Halo models

A brief guide to common dark matter halo models

- Pseudo-isothermal $\rho \sim \text{const} \quad r \to 0$
  $\rho \sim r^{-2} \quad r \to \infty$

  Works well for fitting rotation curves: empirically motivated

- NFW (Navarro - Frenk - White 1997)

  emerges from computer simulations of structure formation $\rho \sim r^{-1} \quad r \to 0$
  $\rho \sim r^{-3} \quad r \to \infty$

  for self-gravitating but otherwise non-interacting dark matter particles in an expanding universe.

  Provides a poor description of real rotation curves

- Burkert

  The Burkert profile is an attempt to reconcile the best features of p-ISO & NFW halos:

  $$\rho \sim \text{const} \quad r \to 0$$
  $$\rho \sim r^{-3} \quad r \to \infty$$

- Einasto profile

  The Einasto profile adds a 3rd parameter to better fit numerical simulation results. Observationally it is indistinguishable from the NFW halo.

Halo models

Many models have been proposed for dark matter halos

A brief guide to some of the more common that one may run across in the literature

- the Pseudo-Isothermal halo

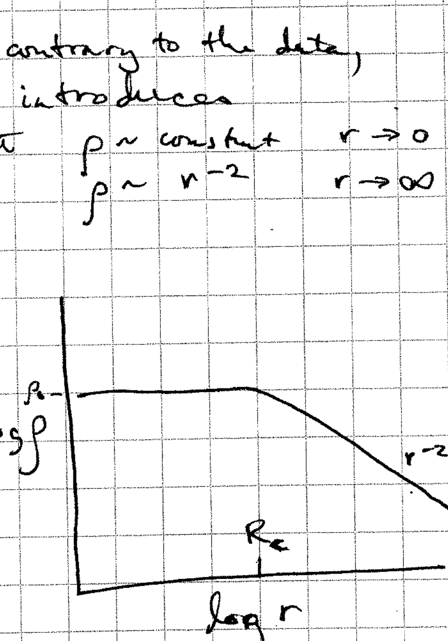
WORKS WELL FOR FITTING OBSERVED ROTATION CURVES

This was the form most commonly assumed after the discovery of flat rotation curves. In order to obtain $V(R) \sim$ constant requires $\rho \sim r^{-2}$. This occurs in an isothermal sphere in which the velocity dispersion of stars can be described in analogy to a perfect gas with $\sigma^2 = \frac{kT}{m}$ (see BT 4.3).
This behavior persists to $r = 0$, contrary to the data, hence the "pseudo" part, which introduces a constant density core so that

$$\rho \sim \text{constant} \quad r \to 0$$
$$\rho \sim r^{-2} \quad r \to \infty$$

$$\rho(r) = \frac{\rho_0}{1 + (r/R_c)}$$

where ρ_0 = core density
R_c = core radius



The rotation curve is

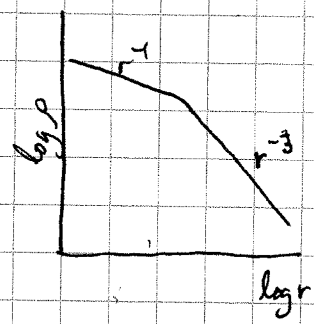
$$V_{p\text{-}iso}(r) = V_\infty \sqrt{1 - \left(\frac{R_c}{r}\right)\tan^{-1}\left(\frac{r}{R_c}\right)}$$

with $\quad V_\infty = \sqrt{4\pi G \rho_0 R_c^2}$

with the obvious temptation to associate $V_f = V_\infty$

— NFW halos

Following the formalism used by Jerry Sellwood in his notes on NFW ~~[illegible]~~ linked on the course web page under review literature,



$$\rho(r) = \frac{\rho_s r_s^3}{r(r+r_s)^2}$$

This leads to the rotation curve

$$V(R) = V_{200} \sqrt{\frac{\ln(1+cx) - \frac{cx}{1+cx}}{x\left[\ln(1+c) - \frac{c}{1+c}\right]}}$$

where $x = \dfrac{r}{R_{200}}$; $c = \dfrac{R_{200}}{r_s}$ ("concentration")

R_{200} is the radius that encloses a density $200\times$ the critical density of the universe.

$$\rho_{crit} = \frac{3H_0^2}{8\pi G} \qquad \Delta \equiv \frac{\rho}{\rho_{crit}}$$

$\Delta \approx 200$ is roughly the "virial" overdensity, i.e., material within this overdensity has had time to settle.
[Strictly speaking $\Delta = 186$ for $\Omega_m = 1$. The virial overdensity is more like $\Delta \approx 100$ in ΛCDM, but we persist in referencing everything to $\Delta = 200$.]

This is all notional.

Note: the NFW density profile diverges, but only logarithmically in mass.

$$V_{esc}^2 = 2|\Phi|$$

The potential is finite, so an escape velocity can be defined (unlike for the pseudo-isothermal halo)