Final Review Spring 2016

Orbital anisotropy

general mass estimator:

$$M(r) = \frac{r\,\sigma_r^2}{G}\left(\gamma_* + \gamma_\sigma - 2\beta\right)$$

$$\gamma_* = -\frac{d \ln n_*}{d \ln r}$$ logarithmic slope of tracer density profile $n_*(r)$

$$\gamma_\sigma = -\frac{d \ln \sigma_r^2}{d \ln r}$$ logarithmic slope of [3D, not line-of-sight] radial velocity dispersion profile $\sigma^2(r)$

$$\beta = 1 - \frac{\sigma_t^2}{\sigma_r^2}$$ anisotropy parameter ratio of tangential vs radial motion

Measures kinetic energy in rotation vs. that in pressure support.

β can be a fcn of radius

Isotropy ($\beta = 0$) implicitly assumed in most virial analyses

Luminosity function

Schechter fcn: $$\Phi(L) = \Phi^* e^{-L/L^*}\left(\frac{L}{L^*}\right)^{\alpha}$$

w/ characteristic density Φ^*, luminosity L^*, & faint end slope α

Integrated luminosity density $j = \Phi^* L^* \Gamma(\alpha+2)$

ΛCDM predicted mass fcn has steep slope $\alpha \approx -1.9$
Observed luminosity " " shallow " $\alpha \approx -1.2$

This is the origin of the mis-named missing satellite problem — lots of low mass halos are predicted (not just as satellites) while relatively few low mass galaxies are observed

Gravitational Lensing

Flavors:
- Strong lensing – multiple lensed images of single source
- Weak lensing – mild distortion of source
- Microlensing – temporary brightening due to unresolved strong lensing of source by object that passes in front of it

Threshhold for strong lensing:

$$\Sigma > \Sigma_{crit} \simeq \frac{c^2}{4\pi G} \frac{D_S}{D_L D_{LS}}$$

Bend angle

$$\alpha_d = \frac{4GM}{bc^2} = \frac{D_S}{D_{LS}} (\theta_I - \theta_S)$$

← true angle between

← apparent angle between source & lens

Mass estimator ~~for weak lensing~~

e.g., within an arc in a cluster of galaxies

$$M(<\theta_0) = (1.1 \times 10^{14} M_\odot)\left(\frac{\theta_0}{30''}\right)^2 \left(\frac{D_L}{D_S}\right)\left(\frac{D_{LS}}{1\,\mathrm{Gpc}}\right)$$

Microlensing optical depth

$$\tau = 2\pi \frac{\sigma_v^2}{c^2} \frac{D_L D_{LS}}{r D_S}$$

Not enough lensing events are observed towards the LMC to account for the dark matter. On the flip side, the lensing optical depth towards the bulge is rather high. Lensing objects are just ordinary stars if the disk is maximal.

Cosmology

Friedmann eqn

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \frac{1}{3}\Lambda c^2$$

$H = \frac{\dot{a}}{a}$ — Hubble expansion; mass density; geometry; cosmological constant (aka dark energy)

Critical density

$$\rho_{crit} = \frac{3H^2}{8\pi G} \qquad \Omega_m = \frac{\rho_m}{\rho_{crit}}$$

$\Omega_m > 1$ eventually halts expansion & recollapses
$\Omega_m \leq 1$ expands forever
$\Lambda > 0$ eventually causes expansion to accelerate

As $t \to 0$ $(z \to \infty)$ all solutions tend to $\Omega_m \to 1$

Cosmology needs non-baryonic dark matter because

1. $\Omega_m > \Omega_b$ from Big Bang Nucleosynthesis
2. Large Scale Structure formation
   - need growth factor of $\sim 10^5$ from CMB to now
     gravity + baryons will only do ~ 100

In addition to the cosmological missing mass, we also have 2 distinct missing baryon problem:

1. Sum of known baryons $< \Omega_b$

2. Halo-by-halo missing baryon problem

$$\frac{M_b}{M_{tot}} < f_{b,\,cosmic}$$ in bound objects like galaxies and groups of galaxies

WIMPs Weakly Interacting Massive Particle

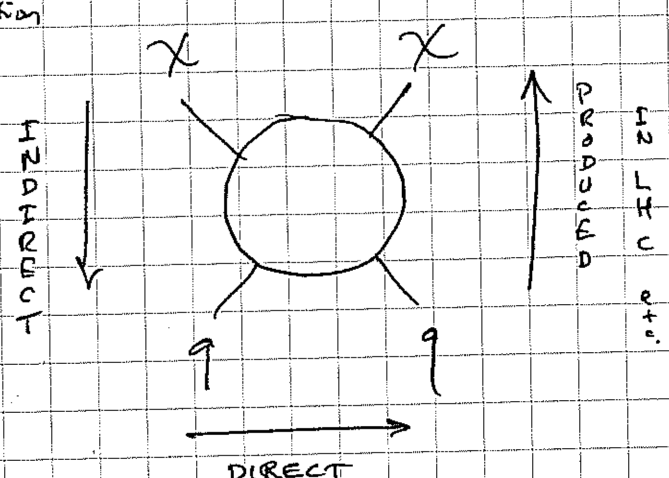
Leading candidate for the non-baryonic dark matter

Usually presumed to be the lightest stable particle in the hypothetical super symmetric sector (SUSY) (e.g., the neutralino)

In SUSY, every normal (standard model) particle has a SUSY partner particle. Minimal SUSY, with the fewest new parameters, called MSSM. (Minimal Super-Symmetric Model)

The WIMP miracle: the weak force interaction scale ($m_\chi \approx 100$ GeV) is about right to leave a relic density that's about right to be the dark matter ($\Omega_\chi \sim 0.1 \pm$ a few dex)

WIMP detection



Direct detection: WIMPs scatter off nuclei in underground labs

Indirect detection: WIMPs are their own anti-particle; can annihilate into standard model particles, creating a source of cosmic rays & γ-rays.

WIMP production: Particle colliders that achieve energies $> M_\chi c^2$ could create WIMPs which might be recognized as a deficit in detected debris mass-energy.

As always, remember the exponential disk:

$$\Sigma = \Sigma_0 e^{-r/R_d}$$

with enclosed mass

$$M(<x) = 2\pi \Sigma_0 R_d^2 \left[1-(1+x)e^{-x}\right] \quad \text{where} \quad x = \frac{r}{R_d}$$

One can have a double-exponential in radius an vertical:

$$\rho = \rho_0 e^{-r/R_d} e^{-z/z_0}$$

which has a vertical velocity dispersion of approximately

$$\sigma_z^2(r) = 2\pi G \Sigma(r) z_0$$

and the virial theorem

$$2\langle K\rangle + \langle W\rangle = 0$$

with mass estimator $\quad M = \frac{2\sigma^2}{G} R_{hms}$

with the harmonic radius usually approximated as

$$R_{hms} = 1.25 R_{eff}$$