

Einstein halo

A better fitting function for simulated halos than NFW
at the expense of an extra parameter

Merritt et al (2006):

$$\rho(r) = \rho_e e^{-d_n \left[\left(\frac{r}{r_e} \right)^{1/n} - 1 \right]}$$

If that looks familiar, it is because the
Einstein profile is the 3D equivalent of
the Sersic profile used for fitting projected
surface densities

ρ_e = density at radius r_e
that contains $\frac{1}{2}$ of the total mass

d_n is a hassle to obtain, but is well approximated by

$$d_n \approx 3n - \frac{1}{3} + \frac{0.0079}{n} \quad \text{for } n > \frac{1}{2}$$

Simulated halos have $4.6 < n < 8.2$

In the notation of Navarro et al. (2004), $\alpha = \frac{1}{n}$
who found the mid-point $\alpha \approx 0.17$

The variable n is sometimes invoked to explain cores
in real galaxies, but this is not correct.

Both the change in slope and its location
(at very small radii) fail to explain observations.
It is too small an effect - really just a tweak
to NFW

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44

Einstein halos do have the nice property of a finite total mass

$$M(r) = 4\pi n r^3 \rho_e e^{dn} d_n^{-3n} \gamma(3n, x)$$

where γ is the incomplete gamma fn

integrating $M(r)$ to ∞ turns γ into Γ

$$M_{\text{tot}} = 4\pi n r_e^3 \rho_e e^{dn} d_n^{-3n} \Gamma(3n)$$

$$x = d_n \left(\frac{r}{r_e}\right)^{1/n}$$

Empirical DM Halo

McGaugh et al (2007)

Walker et al (2009)

Just fit the portion of the data attributable to dark matter. Adjust M_x/L & choose that which minimizes scatter - in both BTFR and $V_{\text{DM}}(R)$, as it happens. Get

$$\log_{10} V(R) = 1.47^{+0.15}_{-0.19} + \frac{1}{2} \log R$$

V in km s^{-1}
 R in kpc

slope $\frac{1}{2}$ fixed to NFW value (best fit 0.49)

this can also be expressed in terms of the enclosed mass

$$M_{\text{DM}}(R) = 200^{+200}_{-120} \left(\frac{R}{\text{pc}}\right)^2$$

M in M_{\odot}
 R in pc this time

note: this is not the same as M_{total} , which increases linearly

or

$$g_{\text{DM}} = 3^{+3}_{-2} \times 10^{-11} \text{ m s}^{-2}$$

12
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48
60
72
84
96
108
120
132
144