\xi}$$

For axis-symmetric, cylindrical potential
$\Phi(R, \phi, z)$, Angular Momentum L_z
is conserved if $\dfrac{\partial \Phi}{\partial \phi} = 0$ not true with Bar!

orbits in Galactic potential
NOT CLOSED



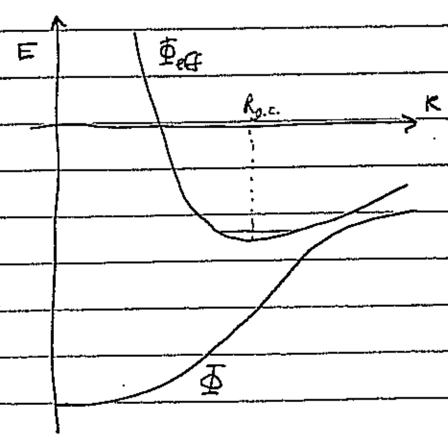
$$\ddot{\xi} = \ddot{R} = R\dot{\phi}^2 \quad \frac{\partial \Phi}{\partial R} = -\frac{\partial \Phi_{eff}}{\partial R}$$

epicycles try to approximate this.

$$(\dot{\phi} \equiv \Omega)$$

$$\Phi_{eff} = \Phi(R, z) + \frac{L_z^2}{2R^2}$$ (eqn 3.65 Sparke & Gallagher)

within this approximation,
a star executes harmonic
motion about the guiding center
for $\xi \ll R_{g.c.}$ Harmonic in x, y, z
z has higher frequency



Despite the Bar, this is a pretty
good approximation in the solar neighborhood

$$\ddot{\xi} = -2\frac{V}{R}\left(\frac{V}{R} + \frac{dV}{dR}\right)_{R_0} \xi$$

$$\ddot{\xi} = K^2 \xi \qquad K^2 = -4B(A-B) = -4B\Omega$$

can divide
ξ into X & Y
components
to compute orbit

epicyclic frequency $K \approx 37$ km s^{-1} kpc^{-1}

NO CLOSED ORBITS:
only slightly higher
than $\Omega \approx 30$ km s^{-1} kpc^{-1}

how many epicycles per orbit

Epicyclic motion BT 3.2.3

$$\xi = R - R_{g.c.} \qquad \Phi_{eff}(R) = \Phi(R) + \frac{L_z^2}{2R^2}$$

expand Φ_{eff} as Taylor series for $\xi \ll R_{g.c.}$:

$$\Phi_{eff} = \Phi_{eff}(R_g) + \frac{1}{2}\left(\frac{\partial^2 \Phi_{eff}}{\partial R^2}\right)_{R_g} + \text{higher orders that we ignore}$$

Note: there would be a term $\frac{\partial^2 \Phi_{eff}}{\partial R \partial z}$ but we assume z symmetry so this disappears

now define the epicyclic frequency

$$\kappa^2 = \left.\frac{\partial^2 \Phi_{eff}}{\partial R^2}\right|_{R_g}$$

One can similarly define a vertical frequency

$$\nu^2 = \left.\frac{\partial^2 \Phi_{eff}}{\partial z^2}\right|_{R_g}$$

So radial & vertical oscillations are harmonic

$$\ddot{\xi} = -\kappa^2 \xi \qquad \ddot{z} = -\nu^2 z$$

$$\kappa^2 = \left.\frac{\partial^2 \Phi}{\partial R^2}\right|_{R_g} + \frac{3L_z^2}{R_g^4} = \left(R\frac{d\Omega^2}{dR} + 4\Omega^2\right)_{R_g}$$

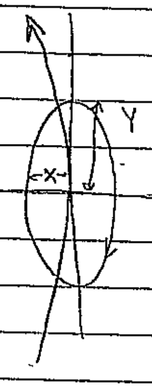
see BT p. 165

or

$$\kappa^2 = -2\frac{V}{R}\left(\frac{V}{R} + \frac{dV}{dR}\right)_{R_g} \xi$$

FOR POP I STARS,

dimensions of epicycle larger in direction we orbit



$Y > X$

$$Y = \frac{2\Omega}{\kappa} X$$

also have velocity ellipsoid

$$\frac{\sigma_y^2}{\sigma_x^2} = \frac{\kappa^2}{4\Omega^2} = \frac{-B}{A-B}$$

In general $\sigma_x > \sigma_y > \sigma_z$ $\sqrt{\Sigma \sigma^2} \approx 30 \text{ km s}^{-1}$ in disk (Pop I).

For a flat rotation curve, $\sigma_x^2 = 2\sigma_y^2$ $V/\sigma \approx 8 \gg 1$

Q: If $\sigma_x^2 > 2\sigma_y^2$, what does this mean about the local shape of $V(r)$?

[measured value: $\sigma_x^2 = 2.2\,\sigma_y^2$]

Linblad resonances

for some pattern (like a bar)
have "pattern speed" Ω_b

corotation when $\Omega = \Omega_b$

inner LR $\Omega_b - \Omega = -\kappa/2$

outer LR $\Omega_b - \Omega = +\kappa/2$

or κ/m for higher modes