Cosmological Framework

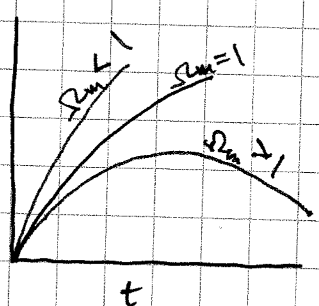
Dark matter halos are thought to form by gravitational collapse of over-dense regions in an otherwise expanding universe.

$a = \frac{1}{1+z}$

A little necessary context:

Friedmann eqn: $\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3}$,

scale size — mass density — curvature — cosmological constant



a is the scale size of the universe — a dimensionless quantity that encodes the physical separation between comoving coordinate tracers (e.g., galaxies w/ zero peculiar motion)

a(t) is the expansion history of the universe following from the solution of Friedmann's eqn.

Note that the Hubble parameter $H = \frac{\dot{a}}{a}$ is the expansion rate.

H must vary with time; it's current measured value is the misnamed Hubble "constant" $H_0 = \left(\frac{\dot{a}}{a}\right)_0$ measured now at $t = t_0$, the age of the U.

It is also convenient to define the density parameter $\Omega_m = \frac{\rho}{\rho_{crit}}$

which is the ratio of the actual mass density to the critical density

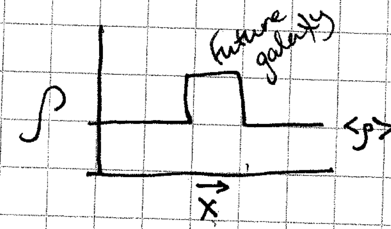
$\rho_{crit} = \frac{3H^2}{8\pi G}$ that defines the over/under between eternal expansion and eventual recollapse.

Note that ρ_{crit} evolves with H, as do ρ & Ω_m

Only if $\Omega_m = 1$ exactly does it remain 1 for eternity

The Friedmann eqn can be derived from the eqn. of motion of a point on the surface of a uniform expanding sphere, at least in the absence of Λ. It does not depend on scale. So a good first approximation to galaxy formation is to treat the volume that will collapse to form a galaxy as a locally overdense universe with $\Omega_m > 1$.

This is often called a top-hat overdensity



Note that $\vec{x}$ represents all 3 spatial dimensions. The over-dense region is spherical, so Friedmann's eqn applies.

Indeed, as $t \to 0$, $\Omega_m \to 1$, so the curvature and Λ terms may be ignored for the mean $\langle\rho\rangle$.

The solution for the uniform background universe (in which the top-hat is embedded) is

$$\frac{a}{a_0} = \left(\frac{3}{2} H_0 t\right)^{2/3}.$$

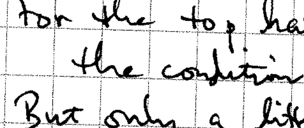
For the top hat, $\Omega_m > 1$, the condition necessary for it to collapse.

But only a little > 1: the initial condition is set by the fluctuations in the CMB when $t \approx 10^5$ yr and $\frac{\delta\rho}{\langle\rho\rangle} \approx 10^{-5}$ — galaxies are thought to arise from the gravitational growth and collapse of these initially tiny over-densities. (Under-densities become voids.)

For $\Omega_m > 1$ the solution is

$$\frac{a}{a_0} = \frac{1}{2} \frac{\Omega_{m,0}}{\Omega_{m,0} - 1} (1 - \cos\eta) \; ; \; H_0 t = \frac{1}{2} \frac{\Omega_{m,0}}{(\Omega_{m,0} - 1)^{3/2}} (\eta - \sin\eta)$$

η is the "development parameter" representing time to recollapse



η runs from 0 to 2π with maximum expansion at $\eta = \pi$

also used in timing argument

Monolithic collapse – hypothesis that the Galaxy formed from the collapse of one big top hat.
Historically important; mostly a straw-man first approximation now.

Consider the Milky Way: $M \approx 2\times10^{12}\ M_\odot$
In order to gather up that much mass, we need to collect from a volume V

$$M = \rho V \qquad \rho = \Omega_{m,0}\,\rho_{crit} \qquad \rho_{crit} = 1.5\times10^{11}\ \frac{M_\odot}{Mpc^3}$$

$$V = \frac{4\pi}{3} R^3_{MW,initial} \qquad \Omega_{m,0} \simeq 0.3$$

corresponding to a current physical radius

$$R_{MW,i} \simeq 2.2\ \text{Mpc}$$

Could also work out volume containing baryonic mass $\Omega_b \approx 0.04$
Gives $R_{MW} \approx 1.3$ Mpc
$(M_b/M_h < f_b)$

This is about a factor a 10 larger than the current radius of the MW encompassing this much mass (~ 200 kpc).

So the collapse factor is ~10 in radius
~1000 in volume.

Important events in galaxy formation

$t \approx 3\times10^5$ yr — Baryons decouple from CMB photons

First ~ ½ Gyr — Dark matter halos grow
Baryons fall into DM halos
A few stars form during collapse: stellar halo

$t \approx 1$ Gyr — Gas settles into rotating disk (dissipational collapse)
Stars start to form in gas disk

$t \approx$ a few Gyr — ⋮ Further mass accretion. Thick disk formation
⋮

$t \approx 9$ Gyr — Sun forms
⋮ other, lesser mass accretions

$t \approx 13.5$ Gyr — present

	12
	12
	24
	36
	48
5	60
6	72
7	84
8	96
9	108
10	120
21	132
132	144